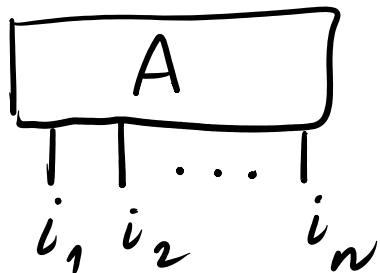


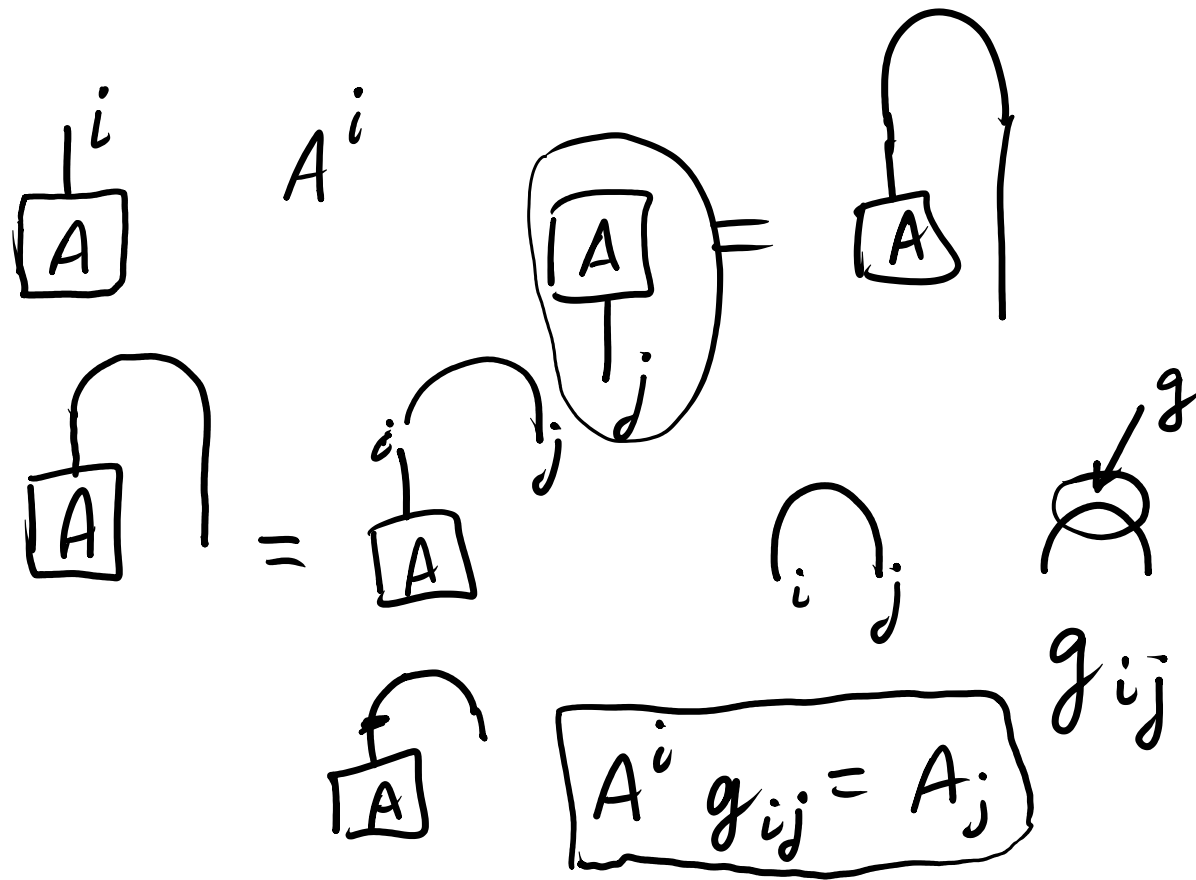
Лекция 2 по курсу "Квантовые тензорные сети"
(14.09.21)

$A_{i_1 \dots i_n}$ — тензор порядка n
 n -мн. форма
 n -мерн. матрица
табл



$$i \downarrow_k = \bigcup_j i \downarrow_k = g^{ij} g_{jk} = \delta^i_k$$

$$\cup = g^{ij}$$

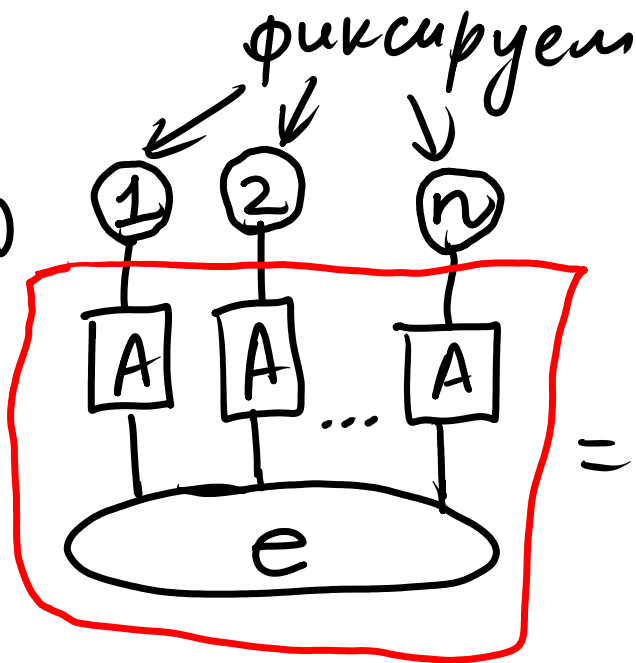




$$\boxed{A}^i_j = A^i_j$$

$$\det A =$$

$\uparrow$ мсно



=

$$F^{ij}$$



$$\frac{1}{2} F_{ij} e^{ijkl} = \tilde{F}^{kl}$$

A diagram showing a square labeled F with two vertical lines extending upwards from its top edge.

$$\frac{1}{2} \boxed{F}^{ij} \boxed{e}_{ijkl} = \boxed{\tilde{F}}^{kl}$$

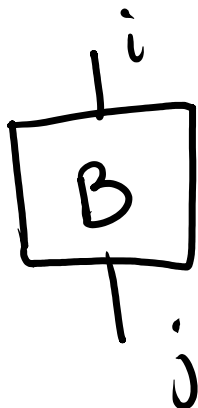


$$e^{i_1 \dots i_n}$$

$$\boxed{A}^{i_1 \dots i_n} = \boxed{A}^{i_1 \dots i_n} = \boxed{A}^{i_1 \dots i_n}$$

$$A^{i_1} \dots A^{i_n}$$

$$\text{tr } A = \boxed{A} = A^i_i$$



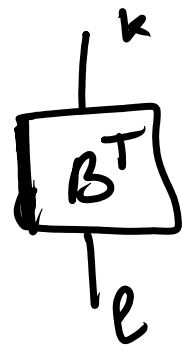
$$B^i_j$$

$$(B^T)^k_e = B^e_k$$

$$\boxed{B} \stackrel{\text{g-ebkangoh}}{=} (B^T)^k_e$$

A square box labeled 'B'. A vertical line enters the top of the box from above, labeled with the index 'e'. A vertical line exits the bottom of the box, labeled with the index 'k'. A curved arrow points from the text 'g-ebkangoh' to the box.

$$= (B^T)^k_e =$$



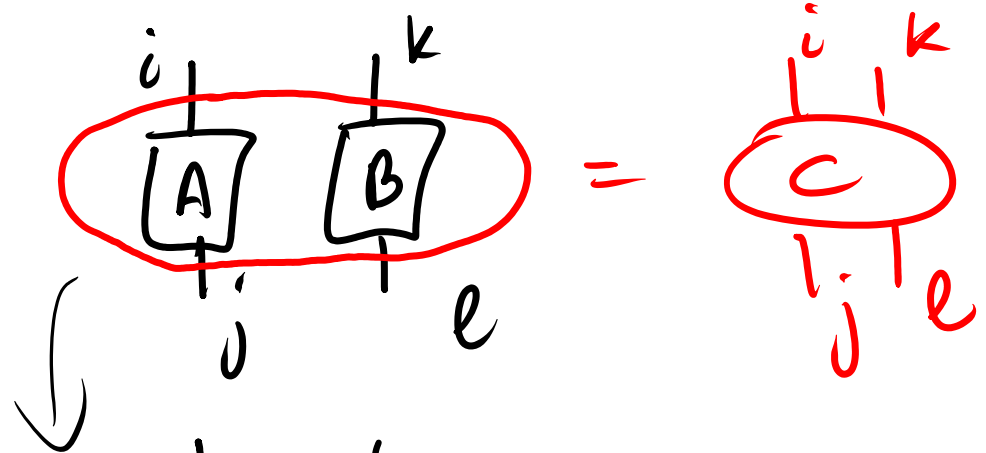
$$\text{tr } (AB) = \boxed{A \overline{B}} = \boxed{A} \boxed{B} = \boxed{A} \boxed{B^T} = \boxed{B \overline{A}} = \text{tr } (BA)$$

The diagrammatic proof shows the following steps:

- $\boxed{A \overline{B}}$: A square box containing 'A' on top and 'B' on the bottom, with a single line entering from the top and exiting from the bottom.
- $= \boxed{A} \boxed{B}$: Two square boxes, 'A' on top and 'B' on the bottom, connected by a single line.
- $= \boxed{A} \boxed{B^T}$: Two square boxes, 'A' on top and 'B^T' on the bottom, connected by a single line. A red circle highlights the 'B' box in the previous step, with an arrow pointing to this step.
- $= \boxed{B \overline{A}}$: A square box containing 'B' on top and 'A' on the bottom, with a single line entering from the top and exiting from the bottom.
- $= \text{tr } (BA)$: The final result, which is the trace of the product BA.

Тензорное произведение (кронекер)

$$A^i_j B^k_l = C^{i,k}_{j,l} = (A \otimes B)^{i,k}_{j,l}$$



$$(A \otimes B)^+ = A^+ \otimes B^+$$

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix}$$

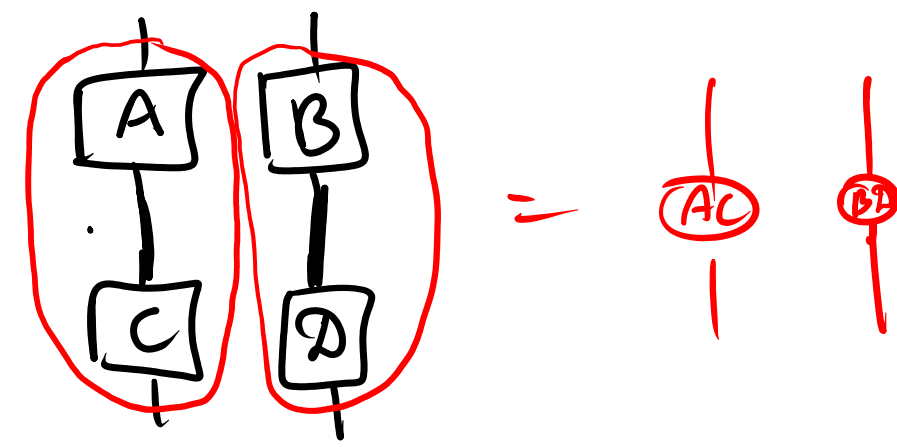
$$A \otimes B = \begin{array}{|c|} \hline \boxed{A} \\ \hline \boxed{B} \\ \hline \end{array} = \underbrace{(A \otimes I) \cdot (I \otimes B)}$$

$$B = \begin{pmatrix} b_{11} & \dots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{m1} & \dots & b_{mm} \end{pmatrix}$$

$$A \otimes B = \begin{pmatrix} a_{11}B & \dots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \dots & a_{mn}B \end{pmatrix}$$

$n m \times n m$

$$\underbrace{(A \otimes B)}_{3 \times 3 \times 4 \times 4} \cdot \underbrace{(C \otimes D)}_{2 \times 2 \times 6 \times 6} = \underbrace{(AC)}_{12 \times 12} \otimes \underbrace{(BD)}_{12 \times 12}$$



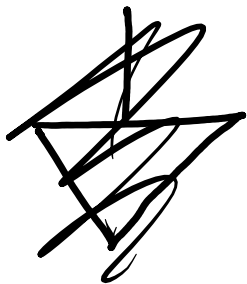
если использов.
определений

$$A \otimes B = C$$

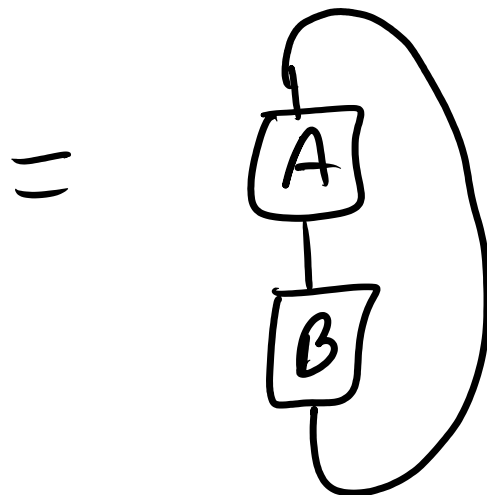
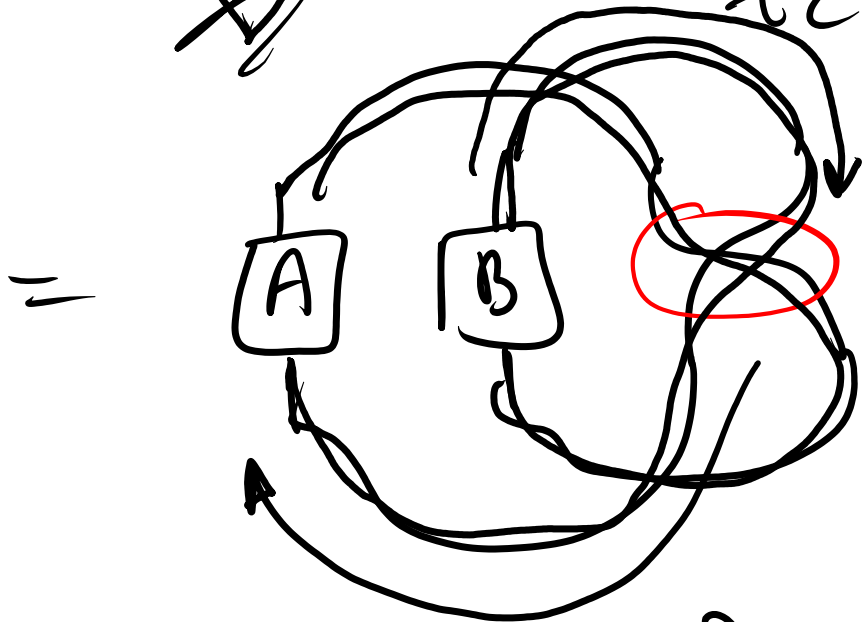
$$C_{ij}^k = A_{ij}^i B_{je}^k$$

$$\text{tr}(A \otimes B) = \text{tr}(A \otimes B \cdot \text{SWAP}) = \text{tr}(A) \cdot \text{tr}(B)$$

$$\text{tr} C = \sum_{i=j} \sum_{k=e} C_{je}^{ik}$$

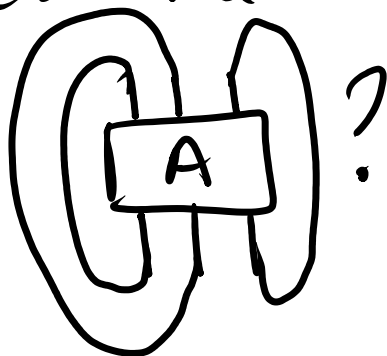
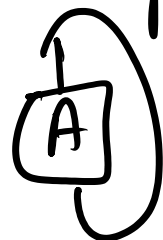
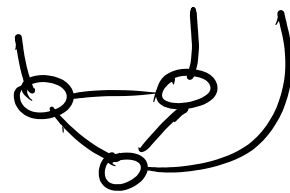


$$\text{tr} (A \otimes B \cdot \text{SWAP}) =$$



$$= \text{tr} (AB).$$

1) Как опред. след для тензора?



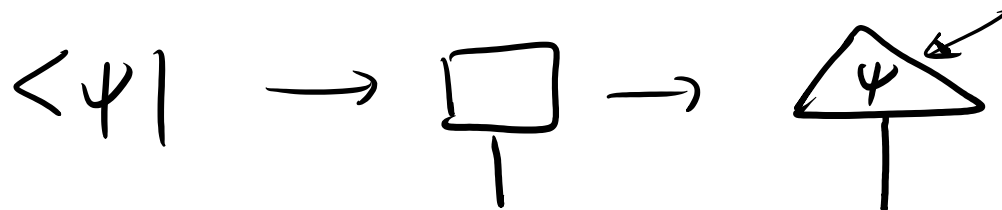
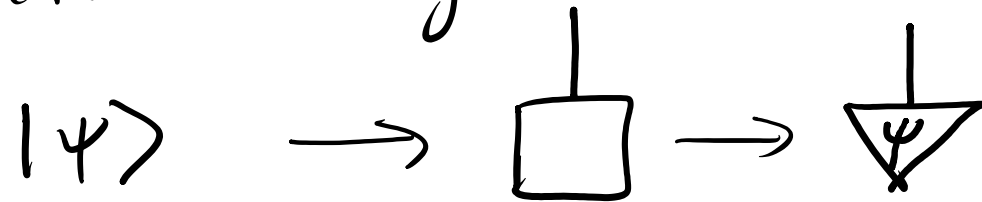
$\text{tTr} = \text{tensor trace}$

2)



$$= \text{tTr} (B \otimes B^+).$$

Квантовый мультиметр



$\begin{array}{c} \downarrow \\ \psi \end{array} \begin{array}{c} \psi \\ \uparrow \end{array} = |\psi\rangle\langle\psi| = \rho_\psi$

$\begin{pmatrix} \begin{array}{c} \psi \\ \uparrow \end{array} \\ \begin{array}{c} \downarrow \\ \psi \end{array} \end{pmatrix} = \langle\psi|\psi\rangle$

$\begin{array}{c} \psi \\ \uparrow \end{array} \begin{array}{c} \downarrow \\ \psi \end{array} = \overline{\begin{pmatrix} \begin{array}{c} \psi \\ \uparrow \end{array} \\ \begin{array}{c} \downarrow \\ \psi \end{array} \end{pmatrix}}$

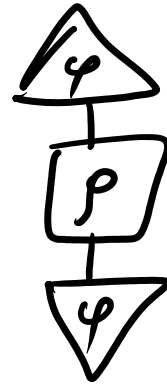
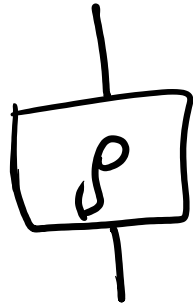
$\begin{array}{c} \psi \\ \uparrow \end{array} \begin{array}{c} \downarrow \\ \psi \end{array} = \langle\psi|\psi\rangle$

$$\cup^i_j = \text{Diagram of a cup with indices } i \text{ and } j = \text{Diagram of a triangle with two vertical lines} = \sum_i |i\rangle \otimes |i\rangle \leftarrow \begin{array}{l} \text{макс.} \\ \text{перен.} \\ \text{состояние} \end{array} = |\psi_+\rangle$$

$$\cap = \sum_i \langle i| \otimes \langle i| = \langle \psi_+|$$

$$\text{tr } A = \text{Diagram of a square with a box labeled } A \text{ and a loop} = \langle \psi_+ | (A \otimes I) | \psi_+ \rangle$$

$$\text{т. н. н. } \langle \varphi | \rho | \varphi \rangle \geq 0$$

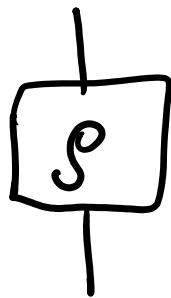


$$\geq 0$$

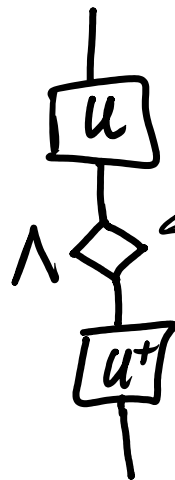
$$\forall \varphi$$

Спектр. разл. для ρ

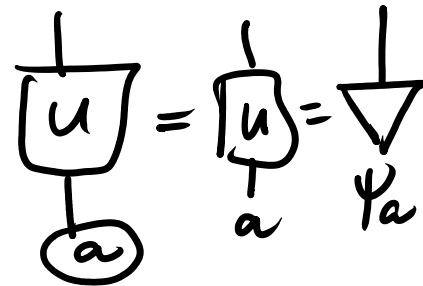
$$\rho = \sum_a \lambda_a |\varphi_a\rangle \langle \varphi_a|$$



$$=$$



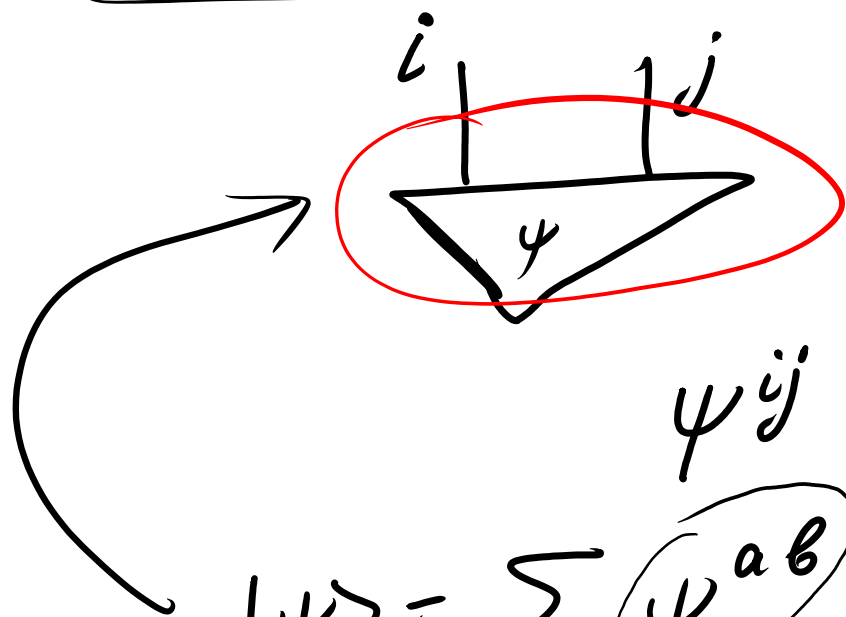
$$\lambda^a_b = \lambda_a \delta^a_b$$



Сост. системы:

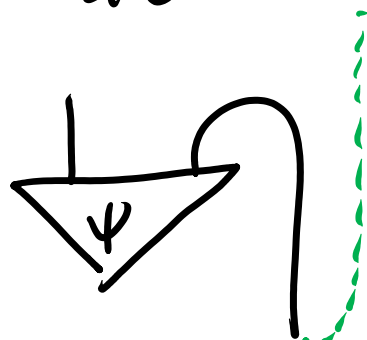
$|\{i\}_i|$ — изм. по индексу i

$|\{j\}_j|$



$$|\psi\rangle = \sum_{a,b} \psi^{ab} |a\rangle \otimes |b\rangle$$

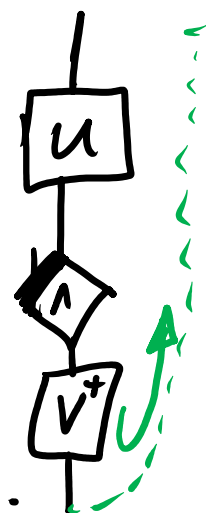
$\psi^{ab} \in \mathbb{C}$



$$= \psi^a \otimes \bar{a}$$

$$M = U \Lambda V^\dagger$$

цикл. разложение



↑
разложение
Шмидта
для ψ .
ранг Λ

$$U^\dagger U = I, \quad V^\dagger V = I.$$

$$\text{rank } \Lambda = 1$$

Если $\text{---} \diamond \text{---} = \text{---} \hookrightarrow \triangleleft \text{---}$

$\text{---} \triangle \text{---} = \text{---} \triangle \text{---} \text{---} \triangle \text{---}$ — сепараб.

Интеграл перенормировки

$$S(\rho_1) = S \left(\text{---} \triangle \text{---} \right) = S \left(\text{---} \square \text{---} \right) = S \left(\text{---} \square \text{---} \right)$$

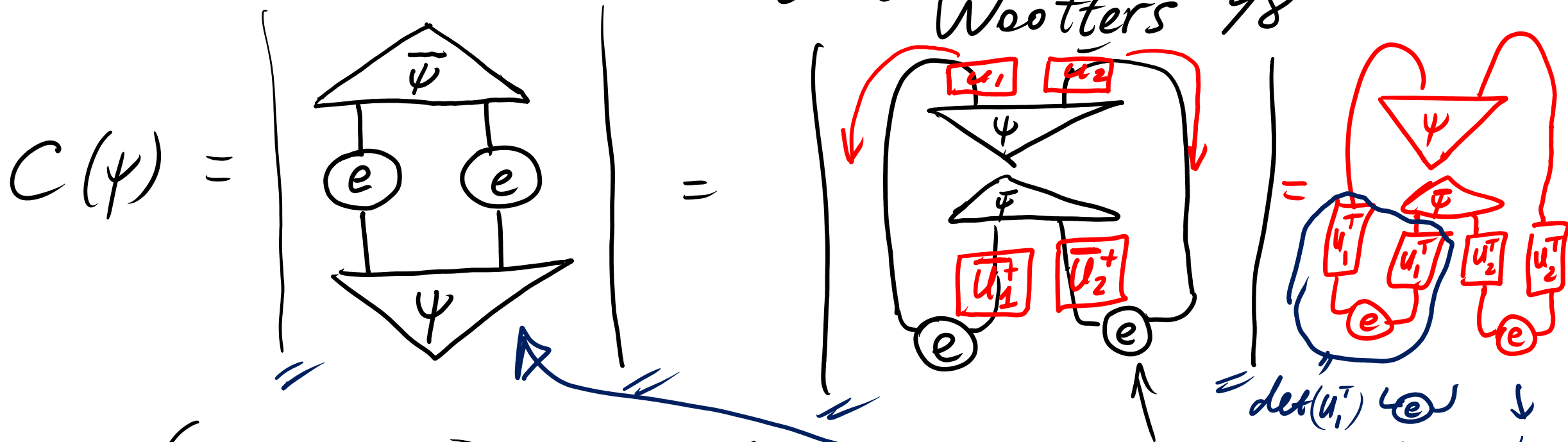
$$S(\rho) = -\text{tr}(\rho \log \rho) =$$

$$= - \sum_k \mu_k \log \mu_k$$

с.з.р.

$$= S \left(\text{---} \diamond \text{---} \right) = S(\rho_2).$$

Concurrence — мера перепутанности
 двухкубитных состояний
 Wootters '98



$$C(u_1 \otimes u_2 | \psi) = C(|\psi\rangle)$$

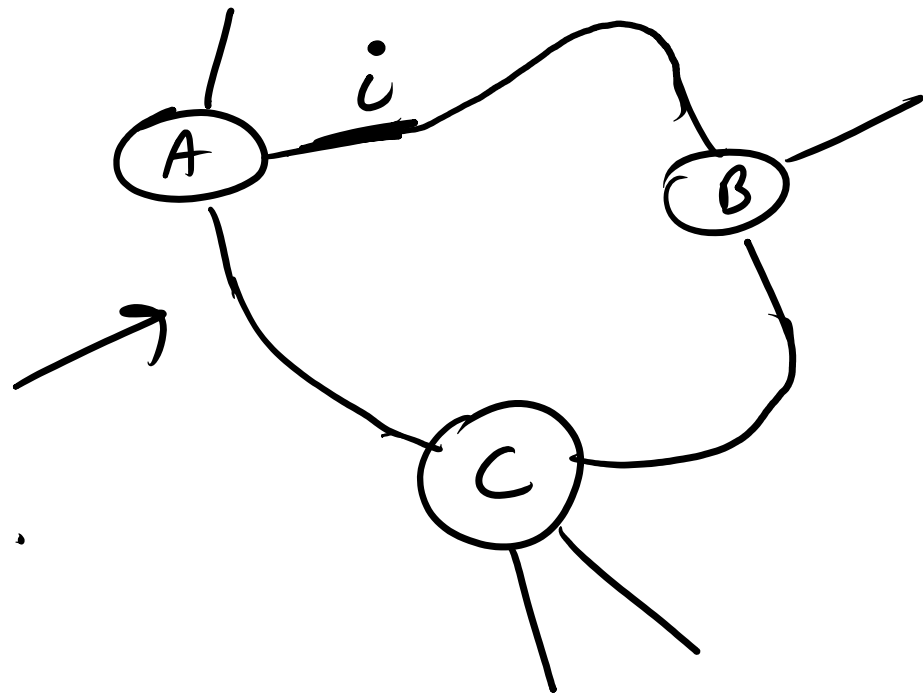
$\forall$ унитар. u_1 и u_2 .

символ Л.-У.
 для $n=2$

$$e^{12} = 1 \quad e^{11} = e^{22} = 0$$

$$e^{21} = -1$$

Дерево



не
тенз.
дерево.

Тензорная сеть — тензор, полученный из тензоров $A, B, C, \dots$ путём взятия свёрток между ними.

Архитектура тензорной сети = "~~зада~~ форма тензоров" + "граф свёрток".

Тензорные деревья — тенз. сети без циклов.

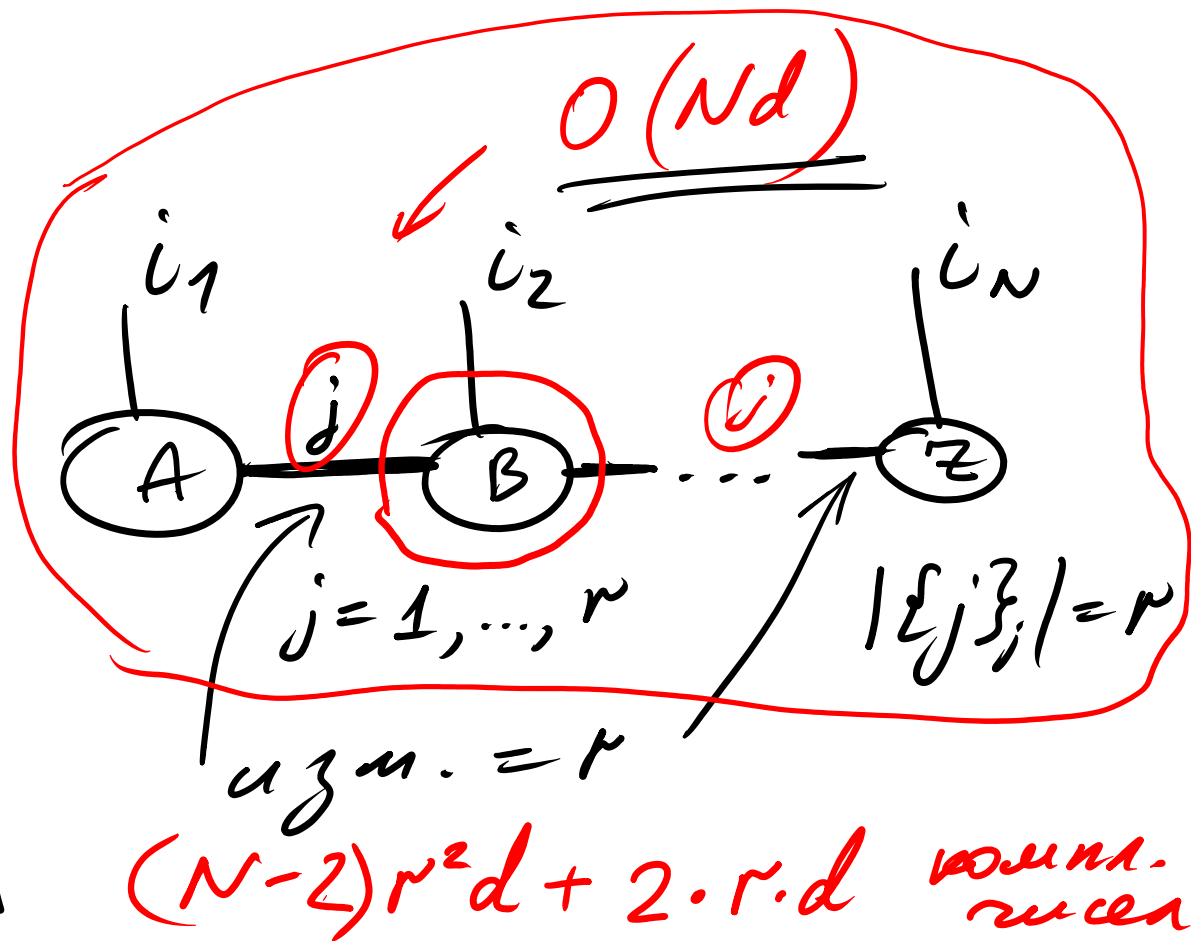
Чистое Состояние из N d -уровневых систем

$$|\psi\rangle = \sum_{i_1, \dots, i_N=1}^d \psi^{i_1 \dots i_N} |i_1\rangle \otimes \dots \otimes |i_N\rangle$$

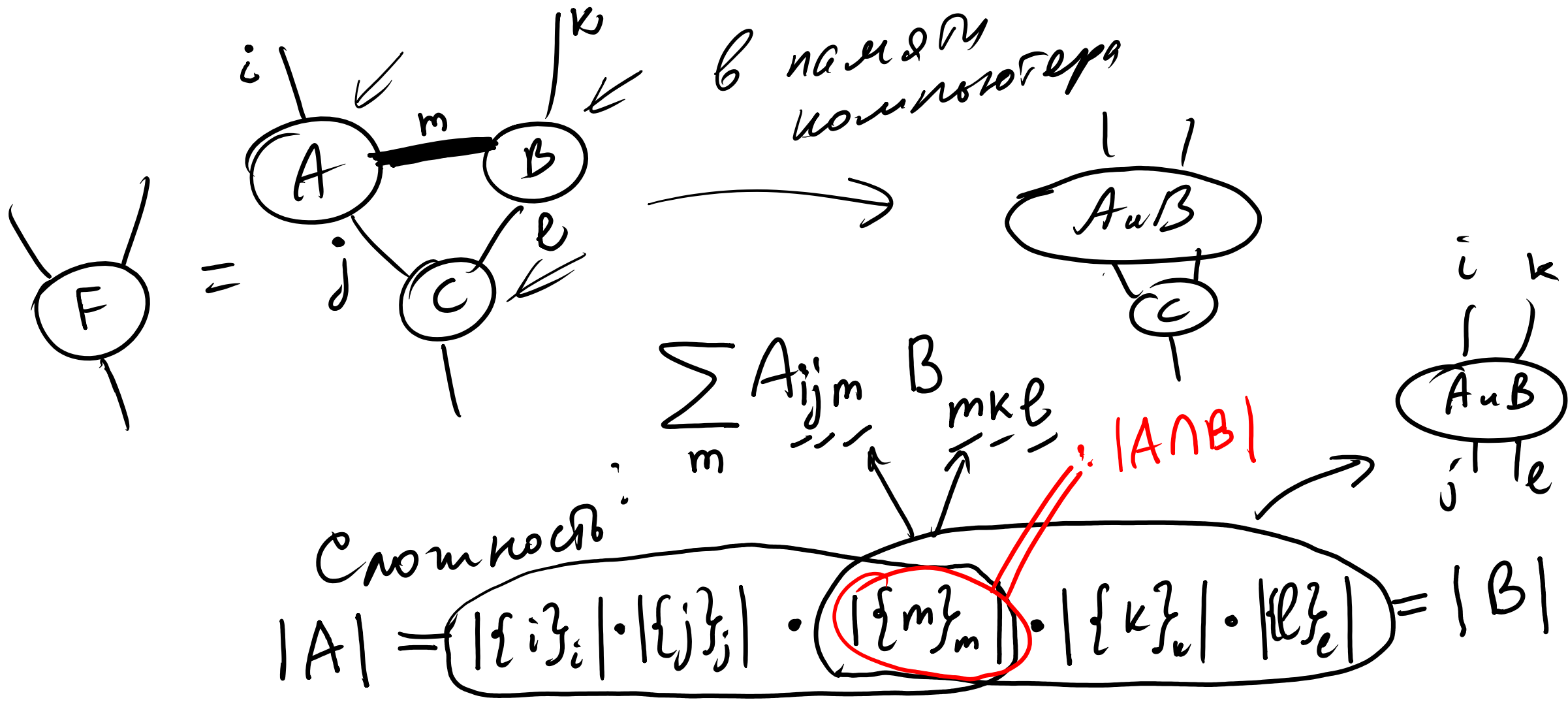


d^N комп. чисел
задают ψ

$O(d^N)$



Вычислительная сложность свёртки



$$\text{cost}(A; B) = \frac{|A| \cdot |B|}{|A \cap B|}$$

