MARINA: Faster Non-Convex Distributed Learning with Compression

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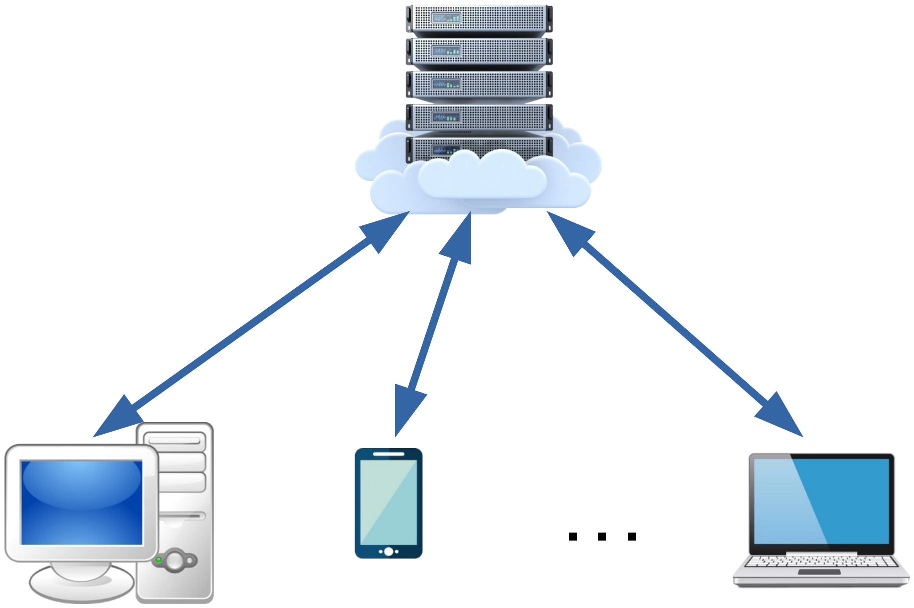
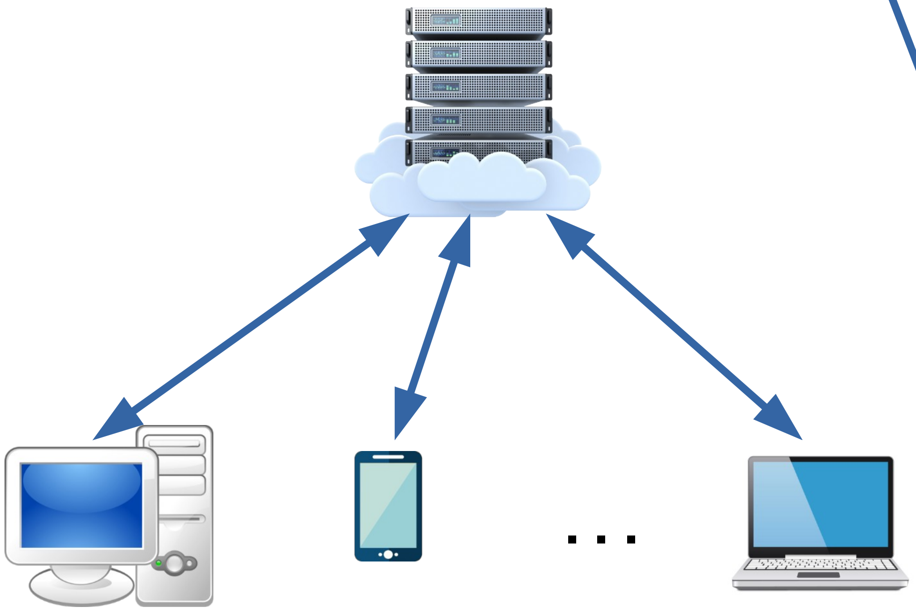
KAUST

Outline

1. The Problem



$$\min_{x\in\mathbb{R}^d} \left\{ f(x) := \frac{1}{n}\sum_{i=1}^{n} f_i(x) \right\}$$



parameters of the model

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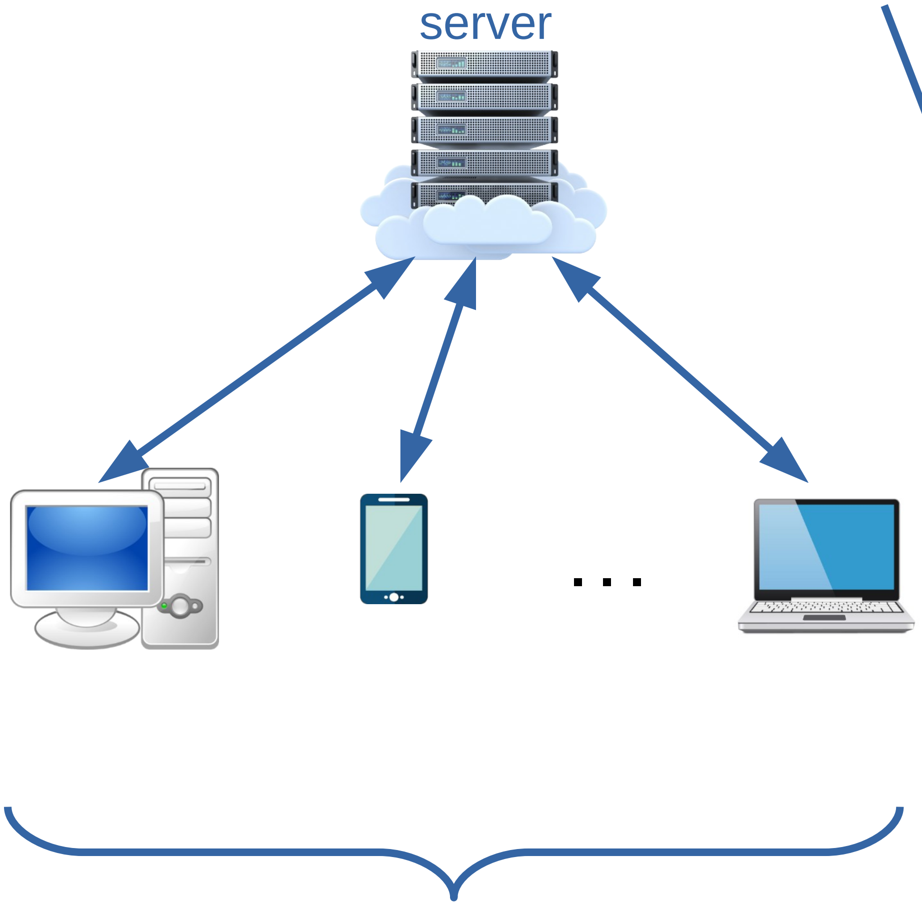
# of parameters



$\boxed{n}$ workers/devices

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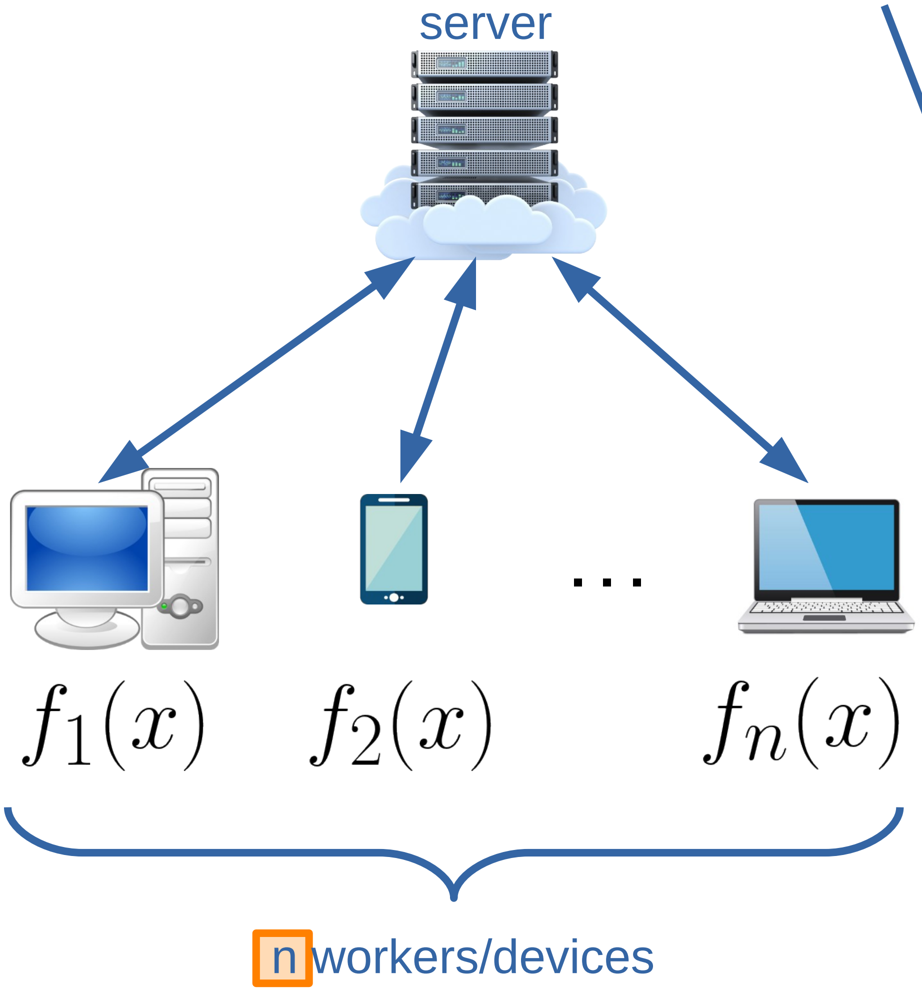

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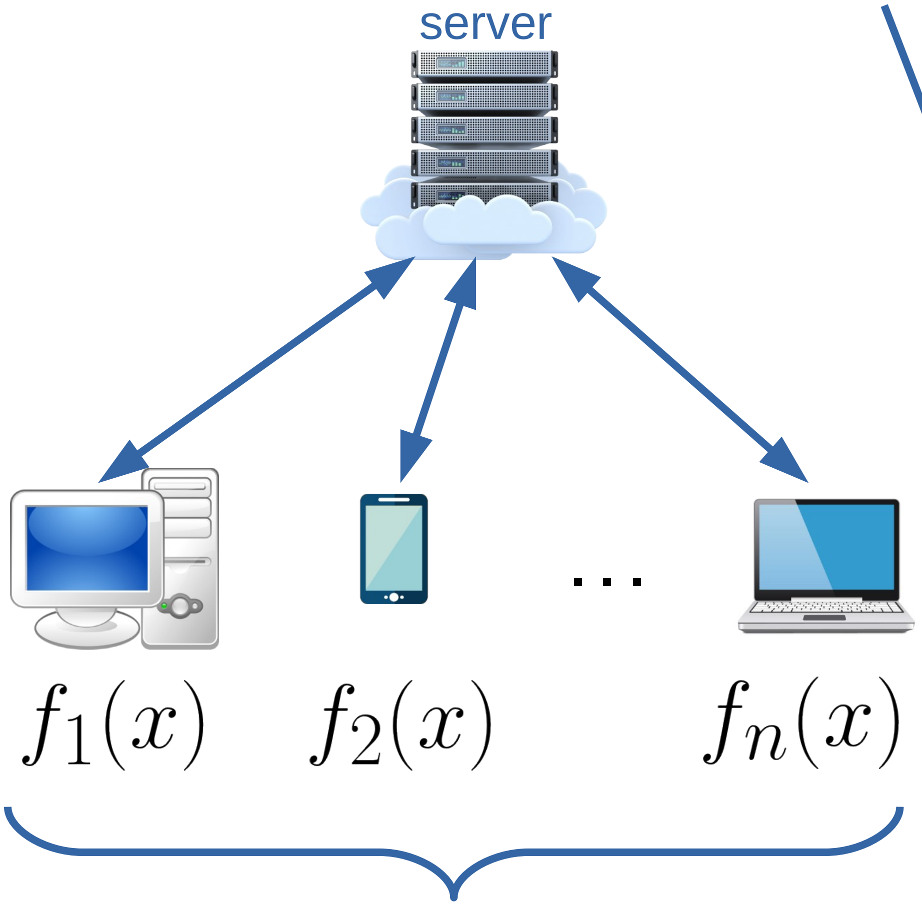

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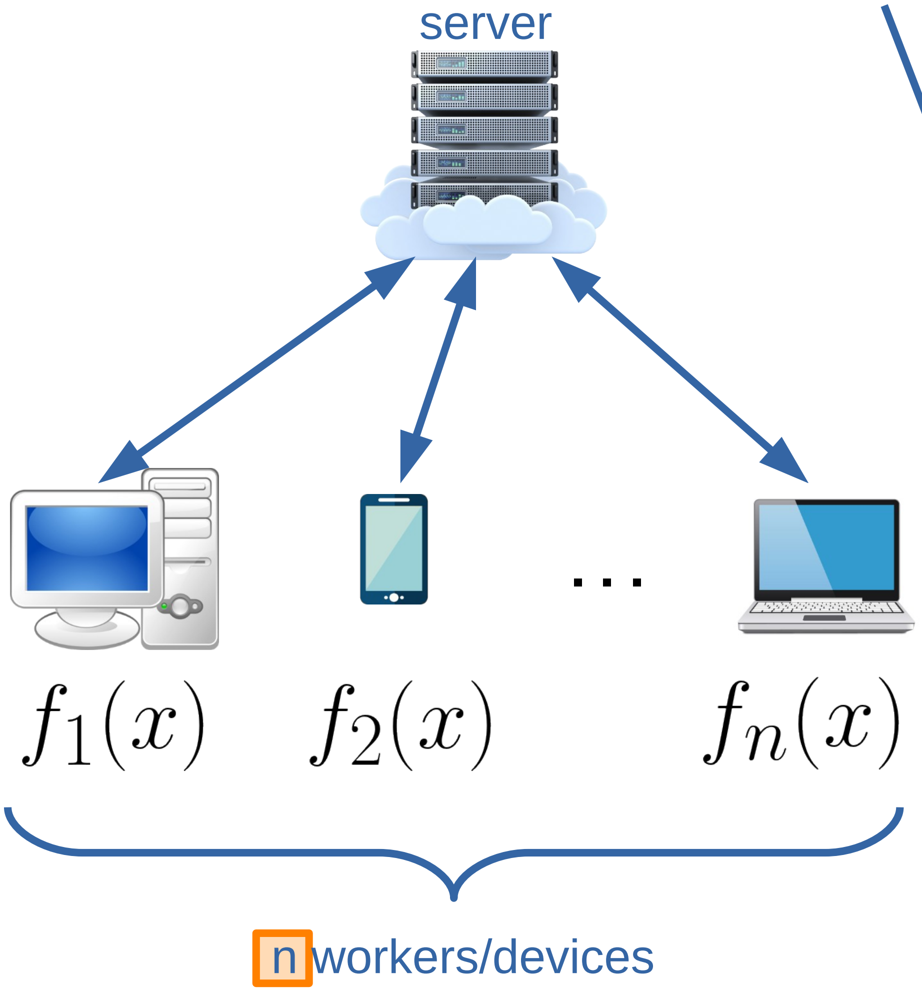

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- $f_i(x) = \mathbf{E}_{\xi_i\sim\mathcal{D}_i}\left[f_{\xi_i}(x)\right]$
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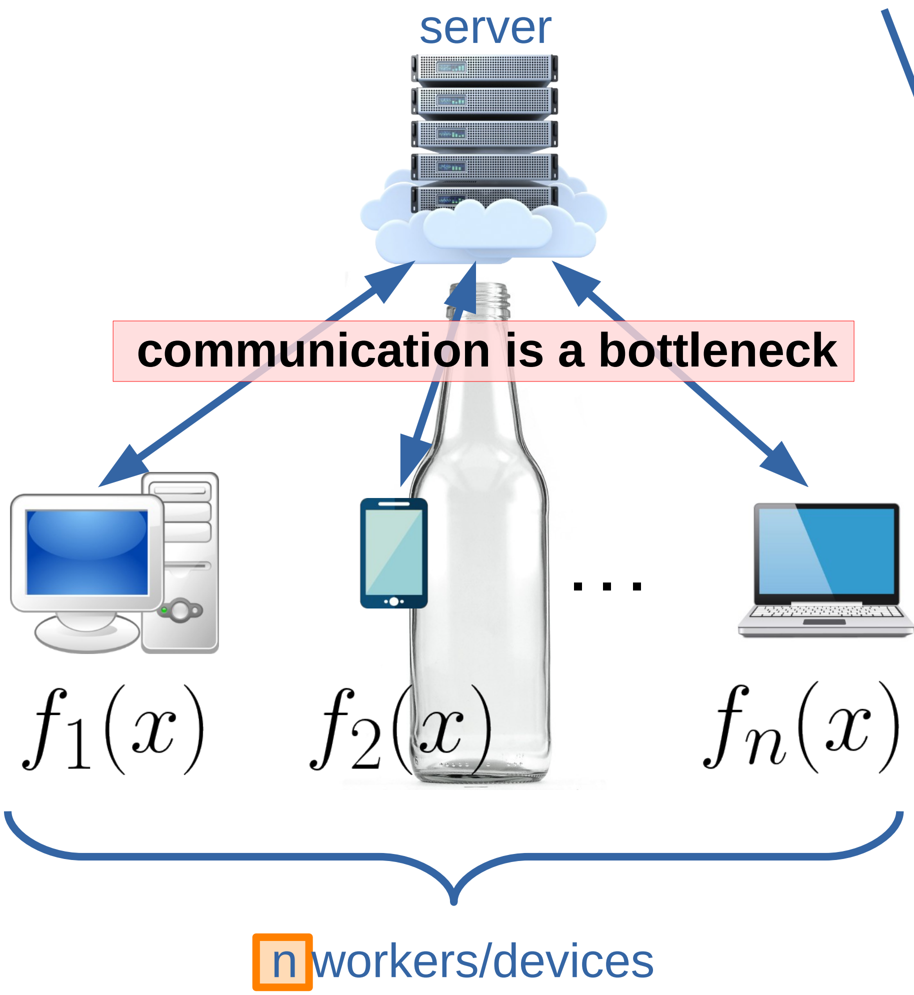

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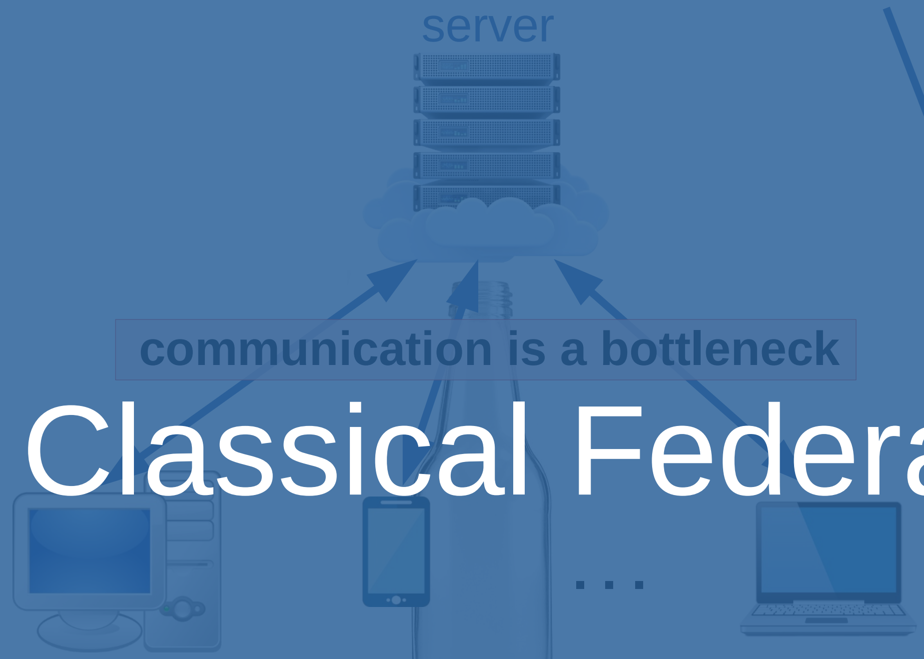
Classical Federated Learning Setup



$f_1(x) \quad f_2(x) \quad \dots \quad f_n(x)$

$\boxed{n}$ workers/devices

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How to Handle Communication Bottleneck?

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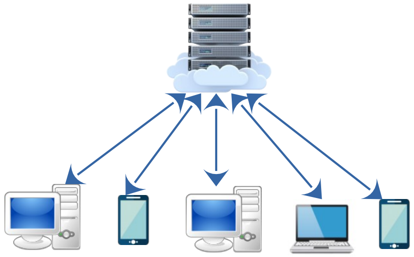
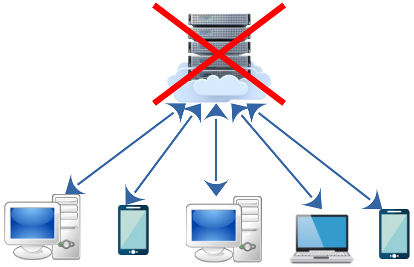
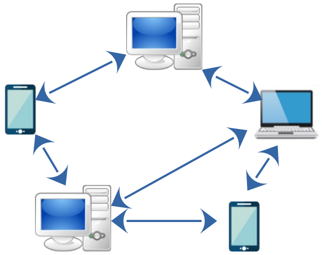
- Change the topology of the network



How to Handle Communication Bottleneck?

- Change the topology of the network → Decentralized optimization





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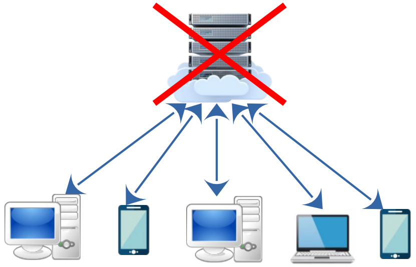

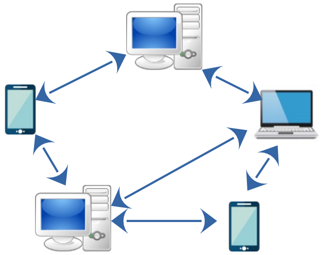

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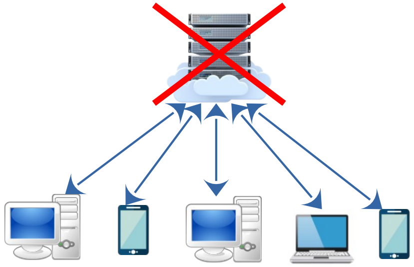

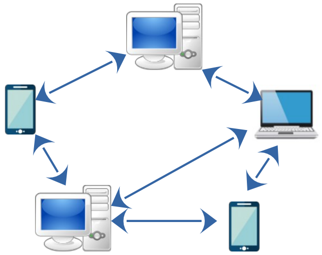

- Do more work on each worker in the hope of communicating less → Local-SGD/Federated Averaging

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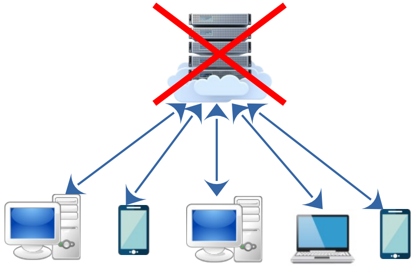

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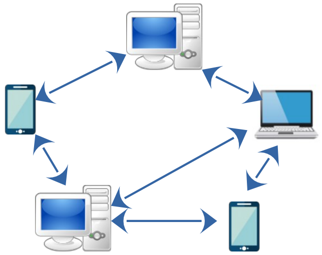

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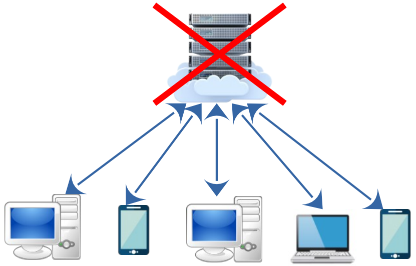

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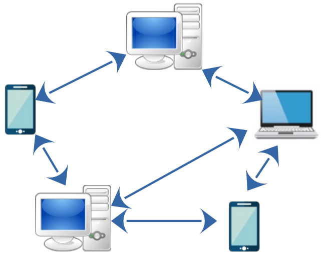

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Workers send dense vectors

$$g = \begin{pmatrix} 1 \\ -15 \\ 0.2 \\ -7 \\ 10 \end{pmatrix}$$

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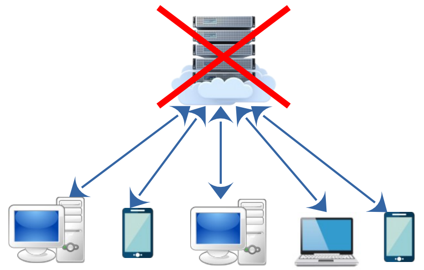

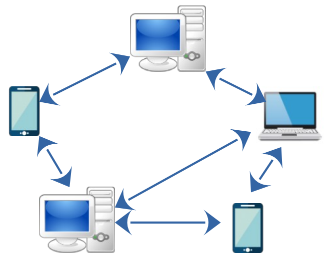

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Workers send dense vectors → Workers send compressed/sparse vectors

$$g = \begin{pmatrix} 1 \\ -15 \\ 0.2 \\ -7 \\ 10 \end{pmatrix} \quad \longrightarrow \quad \mathcal{Q}(g) = \frac{5}{2} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ -7 \\ 0 \end{pmatrix}$$

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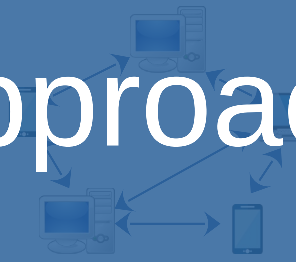

We study this approach

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Unbiased compression (quantization)

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Example: RandK (for K = 2)

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Example: RandK (for K = $\boxed{2}$)

$$\boxed{d = 5}\left\{\begin{pmatrix} 1 \\ -15 \\ 0.2 \\ -7 \\ 10 \end{pmatrix}\right. \Rightarrow \frac{\boxed{5}}{\boxed{2}} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ -7 \\ 0 \end{pmatrix} \qquad \omega = \frac{\boxed{d}}{\boxed{K}} - 1$$

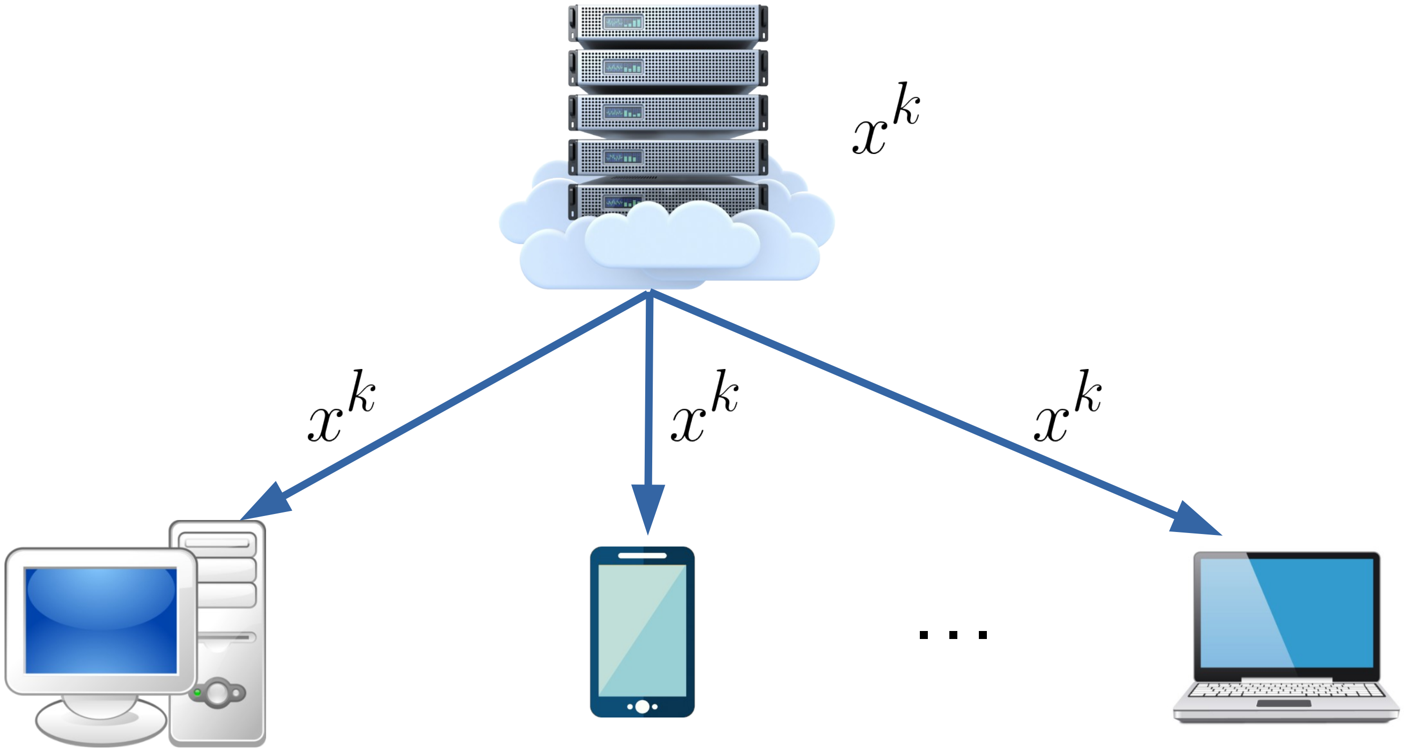
for unbiasedness

Pick K = $\boxed{2}$ components uniformly at random

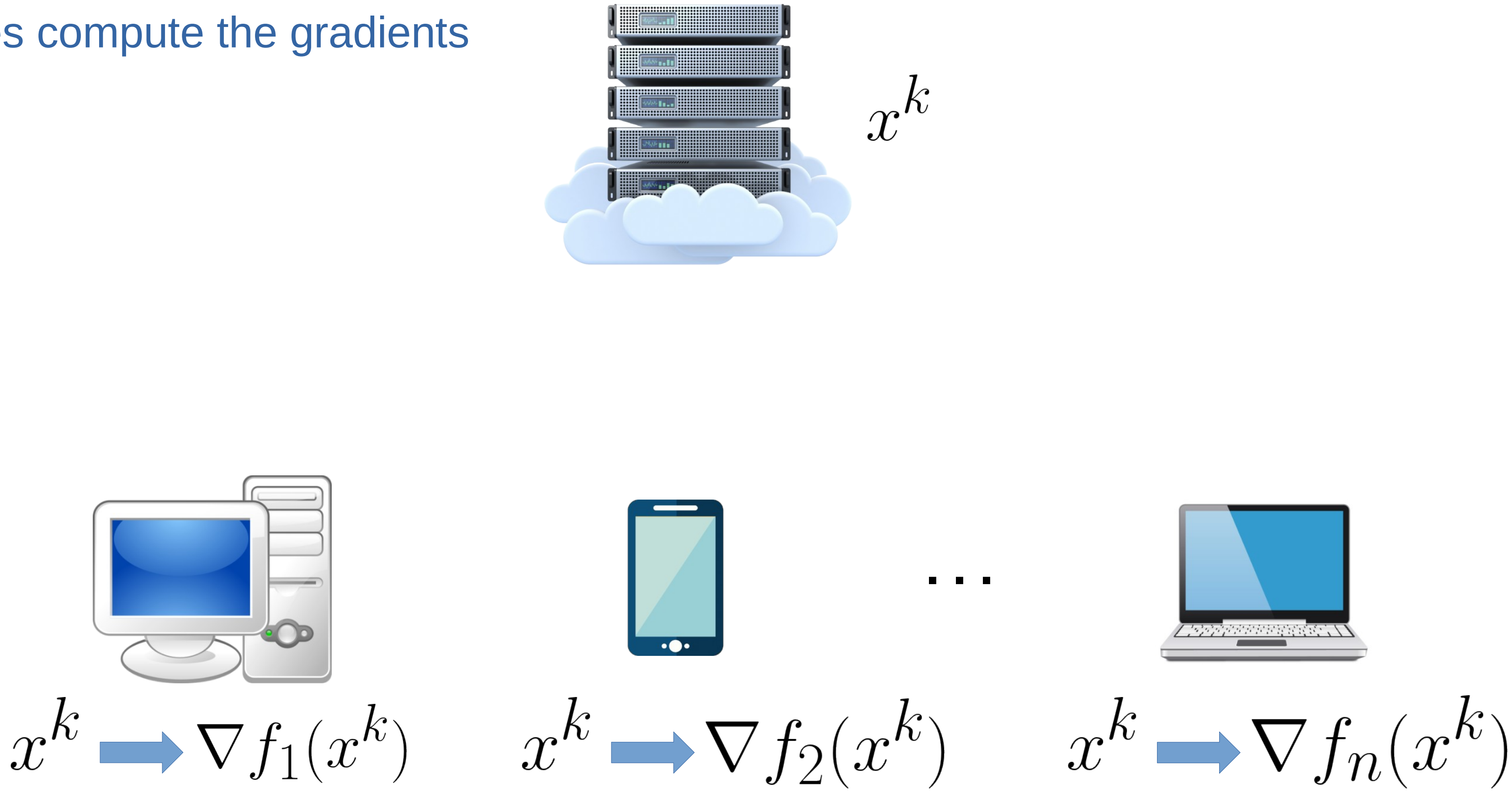
2. Quantized Gradient Descent (QGD)

Alistarh, Dan, Demjan Grubic, Jerry Li, Ryota Tomioka, and Milan Vojnovic. **"QSGD: Communication-efficient SGD via gradient quantization and encoding."** *In Advances in Neural Information Processing Systems*, pp. 1709-1720. 2017.

1 Server broadcasts the parameters

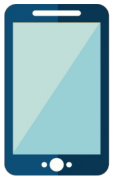


1. Server broadcasts the parameters
2. Devices compute the gradients



1. Server broadcasts the parameters
2. Devices compute the gradients
3. Devices quantize the gradients

x^k

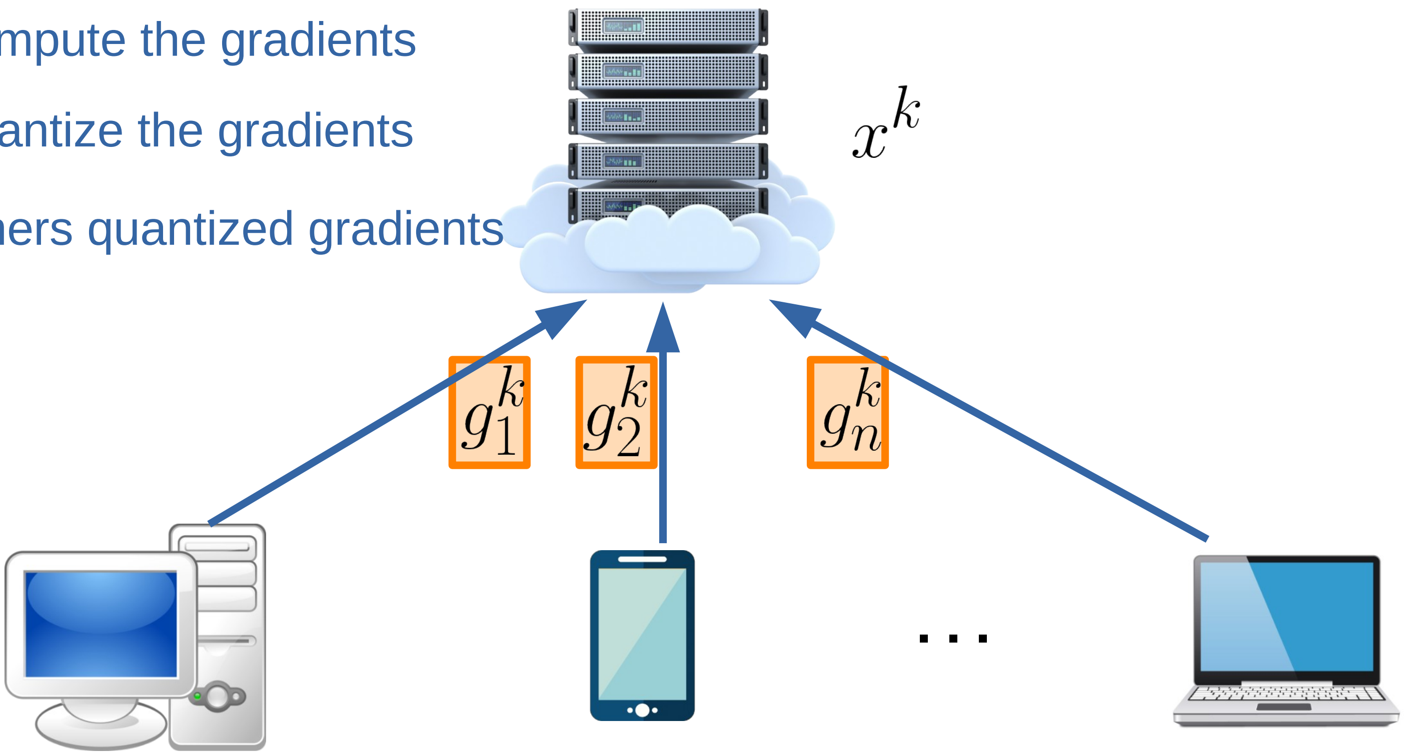


$\cdots$

$x^k \rightarrow \nabla f_1(x^k)$ $x^k \rightarrow \nabla f_2(x^k)$ $x^k \rightarrow \nabla f_n(x^k)$

$\boxed{g_1^k = \mathcal{Q}\left(\nabla f_1(x^k)\right)}$ $\boxed{g_2^k = \mathcal{Q}\left(\nabla f_2(x^k)\right)}$ $\boxed{g_n^k = \mathcal{Q}\left(\nabla f_n(x^k)\right)}$

1. Server broadcasts the parameters
2. Devices compute the gradients
3. Devices quantize the gradients
4. Server gathers quantized gradients



$x^k \rightarrow \nabla f_1(x^k)$ $\quad$ $x^k \rightarrow \nabla f_2(x^k)$ $\quad$ $x^k \rightarrow \nabla f_n(x^k)$

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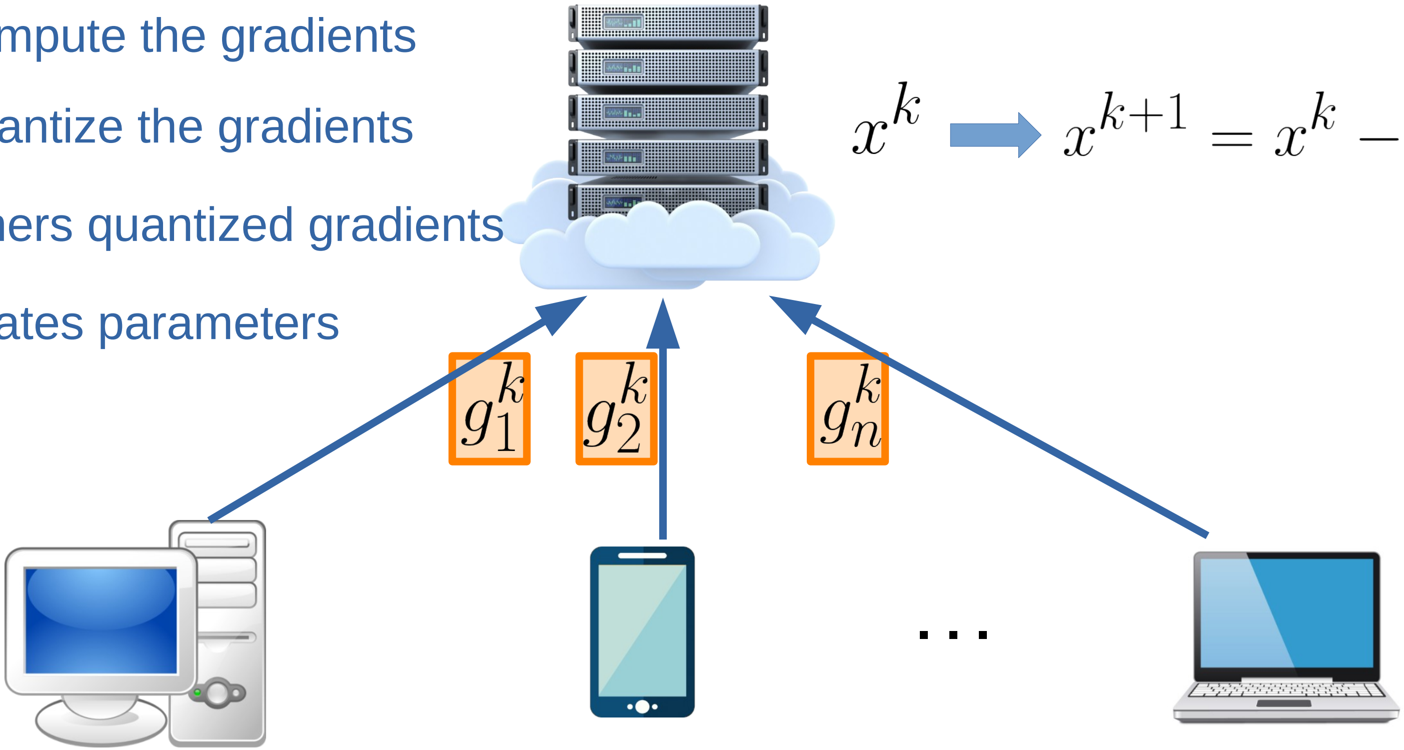
1. Server broadcasts the parameters
2. Devices compute the gradients
3. Devices quantize the gradients
4. Server gathers quantized gradients
5. Server updates parameters

$$x^k \rightarrow x^{k+1} = x^k - \gamma \cdot \frac{1}{n}\sum_{i=1}^{n} \boxed{g_i^k}$$



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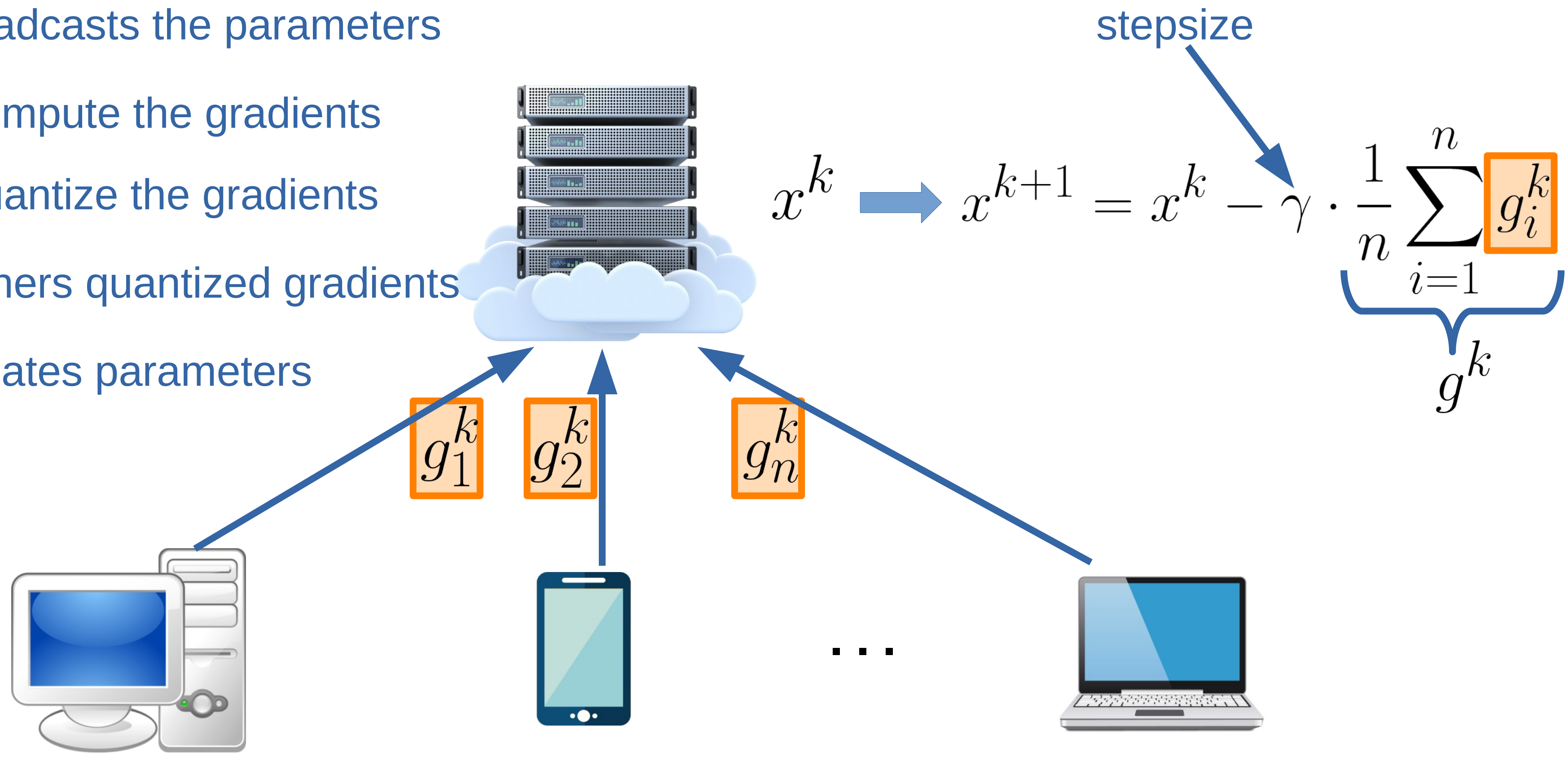

$$x^k \Rightarrow \nabla f_1(x^k) \qquad x^k \Rightarrow \nabla f_2(x^k) \qquad x^k \Rightarrow \nabla f_n(x^k)$$

$$\boxed{g_1^k = \mathcal{Q}\left(\nabla f_1(x^k)\right)} \quad \boxed{g_2^k = \mathcal{Q}\left(\nabla f_2(x^k)\right)} \quad \boxed{g_n^k = \mathcal{Q}\left(\nabla f_n(x^k)\right)}$$

1. Server broadcasts the parameters
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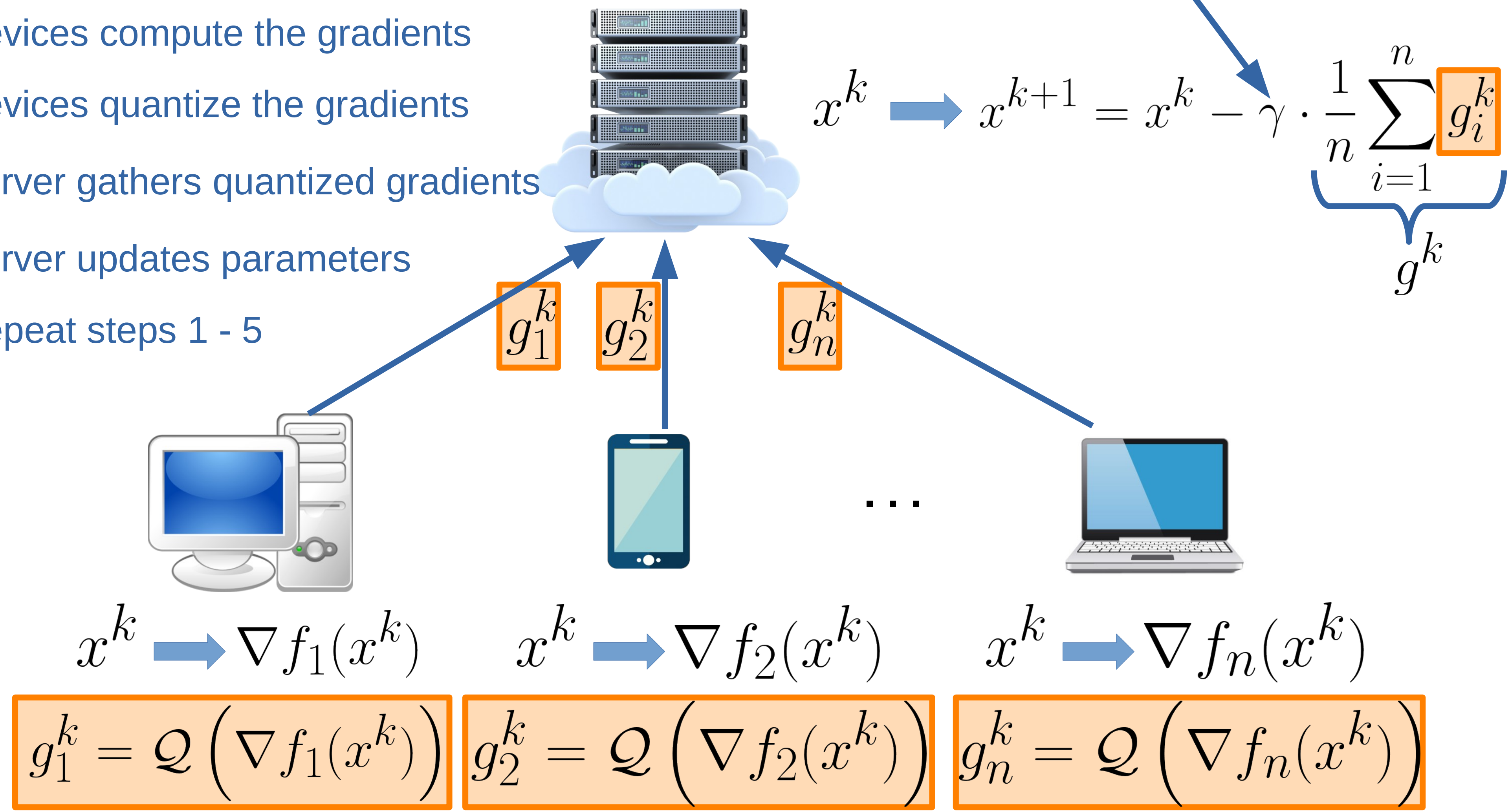
3. Devices quantize the gradients
4. Server gathers quantized gradients
5. Server updates parameters
6. Repeat steps 1 - 5



Assumptions

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Assumptions

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2. Smoothness: $\|\nabla f_i(x) - \nabla f_i(y)\| \leq L_i \|x - y\|$

Complexity Bound for QGD

Khaled, Ahmed, and Peter Richtárik. **"Better theory for SGD in the nonconvex world."** arXiv preprint arXiv:2002.03329 (2020).

QGD finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \varepsilon^2$ after

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$$\mathcal{O}\left(\frac{\Delta_0}{\varepsilon^2} + \frac{(1+\omega)\Delta_0^2}{\varepsilon^4 n} + \frac{(1+\omega)\Delta_0\Delta_f^*}{\varepsilon^4 n}\right)$$

communication rounds

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Hides numerical factors and smoothness constants

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$$\boxed{\mathbb{E}\|Q(x) - x\|^2 \leq \omega \|x\|^2}$$

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$$\mathcal{O}\left(\frac{\boxed{\Delta_0}}{\boxed{\varepsilon^2}} + \frac{(1+\boxed{\omega})\boxed{\Delta_0^2}}{\boxed{\varepsilon^4}n} + \frac{(1+\boxed{\omega})\boxed{\Delta_0}\Delta_f^*}{\boxed{\varepsilon^4}n}\right)$$

communication rounds

$$\boxed{\mathbb{E}\|\mathcal{Q}(x) - x\|^2 \leq \omega\|x\|^2}$$

$$\boxed{\Delta_0 = f(x^0) - f_*}$$

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Khaled, Ahmed, and Peter Richtárik. **"Better theory for SGD in the nonconvex world."** arXiv preprint arXiv:2002.03329 (2020).

QGD finds such $\hat{x}$ that $\mathbb{E}\left[\|\nabla f(\hat{x})\|^2\right] \leq \boxed{\varepsilon^2}$ after

Hides numerical factors and smoothness constants $\longrightarrow$ $\mathcal{O}\left(\frac{\boxed{\Delta_0}}{\boxed{\varepsilon^2}} + \frac{(1+\boxed{\omega})\boxed{\Delta_0^2}}{\boxed{\varepsilon^4} n} + \frac{(1+\boxed{\omega})\boxed{\Delta_0}\boxed{\Delta_f^*}}{\boxed{\varepsilon^4} n}\right)$ communication rounds

$$\boxed{\mathbb{E}\|\mathcal{Q}(x) - x\|^2 \leq \omega \|x\|^2}$$

$$\boxed{\Delta_0 = f(x^0) - f_*}$$

$$\boxed{\Delta_f^* = f_* - \frac{1}{n}\sum_{i=1}^{n} f_{i,*}}$$

3. DIANA

Mishchenko, Konstantin, Eduard Gorbunov, Martin Takáč, and Peter Richtárik. **"Distributed learning with compressed gradient differences."** arXiv preprint arXiv:1901.09269 (2019).

Horváth, Samuel, Dmitry Kovalev, Konstantin Mishchenko, Sebastian Stich, and Peter Richtárik. **"Stochastic distributed learning with gradient quantization and variance reduction."** arXiv preprint arXiv:1904.05115 (2019).

$$x^{k+1} = x^k - \gamma \cdot \frac{1}{n} \sum_{i=1}^{n} g_i^k$$

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learnable local shifts

$$\boxed{h_i^{k+1} = h_i^k + \alpha \mathcal{Q}\left(\nabla f_i(x^k) - h_i^k\right)}$$

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QGD: $$g_i^k = \boxed{\mathcal{Q}\left(\nabla f_i(x^k)\right)}$$

vectors that devices have to send

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Complexity Bound for DIANA

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Complexity Bounds for DIANA and QGD

QGD: $$\mathcal{O}\left(\frac{\Delta_0}{\varepsilon^2} + \frac{(1+\omega)\Delta_0^2}{\varepsilon^4 n} + \frac{(1+\omega)\Delta_0\Delta_f^*}{\varepsilon^4 n}\right)$$

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Complexity Bounds for DIANA and QGD

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Is it possible to get better rates?

DIANA: $$\mathcal{O}\left(\frac{\Delta_0\left(1+(1+\omega)\sqrt{\omega/n}\right)}{\varepsilon^2}\right)$$

4. MARINA

$$x^{k+1} = x^k - \gamma \cdot \frac{1}{n}\sum_{i=1}^{n} g_i^k$$

DIANA: $g_i^k = h_i^k + \mathcal{Q}\left(\nabla f_i(x^k) - h_i^k\right)$

$$h_i^{k+1} = h_i^k + \alpha \mathcal{Q}\left(\nabla f_i(x^k) - h_i^k\right)$$

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MARINA: $g_i^k = \begin{cases} \nabla f_i\left(x^k\right) & \text{w.p. } p \\ g^{k-1} + \mathcal{Q}\left(\nabla f_i\left(x^k\right) - \nabla f_i\left(x^{k-1}\right)\right) & \text{w.p. } 1-p \end{cases}$

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typically small

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vectors that devices have to send

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$$x^{k+1} = x^k - \gamma \cdot \boxed{\frac{1}{n}\sum_{i=1}^{n} g_i^k} = x^k - \gamma \boxed{g^k}$$

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vectors that devices have to send

$$h_i^{k+1} = h_i^k + \alpha \mathcal{Q}\left(\nabla f_i(x^k) - h_i^k\right) \quad \mathbb{E}\left[\boxed{g^k} \mid x^k\right] = \nabla f(x^k)$$

typically small

MARINA: $g_i^k = \begin{cases} \boxed{\nabla f_i\left(x^k\right)} & \text{w.p. } p \\ g^{k-1} + \boxed{\mathcal{Q}\left(\nabla f_i\left(x^k\right) - \boxed{\nabla f_i\left(x^{k-1}\right)}\right)} & \text{w.p. } 1-p \end{cases}$

$$\mathbb{E}\left[\boxed{g^k} \mid x^k\right] \neq \nabla f(x^k)$$

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$$p = \frac{1}{\omega+1} = \Theta\left(\frac{\boxed{\zeta_{\mathcal{Q}}}}{d}\right)$$

$$\boxed{\zeta_{\mathcal{Q}}} = \sup_{x\in\mathbb{R}^d} \mathbb{E}\left[\|\mathcal{Q}(x)\|_0\right]$$

assumption (holds for RandK, l2-quantization)

expected density

Complexity Bounds for MARINA and DIANA

DIANA: $$\mathcal{O}\left(\frac{\Delta_0\left(1+(1+\omega)\sqrt{\omega/n}\right)}{\varepsilon^2}\right)$$

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Complexity Bounds for MARINA and DIANA

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5. Experiments

The Problem

$$\min_{x \in \mathbb{R}^d} \left\{ f(x) = \frac{1}{N} \sum_{t=1}^{N} \left(1 - \frac{1}{1 + \exp(-y_t a_t^\top x)} \right)^2 \right\}$$

The Problem

$$\min_{x\in\mathbb{R}^d}\left\{f(x)=\frac{1}{N}\sum_{t=1}^{N}\left(1-\frac{1}{1+\exp(-y_t a_t^\top x)}\right)^2\right\}$$

$y_t \in \{-1,1\}$ $\qquad$ $a_t \in \mathbb{R}^d$

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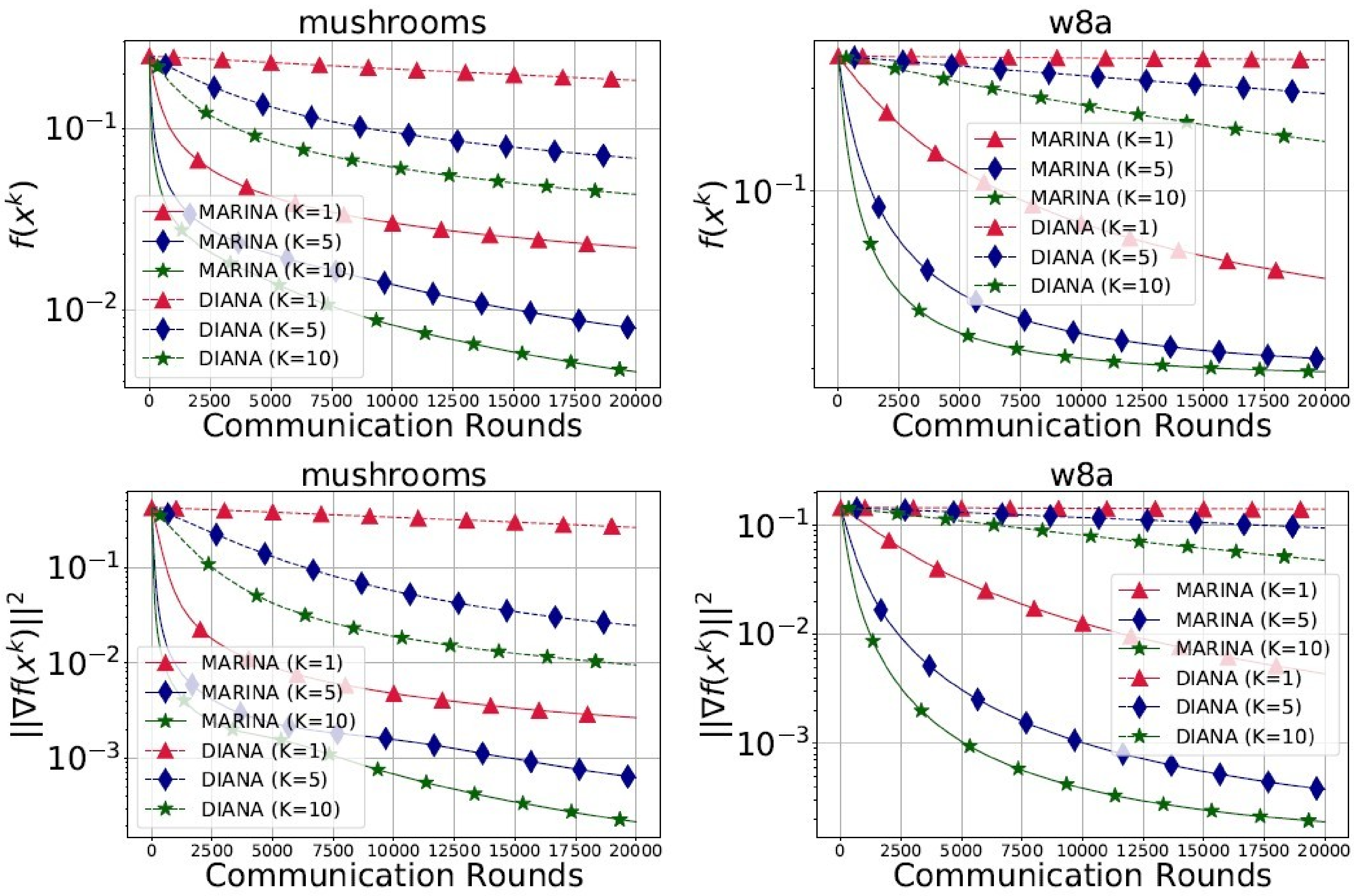
- The dataset was split into 5 equal parts among 5 clients
- Theoretical stepsizes

MARINA vs DIANA (RandK)



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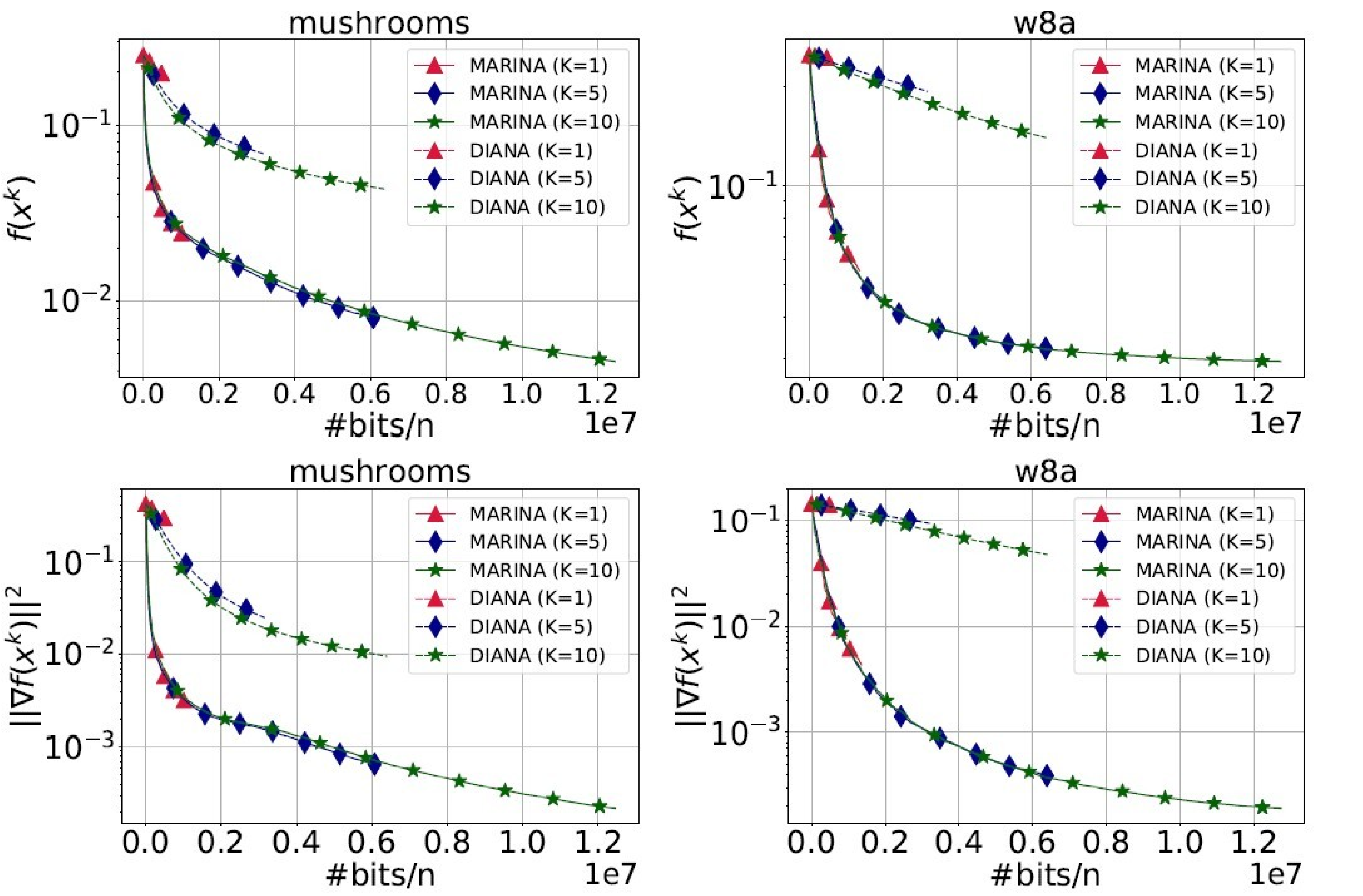

6. Extra Results

In the paper, we also have:

- Variance Reduced MARINA for the problems with $f_i(x) = \frac{1}{m}\sum_{j=1}^{m} f_{ij}(x)$

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- Rates under Polyak- Lojasiewicz Condition
- Explicit dependencies on smoothness constants, non-uniform sampling
- Simple proofs

https://arxiv.org/pdf/2102.07845.pdf