ASYPMTOTICAL BEHAVIOR OF TRAJECTORIES GENERATED BY GONOSOMAL EVOLUTION OPERATOR

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Outline

Introduction

A population is a summation of all the organisms of the same group or species, which live in a particular geographical area, and have the capability of interbreeding.

A gene is the molecular unit of heredity of a living organism.

A free population means random mating in the population: where all individuals are potential partners. This assumes that there are no mating restrictions, neither genetic or behavioural, therefore all recombination is possible.

Bisexual population (BP) means all the organisms of one type must belong to the same sex. Thus, it is possible to speak of male and female types.

In the life sciences the *population dynamics* branch studies the size and age composition of populations as dynamical systems. These investigations are motivated by their application to population growth, ageing populations, or population decline.

The population dynamics is a well developed branch of mathematical biology, which has a history of more than two hundred years [1], although more recently the branch of mathematical biology has greatly increased. Many concrete models of mathematical biology described by corresponding non-linear evolution operator.

We study dynamical system generated by a concrete non-linear multidimensional operator describing a gonosomal evolution. Our model is related to a bisexual population. We note that investigation of dynamical systems generated by evolution operators of free and bisexual population can be reduced to the study of nonlinear dynamical systems [2], [3], [4].

[1] Bacaër N. *A short history of mathematical population dynamics*. Springer-Verlag, London. (2011)

[2] Ganikhodzhaev R.N., Mukhamedov F.M. and Rozikov U.A. Inf. Dim. Anal. Quant. Prob. Rel. Fields. **14**(2), (2011), 279–335.

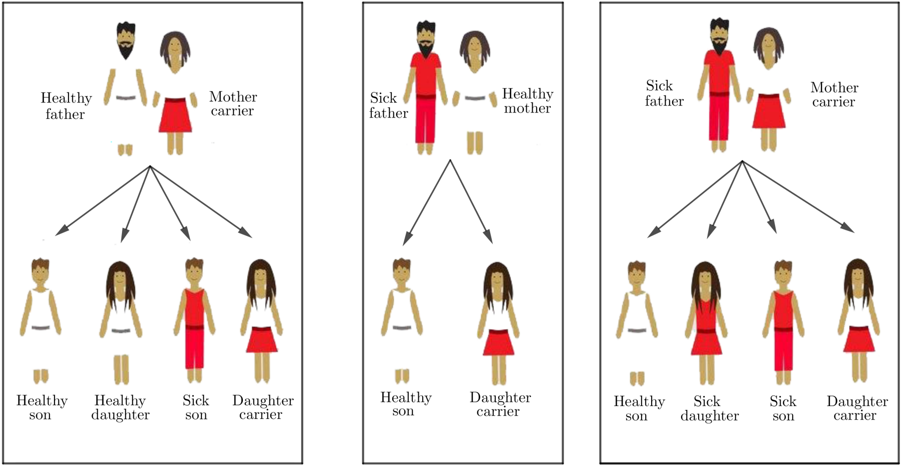
[3] Kesten H. Adv. Appl. Probab. **2**(2) (1970), 1–82; 179–228.

[4] Rozikov U.A. Asia Pacific Math. Newsletter. **3**(1) (2013), 6–11.

§ 1.1 Bisexual population: Gonosomal evolution operator

Hemophilia is a genetic disorder linked to the X chromosome, it is due to mutations in two genes located at the end of the long arm of gonosome X. This is a lethal recessive genetic disease that is lethal in the homozygous state, it follows that if X^h denotes the X chromosome carrying hemophilia, there are only two female genotypes: XX and XX^h (genotype X^hX^h is lethal) and two male genotypes: XY and X^hY. The results of the four types of crosses are:

$$XX \times XY \to \frac{1}{2}XX,\ \frac{1}{2}XY,$$
$$XX \times X^hY \to \frac{1}{2}XX^h,\ \frac{1}{2}XY,$$
$$XX^h \times XY \to \frac{1}{4}XX,\ \frac{1}{4}XX^h,\ \frac{1}{4}XY, \frac{1}{4}X^hY,$$
$$XX^h \times X^hY \to \frac{1}{3}XX^h,\ \frac{1}{3}XY,\ \frac{1}{3}X^hY.$$


Healthy father
Mother carrier
Healthy son
Healthy daughter
Sick son
Daughter carrier
Sick father
Healthy mother
Healthy son
Daughter carrier
Sick father
Mother carrier
Healthy son
Sick daughter
Sick son
Daughter carrier

To define an evolution operator of Hemophilia, we introduce the following

$$S^3 = \left\{ s = (x, y, u, v) \in \mathbb{R}^4 : \ x \geq 0, \ y \geq 0, \ u \geq 0, \ v \geq 0, \ x + y + u + v = 1 \right\}$$

the three-dimensional simplex;

$$\mathcal{O} = \{(x, y, u, v) \in S^3 : (x, y) = (0, 0) \ or \ (u, v) = (0, 0)\};$$

$$S^{2,2} = S^3 \setminus \mathcal{O}.$$

$$Int\ S^{2,2} = Int\ S^3, \qquad \partial S^{2,2} \subsetneq \partial S^3.$$

Let $F = \{XX, XX^h\}$ and $M = \{XY, X^hY\}$ be sets of genotypes. Assume that state of the set F is given by a real vector (x, y) and state of M by a real vector (u, v). Then a state of the set $F \cup M$ is given by the vector $s = (x, y, u, v)$. If $s' = (x', y', u', v')$ is a state of the system $F \cup M$ in the next generation, then by the above rule we get the evolution operator $W : S^{2,2} \to S^{2,2}$ defined [5] by

$$W : \begin{cases} x' = \dfrac{2xu + yu}{4(x+y)(u+v)}, \\ y' = \dfrac{6xv + 3yu + 4yv}{12(x+y)(u+v)}, \\ u' = \dfrac{6xu + 6xv + 3yu + 4yv}{12(x+y)(u+v)}, \\ v' = \dfrac{3yu + 4yv}{12(x+y)(u+v)}. \end{cases} \tag{1}$$

[5] Kesten H. Adv. Appl. Probab. **2**(2) (1970), 1–82; 179–228.

In their work [6] U.A. Rozikov and R. Varro considered normalized gonosomal evolution operator (1) of a sex linked inheritance. They proved that the operator W has a unique nonhyperbolic fixed point $s_0 = (\frac{1}{2}, 0, \frac{1}{2}, 0)$ and there is an open neighborhood $U(s_0) \subset S^{2,2}$ of s_0 such that for any initial point $s \in U(s_0)$, the limit point of trajectories $\{W^m(s)\}$ tends to s_0.

Conjecture of U.A. Rozikov and R. Varro.
For any initial point $s \in S^{2,2}$, it holds that

$$\lim_{m\to\infty} W^m(s) = s_0 = (\frac{1}{2}, 0, \frac{1}{2}, 0).$$

[6] Rozikov U.A., Varro R. Discont. Nonlinear. and Complexity., **V.5**, N.2, p.173-185. (2016)

In our work [7] we proved that conjecture and the following Theorem is one of the main results of the chapter I.

Theorem 1.2.1

The operator $W : S^{2,2} \to S^{2,2}$ given by (1) has unique nonhyperbolic fixed point $s_0 = (\frac{1}{2}, 0, \frac{1}{2}, 0)$ and for any initial point $s \in S^{2,2}$ we have

$$\lim_{m\to\infty} W^m(s) = s_0 = (\frac{1}{2}, 0, \frac{1}{2}, 0). \tag{2}$$

The proof of this theorem also available at the recent book[8].

[7] Absalamov A.T. Discont. Nonlinear. and Complexity. V.10, N.1, p.143-149. (2021)

[8] Rozikov U.A., *Population dynamics: algebraic and probabilistic approach*. World Sci. Publ. Singapore. (2020), 460 pages.

In this section, we have investigated the velocity of convergence to the fixed point of the operator (1)

Theorem 1.3.1.

For the trajectory $(x^{(m)}, y^{(m)}, u^{(m)}, v^{(m)})$ of the operator (1) for the sufficiently large m the following hold [a]:

$$\left|\frac{1}{2} - x^{(m)}\right| \leq \frac{c_1}{m}, \quad \frac{c_2}{m} \leq y^{(m)} \leq \frac{c_3}{m},$$
$$\left|\frac{1}{2} - u^{(m)}\right| \leq \frac{c_4}{m}, \quad \frac{c_5}{m} \leq v^{(m)} \leq \frac{c_6}{m}.$$

where c_1, c_2, c_3, c_4, c_5, c_6 are positive constants.

[a] Absalamov A.T. Uzbek Math. Jour. No 4. p. 4-11. (2019)

Suppose that the set of female types is $F = \{1, 2, ..., \eta\}$ and the set of male types is $M = \{1, 2, ..., \nu\}$. Let $x = (x_1, x_2, ..., x_\eta) \in \mathbb{R}^\eta$ be a state of F and $y = (y_1, y_2, ..., y_\nu) \in \mathbb{R}^\nu$ be a state of M. Consider $p^{(f)}_{ir,j}$ and $p^{(m)}_{ir,l}$ as some inheritance non-negative real coefficients (not necessarily probabilities) with

$$\sum_{j=1}^{\eta} p^{(f)}_{ir,j} + \sum_{l=1}^{\nu} p^{(m)}_{ir,l} = 1$$

and the corresponding evolution operator

$$W : \begin{cases} x'_j = \sum_{i,r=1}^{\eta,\nu} p^{(f)}_{ir,j} x_i y_r, & j = 1, ..., n \\ y'_l = \sum_{i,r=1}^{\eta,\nu} p^{(m)}_{ir,l} x_i y_r, & l = 1, ..., \nu. \end{cases} \tag{3}$$

This operator is called gonosomal evolution operator.

The main problem for a given discrete-time dynamical system is to describe the limit points of the trajectory $\{t^{(n)}\}_{n=0}^{\infty}$ for arbitrarily given $t^{(0)} = (x, y) \in \mathbb{R}^{\eta+\nu}$, where

$$t^{(n)} = W^n(t) = \underbrace{W(W(...W(t^{(0)}))...)}$$

The problem of describing the ω-limit set of a trajectory is of great importance in the theory of dynamical systems. The following Propositions are the main results of the section 2.2.

Proposition 2.2.1.

The point $s = (0, 0, ..., 0) \in \mathbb{R}^{\eta+\nu}$ is a fixed point for the operator (3). If $\delta \in [0, 4)$ and the coefficients of the operator (3) are nonnegative real numbers, then for any initial point $t \in Q_\delta$, we have

$$\lim_{n\to\infty} W^n(t) = \underbrace{(0, 0, ..., 0)}_{\eta+\nu}, \tag{4}$$

where $Q_\delta = \{(x_1, ..., x_\eta, y_1, ..., y_\nu) \in \mathbb{R}^{\eta+\nu} : \sum_{j=1}^{\eta} x_j + \sum_{l=1}^{\nu} y_l \leq \delta,\ x_j \geq 0, y_l \geq 0, j = \overline{1, \eta}, l = \overline{1, \nu}\}$

Proposition 2.2.2.

Let the coefficients of the operator (3) and the coordinates of an initial point $t=(x_1,...,x_\eta,y_1,...,y_\nu)\in\mathbb{R}^{\eta+\nu}$ be nonnegative real numbers. If

$$\max_{\substack{1\leq i,\, r\leq\eta,\\ 1\leq j,\, l\leq\nu}}\left\{p_{ij,r}^{(f)}p_{ij,l}^{(m)}x_r y_l\right\}>1,$$

then $\lim\limits_{n\to\infty} W^n(t)=\infty$, i.e. at least one coordinate of $W^n(t)$ tends to ∞ as $n\to\infty$.

when $\eta = \nu = 2$

$$W_0 : \begin{cases} x' = a_1 xu + b_1 yu, \\ y' = c_1 xv + b_2 yu + d_1 yv, \\ u' = a_2 xu + c_2 xv + b_3 yu + d_2 yv, \\ v' = b_4 yu + d_3 yv. \end{cases} \tag{5}$$

and for this operator we have the following results.

Lemma 2.2.1.

Let

$$Q_4 = \{(x, y, u, v) \in \mathbb{R}^4 : x \geq 0, y \geq 0, u \geq 0, v \geq 0, \ x + y + u + v \leq 4\}.$$

For any initial point $t \in Q_4$ if there exists $k \geq 0$ such that

$$\left(a_1 - \frac{1}{2}\right)x^{(k)}u^{(k)} + \left(c_1 - \frac{1}{2}\right)x^{(k)}v^{(k)} + \left(b_1 + b_2 - \frac{1}{2}\right)y^{(k)}u^{(k)} + \left(d_1 - \frac{1}{2}\right)y^{(k)}v^{(k)} \neq 0$$

then

$$\lim_{n\to\infty} W_0^n(t) = (0, 0, 0, 0). \tag{6}$$

we define the following sets

$$O = \{(0,0,u,v) \in \mathbb{R}^4 : u, v \in \mathbb{R}\} \cup \{(x,y,0,0) \in \mathbb{R}^4 : x, y \in \mathbb{R}\}$$
$$I = \{(x,y,u,v) \in \mathbb{R}^4 : y = v = 0\}$$
$$J = \{(x,y,u,v) \in I : x = u\}$$
$$P = \{(x,y,u,v) \in \mathbb{R}^4 : x \geq 0, y \geq 0, u \geq 0, v \geq 0\}$$
$$P_0 = \{(x,y,u,v) \in P : (x+y)(u+v) < 4\}$$
$$Q_a = \{(x,y,u,v) \in P : x+y+u+v \leq a\}, \quad a \in [0,4]$$
$$N = \{(x,y,u,v) \in \mathbb{R}^4 : x \leq 0, y \leq 0, u \leq 0, v \leq 0\}$$
$$N_0 = \{(x,y,u,v) \in \mathbb{R}^4 : x \leq 0, y \leq 0, u \geq 0, v \geq 0\}$$
$$N_1 = \{(x,y,u,v) \in \mathbb{R}^4 : x \geq 0, y \geq 0, u \leq 0, v \leq 0\}$$
$$\Delta_0 = \{(x,y,u,v,) \in P : x+y+u+v > 4, \quad max\{a_1 a_2 xu,\ b_2 b_3 yu,\ d_1 d_3 yv\} > 1\}.$$

Theorem 2.2.1.

If $t=(x,y,u,v)\in\mathbb{R}^4$ is such that

(i) one of the following conditions is satisfied

1) $t\in P_0$,
2) $t\in Q_4$ and Lemma 2.2.1 hold,
3) $t\in N,\quad W_0(t)\ \in P_0$,
4) $t\in N_0,\quad W_0^2(t)\in P_0$,
5) $t\in N_1,\quad W_0^2(t)\in P_0$,

then

$$\lim_{n\to\infty} W_0^n(t)=(0,0,0,0).$$

(ii) one of the following conditions is satisfied

1) $t\in\Delta_0$,
2) $t\in N,\quad W_0(t)\ \in\Delta_0$,
4) $t\in N_0,\quad W_0^2(t)\in\Delta_0$,
5) $t\in N_1,\quad W_0^2(t)\in\Delta_0$,

then

$$\lim_{n\to\infty} W_0^n(t)=\infty,$$

i.e. at least one coordinate of $W_0^n(t)$ tends to ∞.

Definition 2.1.1.

A fixed point s of the operator V_1 is called hyperbolic if its Jacobian J at s has no eigenvalues on the unit circle.

Definition 2.1.2

A hyperbolic fixed point s is called:
i) attracting if all the eigenvalues of the Jacobi matrix $J(s)$ are less than 1 in absolute value;
ii) repelling if all the eigenvalues of the Jacobi matrix $J(s)$ are greater than 1 in absolute value;
iii) a saddle otherwise.

In the section 2.3. we have considered two class of gonosomal evolution operators which are (3) and:

$$W: \begin{cases} x'_{j_1} = \sum_{i_1,i_2,i_3=1}^{\eta_1,\eta_2,\nu_1} p^{(m_1)}_{i_1 i_2 i_3, j_1} x_{i_1} y_{i_2} z_{i_3}, & j_1 = 1, ..., \eta_1, \\ y'_{j_2} = \sum_{i_1,i_2,i_3=1}^{\eta_1,\eta_2,\nu_1} p^{(m_2)}_{i_1 i_2 i_3, j_2} x_{i_1} y_{i_2} z_{i_3}, & j_2 = 1, ..., \eta_2, \\ z'_{j_3} = \sum_{i_1,i_2,i_3=1}^{\eta_1,\eta_2,\nu_1} p^{(m_3)}_{i_1 i_2 i_3, j_3} x_{i_1} y_{i_2} z_{i_3}, & j_3 = 1, ..., \nu_1. \end{cases} \quad (7)$$

where

$$\sum_{j_1=1}^{\eta_1} p^{(m_1)}_{i_1 i_2 i_3, j_1} + \sum_{j_2=1}^{\eta_2} p^{(m_2)}_{i_1 i_2 i_3, j_2} + \sum_{j_3=1}^{\nu_1} p^{(m_3)}_{i_1 i_2 i_3, j_3} = 1.$$

For those operators it is proven the following theorem.

Theorem 2.3.1.

Any nonzero fixed point of the evolution operator (7) can not be attracting.

§ 3.1. Definition of the evolution operator and its fixed points

Define evolution operator $V_1 : S^{2,2} \to S^{2,2}$ by

$$V_1 : \begin{cases} x' = \dfrac{axu}{(x+y)(u+v)} \\ y' = \dfrac{\sigma_1 xv + ayu + ayv}{(x+y)(u+v)} \\ u' = \dfrac{\sigma_2 xv + bxu + byu}{(x+y)(u+v)} \\ v' = \dfrac{byv}{(x+y)(u+v)}, \end{cases} \tag{8}$$

where coefficients satisfy

$$a + b = \sigma_1 + \sigma_2 = 1, \quad a > 0,\ b > 0,\ \sigma_1 \geq 0,\ \sigma_2 \geq 0.$$

In the first section of the chapter III we have discribed all possible fixed points and type of the fixed points of the operator (8).
A point s is called a fixed point of the operator V_1 if $s = V_1(s)$. The set of all fixed points denoted by $\mathrm{Fix}(V_1)$.
The set of all fixed points of operator (8) is $\mathrm{Fix}(V_1) = F_{11} \cup F_{12}$, where

$$F_{11} = \Big\{(0, a, u, v) : \quad u + v = b, \quad u,\, v \in [0, b]\Big\}$$

and

$$F_{12} = \Big\{(x, y, b, 0) : \quad x + y = a, \quad x,\, y \in [0, a]\Big\}.$$

$\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 1 - \frac{v}{b}$ and $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 1 - \frac{x}{a}$ are eigenvalues of the fixed points of the forms F_{11} and F_{12} respectively. By these definitions 2.1.1 , and 2.1.2 we see that all fixed points of the operator (8) are nonhyperbolic fixed points.

In section 3.2 for the operator V_1 and for arbitrarily initial point $s \in S^{2,2}$, we have studied [9] the trajectory $\{s^{(m)}\}_{m=0}^{\infty}$, where

$$s^{(m)} = V_1^m(s) = \underbrace{V_1(V_1(\ldots V_1}_{m}(s))\ldots).$$

Theorem 3.2.1.

For any initial point $s = (x, y, u, v) \in S^{2,2}$ the sequence

$$V_1^m(s) = V_1^m(x, y, u, v) = (x^{(m)}, y^{(m)}, u^{(m)}, v^{(m)})$$

is convergent and

$$\lim_{m\to\infty} x^{(m)} \cdot v^{(m)} = 0.$$

[9] Absalamov A.T., Rozikov U.A. Bulletin of the Institute of Mathematics. Vol.4, ź2, pp. 1-13. (2021)

Define the following sets:

$$T_0 = \left\{(x,y,u,v) \in S^{2,2}: \lim_{m\to\infty} x^{(m)} = \lim_{m\to\infty} v^{(m)} = 0 \right\},$$
$$T_1 = \left\{(x,y,u,v) \in S^{2,2}: \lim_{m\to\infty} v^{(m)} = 0, \quad \lim_{m\to\infty} x^{(m)} \in (0,a]\right\},$$
$$T_2 = \left\{(x,y,u,v) \in S^{2,2}: \lim_{m\to\infty} x^{(m)} = 0, \quad \lim_{m\to\infty} v^{(m)} \in (0,b]\right\}.$$

Corollary 3.2.1.

For any initial point $s = (x,y,u,v) \in S^{2,2}$ the ω-limit set $\omega(s)$ of the operator (8) consists a single point and

$$\omega(s) \in \begin{cases} \{(0,a,b,0)\} & \text{if } s = (x,y,u,v) \in T_0, \\ F_{12} & \text{if } s = (x,y,u,v) \in T_1, \\ F_{11} & \text{if } s = (x,y,u,v) \in T_2. \end{cases} \tag{9}$$

Definition 3.2.1.

An operator V_1 is called regular if for any initial point $s \in S^{2,2}$, the limit

$$\lim_{m \to \infty} V_1^m(s)$$

exists.

The following is a corollary of Theorem 3.2.1.

Corollary 3.2.2.

The operator (8) is regular.

A set E is called invariant with respect to the operator V_1 if $V_1(E) \subset E$.

Theorem 3.3.1.

The following set is invariant surface respect to the operator V_1

$$\Omega = \Big\{(x, y, u, v) \in S^{2,2} : \quad xv\Big[1 + (p_1 - 1)f(\frac{x}{x+y})\Big] = = \Big[\frac{x}{x+y} - f(\frac{x}{x+y})\Big](u+v)(x+y)\Big\}$$

where $f : [0,1] \to [0,1]$ is a smooth function which is the solution of the following iterative functional equation:

$$f(\alpha)\big(\alpha - f(\alpha)\big)(1-\alpha)[1 + (p_1 - 1)f(f(\alpha))] = = \alpha\big(f(\alpha) - f(f(\alpha))\big)[1 + (p_2 - 1)\alpha + (p_1 - p_2)f(\alpha)] \tag{10}$$

We would like to describe the sets T_0, T_1 and T_2 implicitly.
There are three cases for p_1, p_2.

$$\begin{aligned} &1. \quad p_1 = p_2 = 1, \\ &2. \quad p_1 > 1 > p_2 \geq 0, \\ &3. \quad p_2 > 1 > p_1 \geq 0. \end{aligned} \tag{11}$$

where

$$p_1 = \frac{\sigma_1}{a}, \qquad p_2 = \frac{\sigma_2}{b}.$$

when $p_1 = p_2 = 1$ i.e. when $\sigma_1 = a$, $\sigma_2 = b$ we have

$$f(\alpha)\big(\alpha - f(\alpha)\big)(1-\alpha) = \alpha\big(f(\alpha) - f(f(\alpha))\big) \tag{12}$$

Theorem 3.3.2.

The solutions of the functional equation (12) are

$$f(\alpha) = 0, \quad f(\alpha) = \alpha \quad \text{and} \quad f(\alpha) = \theta\alpha - \alpha^2$$

where θ is an arbitrary constant.
In particular,

$$\Omega_\theta = \Big\{(x, y, u, v) \in S^{2,2} : \quad \frac{v}{u+v} = \frac{x}{x+y} + 1 - \theta\Big\}$$

is a one parametric family of invariant surface respect to the operator (8)

Lemma 3.3.2.

For the Ω_θ invariant surface it holds that

$$\bigcup_{\theta\in[0,1)} \Omega_\theta = T_2 = \Big\{(x,y,u,v) \in S^{2,2} : yv > xu\Big\},$$

$$\bigcup_{\theta\in(1,2]} \Omega_\theta = T_1 = \Big\{(x,y,u,v) \in S^{2,2} : yv < xu\Big\},$$

$$\Omega_1 = T_0 = \Big\{(x,y,u,v) \in S^{2,2} : yv = xu\Big\}$$

and that

$$\Omega_{\theta_1} \cap \Omega_{\theta_2} = \emptyset \ \text{ for any } \ \theta_1 \neq \theta_2.$$

Thus it suffices to study the dynamical system on each invariant surfaces Ω_θ, we have the following

Theorem 3.3.4.

The following assertions hold

(i) For any initial point $s = (x, y, u, v) \in T_0$, we have

$$\lim_{m\to\infty} V_1^m(s) = (0; a; b; 0).$$

(ii) If $\theta \in (1, 2]$ then for any initial point $s = (x, y, u, v) \in \Omega_\theta$ the following holds

$$\lim_{m\to\infty} V_1^m(s) = (a(\theta - 1); a(2 - \theta); b; 0).$$

(iii) If $\theta \in [0, 1)$ then for any initial point $s = (x, y, u, v) \in \Omega_\theta$ the following holds

$$\lim_{m\to\infty} V_1^m(s) = (0; a; b\theta; b(1 - \theta)).$$

Corollary 3.3.1.

The operator (8) has infinitely many fixed points and for each such fixed point there are disjointed trajectories which converge to those fixed points, i.e. any trajectory started at a point of the invariant set converges to the corresponding fixed point. Thus there is one-to-one correspondence between such invariant sets and the set of fixed points.

Conjecture.

If $p_1 > 1 > p_2 \geq 0$ (or $p_2 > 1 > p_1 \geq 0$) then for each fixed point $p \in \mathrm{Fix}(V_1)$ there exists unique invariant surface $\Gamma_p \subset S^{2,2}$, such that for any initial point $s^{(0)} \in \Gamma_p$ the limit of its trajectory (under operator (8)) converges to the fixed point p. Moreover,

$$\bigcup_{p \in \mathrm{Fix}(V_1)} \Gamma_p = S^{2,2}.$$

Let $s = (x^{(0)}, y^{(0)}, u^{(0)}, v^{(0)}) \in S^{2,2}$ be an initial state, i.e. the probability distribution on the set of female and male types.
The following are interpretations of our results:

- The set of all fixed points is subset of the boundary of $S^{2,2}$ means that at least one type of female or male in future of population will surely disappear.
- The existence of invariant surfaces means that if states of the population initially satisfied a relation (described the invariant set) then the future of the population remains in the same relation.
- Regularity of the operator means that for any initial state of the population we can explicitly determine its limit (final) state.
- For any $s \in T_0$ as time goes to infinity the type 1 of female and type 2 of males will disappear (die).
- For any $s \in T_1$ as time goes to infinity the type 2 of males will disappear.
- For any $s \in T_2$ as time goes to infinity the type 1 of females will disappear.

Part I. Articles

1) Absalamov A.T. Asymptotical behavior of trajectories for an evolution operator.Uzbek Mathematical Journal. No 4. p. 4-11. (2019)
2) (IF=1.0) Absalamov A.T., Rozikov U.A. The dynamics of gonosomal evolution operators. Journal of Applied Nonlinear Dynamics. V.9., N.2, p.247-257. (2020)
3) Absalamov A.T. On the eigenvalues of a gonosomal evolution operator. Uzbek Mathematical Journal. No 4, p. 4-10. (2020)
4) (IF=0.60) Absalamov A.T. The global attractiveness of the fixed point of a gonosomal evolution operator. Discontinuity Nonlinearity and Complexity. V.10., N.1, p.143-149. (2021)
5) Absalamov A.T., Rozikov U.A. A regular gonosomal evolution operator with uncountable set of fixed points. Bulletin of the Institute of Mathematics. Vol.4, N.2, pp. 1-13. (2021)

Part II. Abstracts in conference proceedings

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Thank you for your attention!