APPLICATIONS OF PROOF THEORY TO CORE MATHEMATICS: RECENT DEVELOPMENTS

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In this talk we survey some recent developments in the project of applying proof-theoretic transformations to obtain new quantitative and qualitative information from given proofs in areas of core mathematics such as nonlinear analysis, convex optimization and geodesic geometry ([2, 3]). We will discuss some of the following items:

(1) The recent extraction of uniform rates of convergence for the ε-capture in the Lion-Man game in the context metric spaces X satisfying a suitable 'betweennes property' ([7]). Here a low complexity rate of convergence is extracted from a proof that made iterated use of sequential compactness arguments (i.e. arithmetical comprehension). The extraction also replaced the strong assumption of the compactness of X by its boundedness.

(2) In [1], we extracted the first explicit moduli of uniform continuity from noneffective continuity proofs for concepts of proximal maps in uniformly convex Banach spaces. It turned out, that in order to get a modulus which (as desired) is independent of the scalar involved one has to modify the previously suggested definition of such maps giving rise to a new definition.

(3) Most recently, in [8] together with P. Pinto, we analyzed proofs due to T. Suzuki which reduce viscosity generalizations of convergence proofs in optimization to the usual versions in terms of rates of convergence and metastability.

(4) In [6], it is shown how the assumption of metric regularity gives low complexity rates of convergence for algorithms which in general do not have computable rates of convergence. In the particular case of the uniqueness of the solution such an approach is used in [9] for a wide range of algorithms computing zeros of accretive set-valued operators as used for abstract Cauchy problems.

(5) An arithmetization of a highly noneffective convergence proof for the computation of so-called sunny nonexpansive retractions

has recently led to effective bounds in [10] and is used in [5] to obtain metastability of a strongly convergent Halpern-type form of the proximal point algorithm in Banach spaces.

(6) A polynomial rate of convergence in Bauschke's celebrated solution of the 'zero displacement conjecture' has been extracted from Bauschke's proof which heavily uses the machinery of maximal monotone operators ([4]). Very recently, this was much generalized to cover so-called averaged mappings by Sipoş [11].

References

[1] M. Bačák, U. Kohlenbach. On proximal mappings with Young functions in uniformly convex Banach spaces. *J. Convex Anal.*, Vol. 25:1291–1318, 2018.

[2] U. Kohlenbach. Applied Proof Theory: Proof Interpretations and their Use in Mathematics. *Springer Monographs in Mathematics*. xx+536 pp., Springer Heidelberg-Berlin, 2008.

[3] U. Kohlenbach. Proof-theoretic Methods in Nonlinear Analysis. In: *Proc. ICM 2018, B. Sirakov, P. Ney de Souza, M. Viana (eds.)*, Vol. 2, pp. 61–82. World Scientific, 2019.

[4] U. Kohlenbach. A polynomial rate of asymptotic regularity for compositions of projections in Hilbert space. *Foundations of Computational Mathematics*, Vol. 19:83–99, 2019.

[5] U. Kohlenbach. Quantitative analysis of a Halpern-type Proximal Point Algorithm for accretive operators in Banach spaces. *J. Nonlin. Convex Analysis*, 21:2125–2138 (2020).

[6] U. Kohlenbach, G. López-Acedo, A. Nicolae. Moduli of regularity and rates of convergence for Fejér monotone sequences. *Israel Journal of Mathematics*, 232:261–297, 2019.

[7] U. Kohlenbach, G. López-Acedo, A. Nicolae. A quantitative analysis of the "Lion-Man" game. *Pacific J. Mathematics*, 310:181–212 (2021).

[8] U. Kohlenbach, P. Pinto. Quantitative translations for viscosity approximation methods in hyperbolic spaces. Preprint, 2021.

[9] U. Kohlenbach, T. Powell. Rates of convergence for iterative solutions of equations involving set-valued accretive operators. *Computers & Mathematics with Applications*, 80:490–503 (2020).

[10] U. Kohlenbach, A. Sipoş. The finitary content of sunny nonexpansive retractions. *Communications in Contemporary Mathematics*, 23:1950093, 63 pp. (2021).

[11] A. Sipoş. Quantitative inconsistent feasibility for averaged mappings. Submitted, 2020. arXiv:2001.01513

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