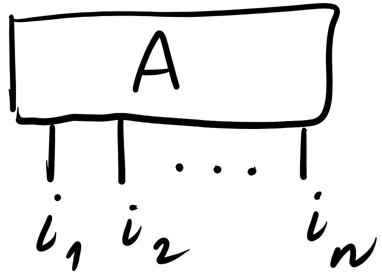
Лекция 2 по курсу "Квантовые тензорные сети" (14.09.21)

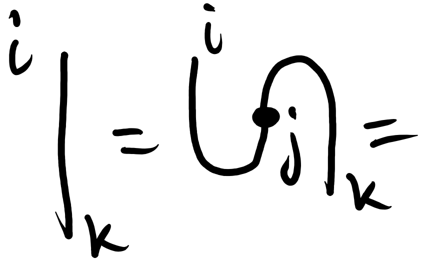
$A_{i_1 \dots i_n}$ — тензор порядка n

n – лин. форма

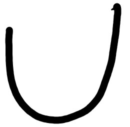
n-мерн. матрица

табл

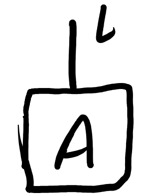




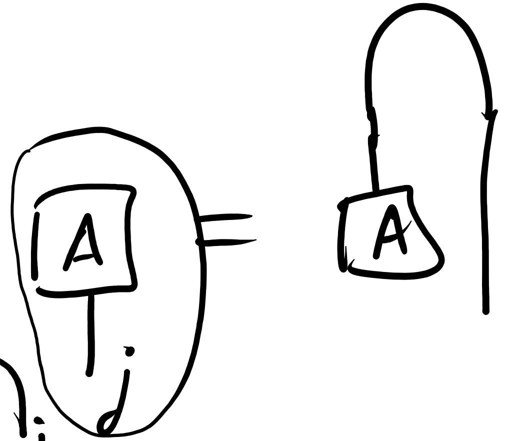
$= g^{ij} g_{jk} = \delta^i_k$

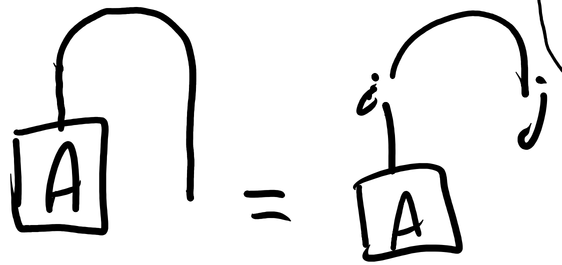


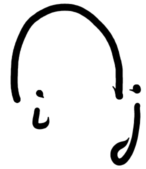
$= g^{ij}$

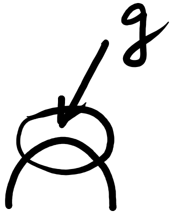


A^i

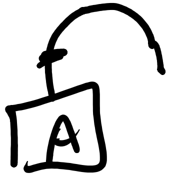




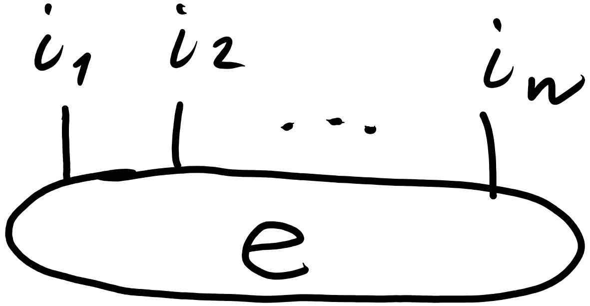


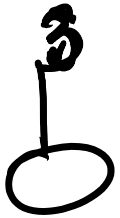


g_{ij}



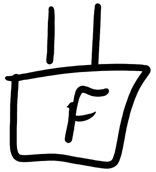
$\boxed{A^i g_{ij} = A_j}$

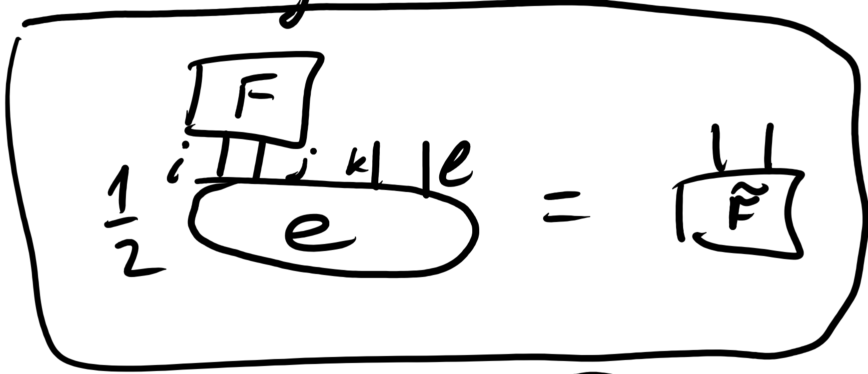


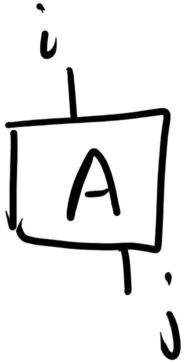


F^{ij}

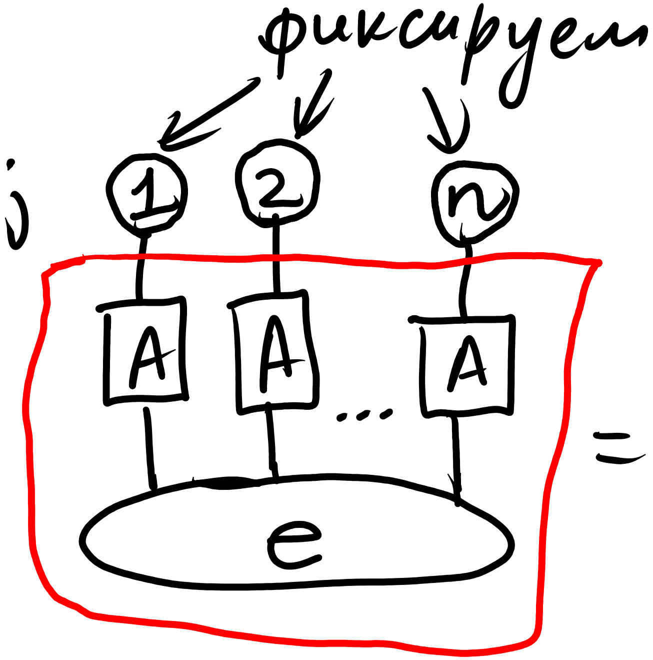
$\frac{1}{2}F_{ij}\, e^{ijkl} = \tilde{F}^{kl}$





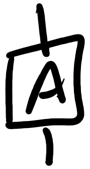


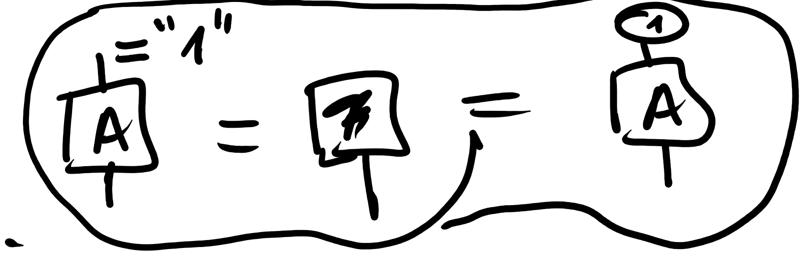
$= A^i_j$

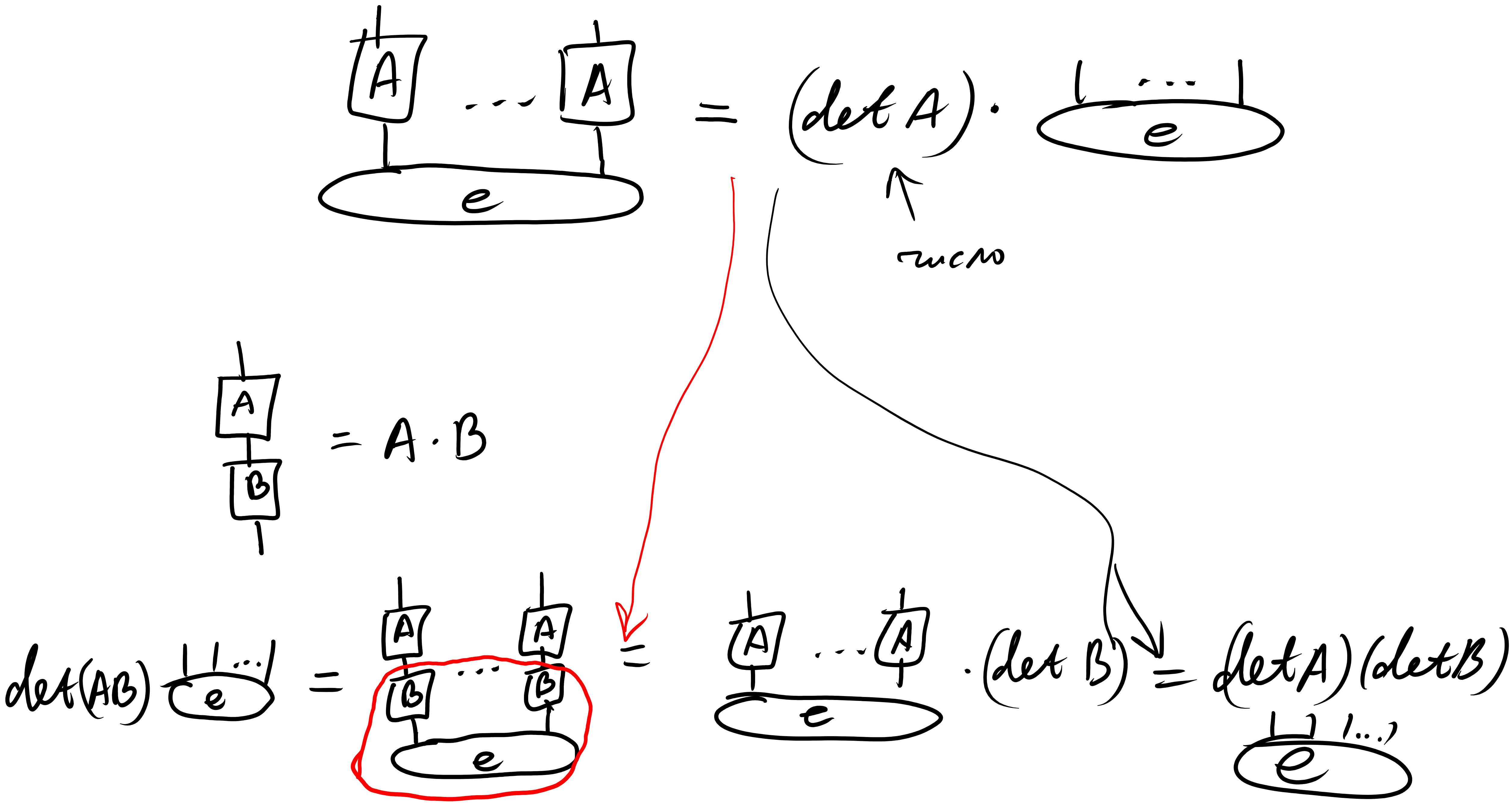
det A = 

$= e^{i_1\dots i_n} A^1_{i_1}\dots A^n_{i_n}$

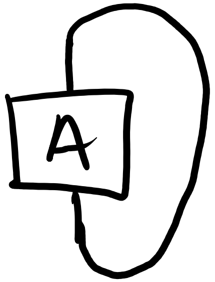
число

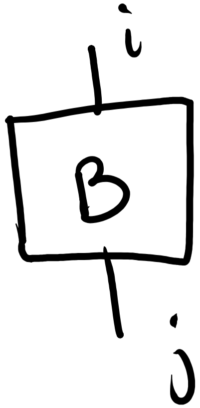




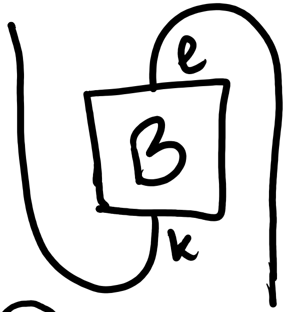


A
A
e
= (det A) ·
e
число
A
B
= A · B
det(AB)
e
=
A
A
B
B
e
=
A
A
e
· (det B) = (det A)(det B)
e

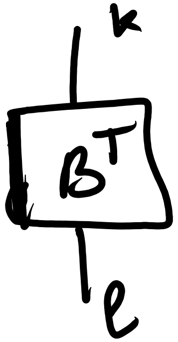
$\operatorname{tr} A =$  $= A^i{}_i$

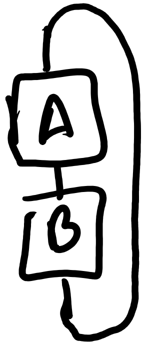


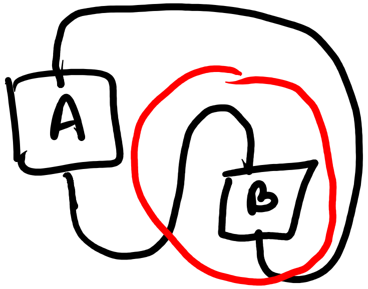
$B^i{}_j$ $\qquad (B^T)^k{}_e = B^e{}_k$

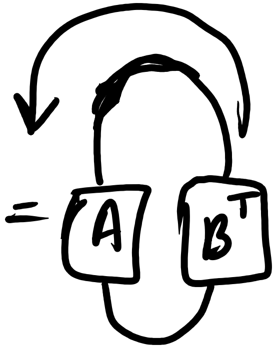


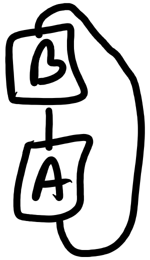
← g- евклидова

$= (B^T)^k{}_e =$ 

$\operatorname{tr}(AB) =$ 

$=$ 

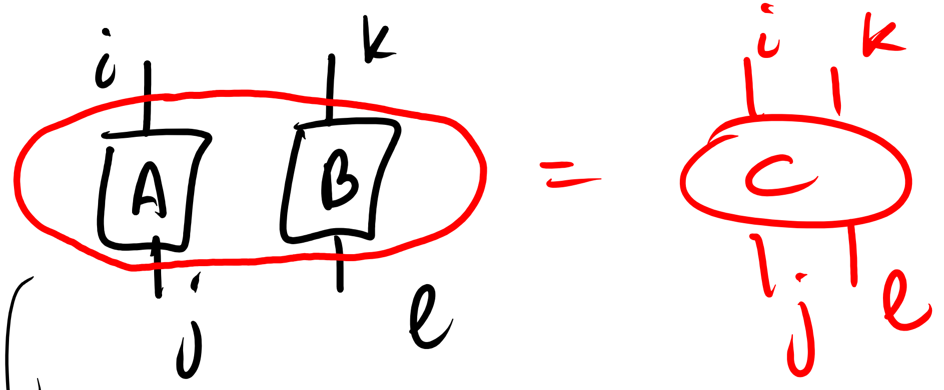
$=$ 

$=$ 

$= \operatorname{tr}(BA)$

Тензорное произведение (кронекерово

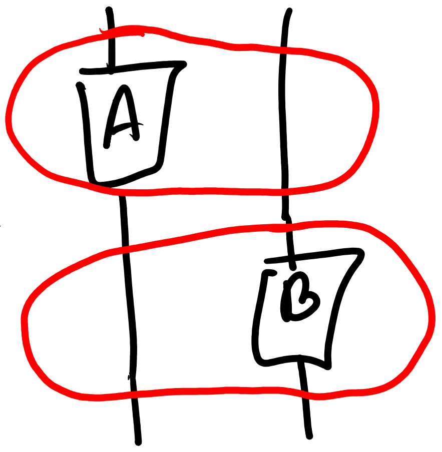
$$A^{i}{}_{j} B^{k}{}_{l} = C^{i\,k}{}_{j\,l} = (A\otimes B)^{ik}{}_{jl}$$



$$(A\otimes B)^{+} = A^{+}\otimes B^{+}$$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

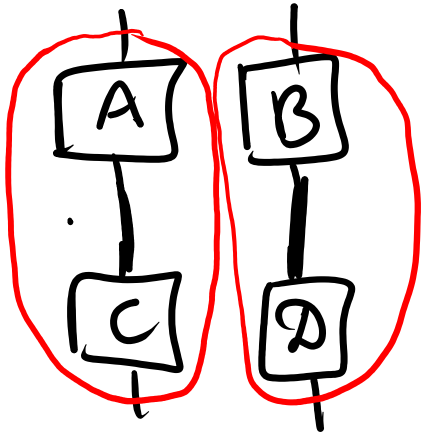
$$B = \begin{pmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mm} \end{pmatrix}$$

$A\otimes B =$  $= \underline{(A\otimes I)\cdot(I\otimes B)}$

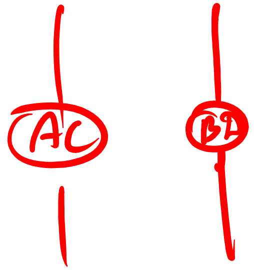
$$\underset{nm\times nm}{A\otimes B} = \begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{n1}B & \cdots & a_{nn}B \end{pmatrix}$$

$$(\underbrace{\underset{\cancel{3\times3}}{A}\otimes\underset{\cancel{4\times4}}{B}}_{\cancel{12\times12}})\cdot(\underbrace{\underset{\cancel{2\times2}}{C}\otimes\underset{\cancel{6\times6}}{D}}_{\cancel{12\times12}}) = (AC)\otimes(BD)$$

если произв. определена

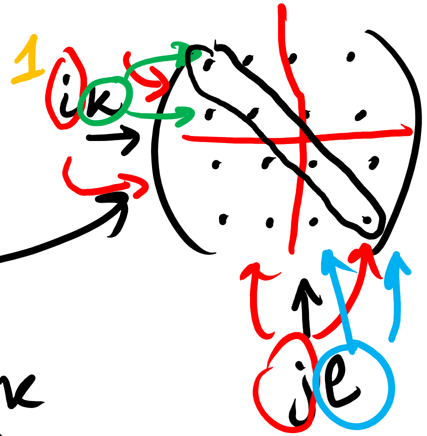


=



$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

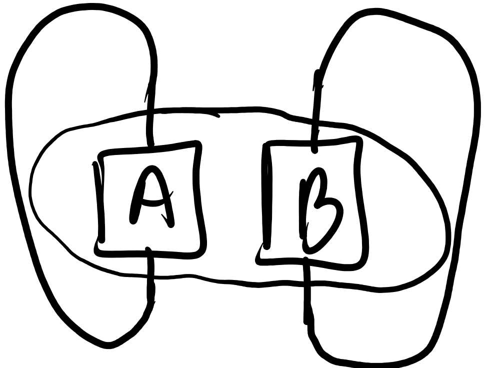
$$A\otimes B = \underset{4\times4}{C}$$



$$C^{ik}_{jl} = A^i_j B^k_l$$

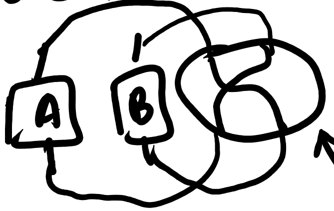
$$\operatorname{tr} C = \sum_{\substack{i=j\\k=l}} C^{ik}_{jl}$$

$\operatorname{tr}(A\otimes B) =$



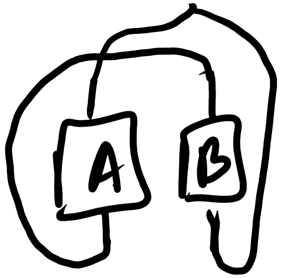
$= \operatorname{tr} A \cdot \operatorname{tr} B.$

$\operatorname{tr}(A\otimes B\cdot \mathrm{SWAP}) =$

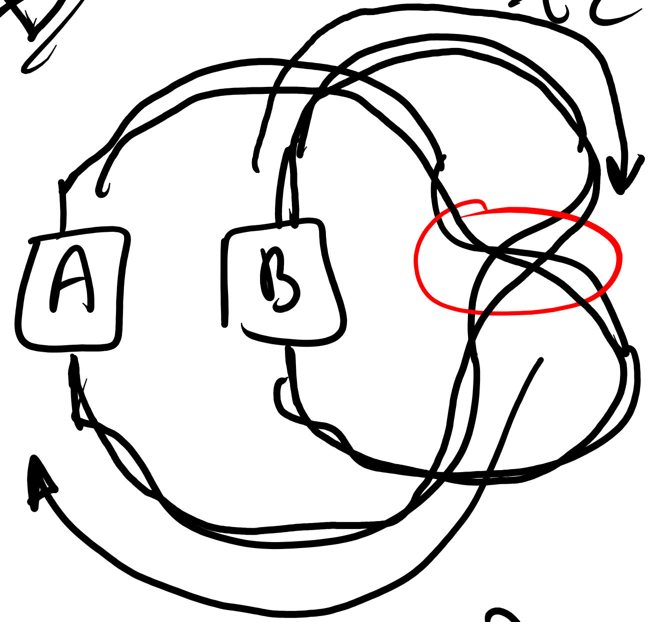
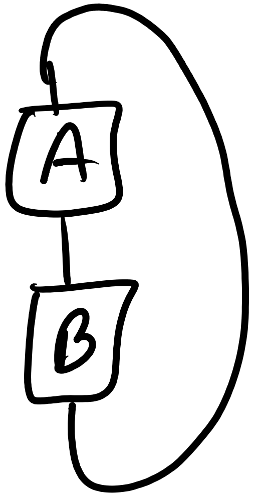


SWAP

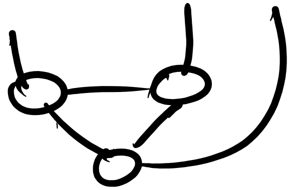
$=$

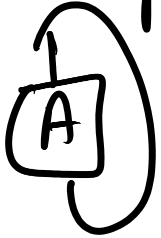


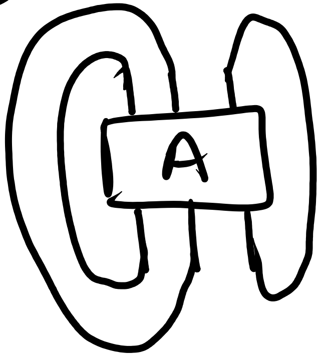
$\mathrm{tr}(A \otimes B \cdot \mathrm{SWAP}) =$

$=$  $=$  $= \mathrm{tr}(AB).$

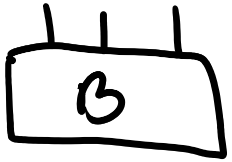
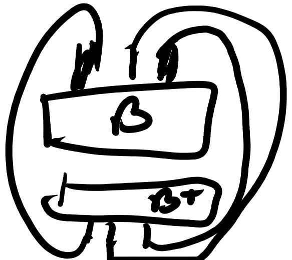
1) Как опред. след для тензора?



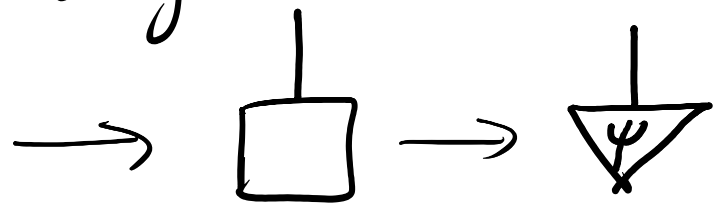


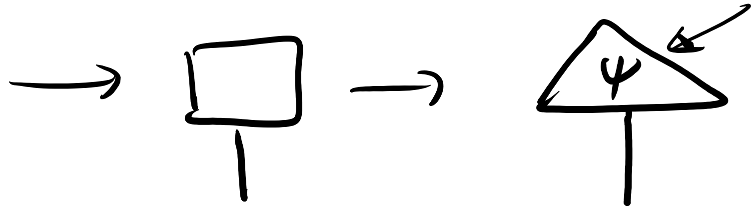
 ?

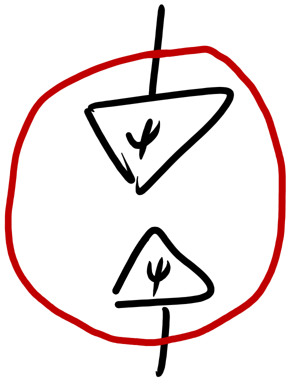
tTr = tensor trace

2)  $\longrightarrow$  $= \mathrm{tTr}(B \otimes B^{+}).$

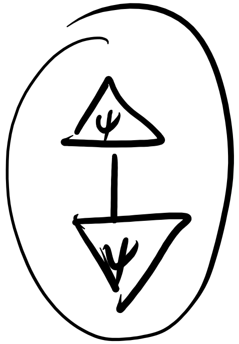
Квантовый случай

$|\psi\rangle \longrightarrow$ 

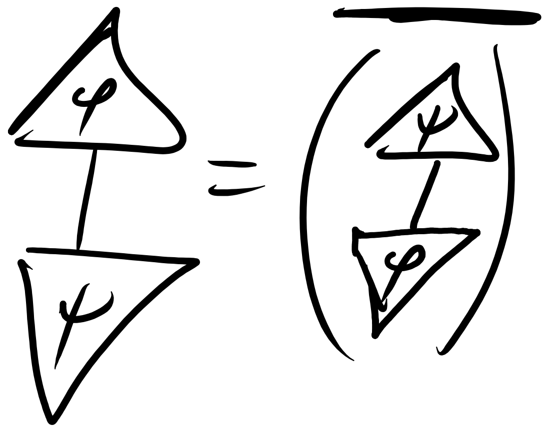
$\langle\psi| \longrightarrow$ 

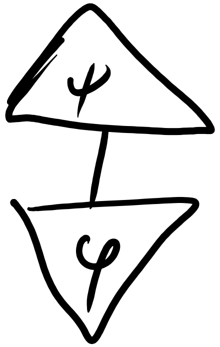


$= |\psi\rangle\langle\psi| = \rho_\psi$

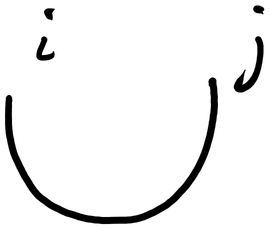


$= \langle\psi|\psi\rangle$

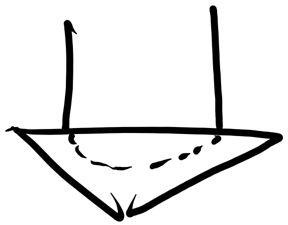




$= \langle\psi|\varphi\rangle.$

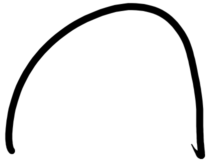


$=$



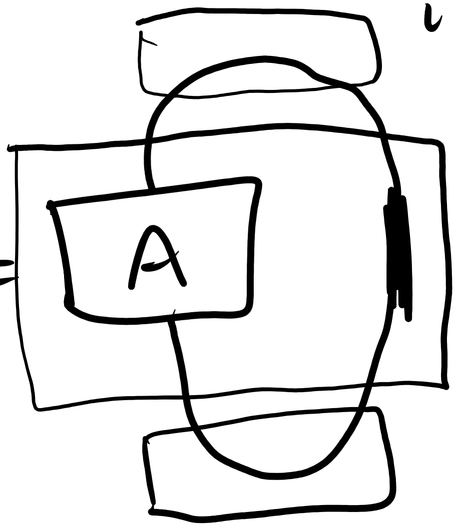
$= \sum_i |i\rangle \otimes |i\rangle$ ← макс. переп. состояни

$= |\Psi_+\rangle$



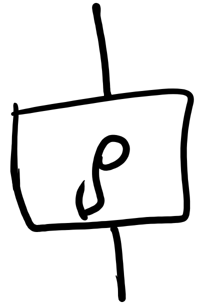
$= \sum_i \langle i| \otimes \langle i| = \langle \Psi_+|$

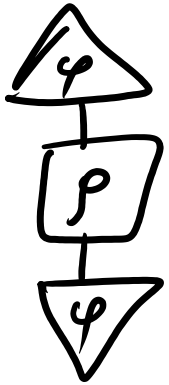
$\mathrm{tr} A =$



$= \langle \Psi_+ | (A \otimes I) | \Psi_+ \rangle$

М. п-ти $\langle\varphi|\rho|\varphi\rangle \geq 0$

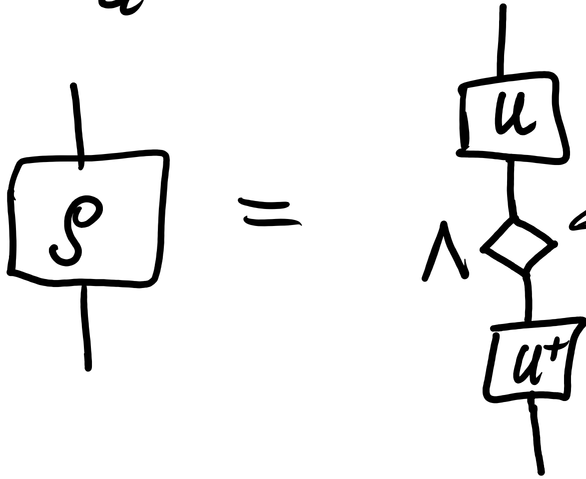




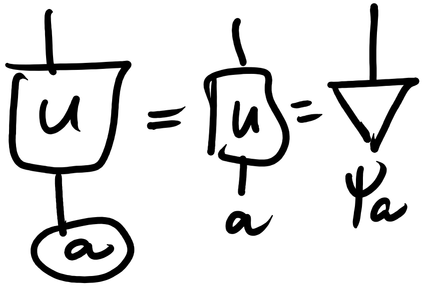
$\geq 0 \quad \forall \varphi$

Спектр. разл. для ρ

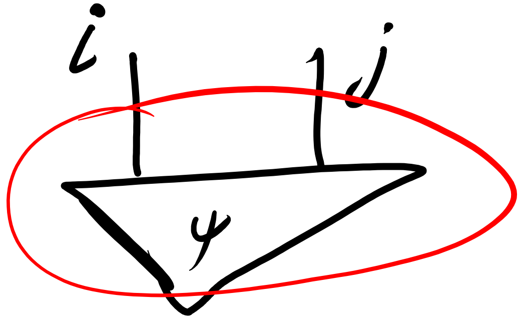
$$\rho = \sum_a \lambda_a |\psi_a\rangle\langle\psi_a|$$



$$\Lambda^a{}_b = \lambda_a \delta^a{}_b$$



Сост. системы:



$|\{i\}_i|$ — изм. по индексу i

$|\{j\}_j|$

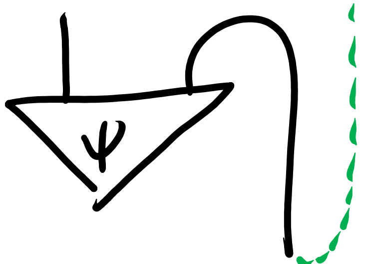
ψ^{ij}

$\psi^{ab} \in \mathbb{C}$

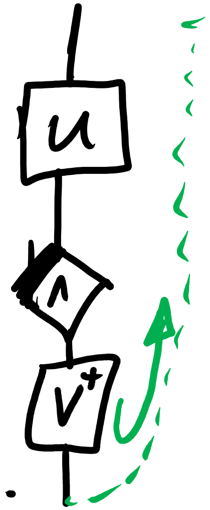
$$|\psi\rangle = \sum_{ab} \psi^{ab} |a\rangle \otimes |b\rangle$$

разложение Шмидта для ψ.

ранг Λ



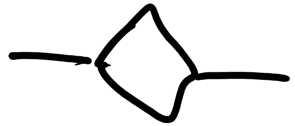
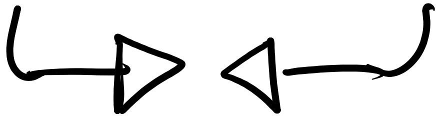
$= \psi^{a}{}_{b} =$



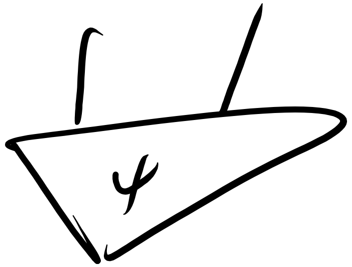
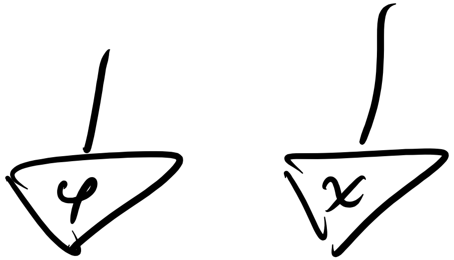
$\boxed{M = U \Lambda V^{+}}$ ← сингул. разложение

$U^{+}U = I, \quad V^{+}V = I.$

$\operatorname{rank} \Lambda = 1$

Если  $=$ 

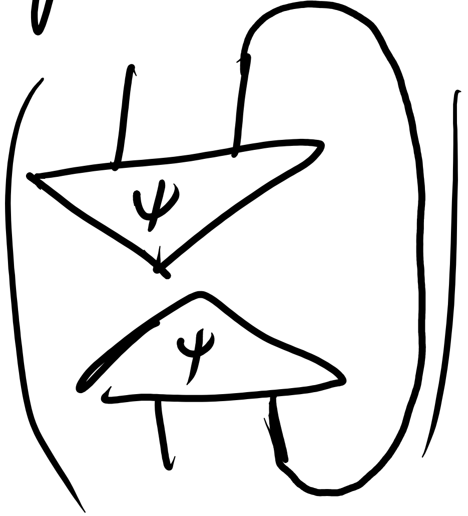
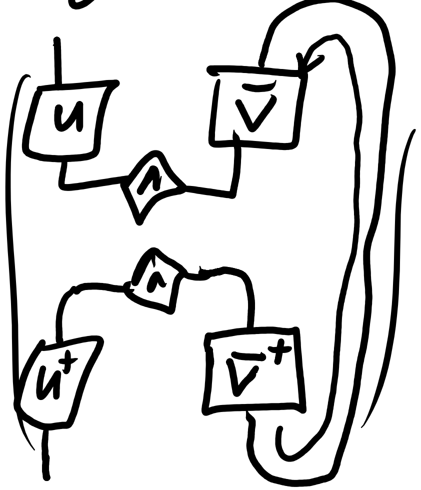
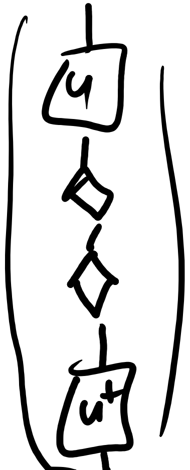
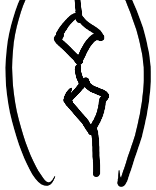
Λ

 $=$  — сепараб.

Энтропия перепутанности

$$S(\rho) = -\operatorname{tr}(\rho \log \rho) = -\sum_k \mu_k \log \mu_k$$

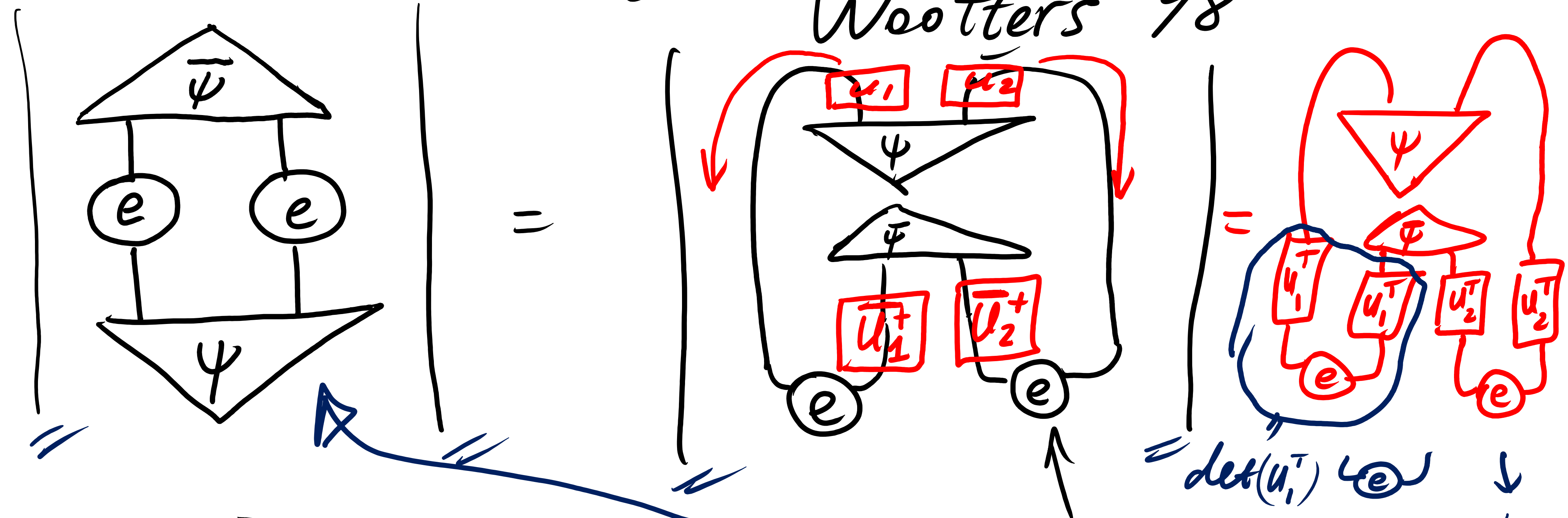
с.з. ρ.

$S(\rho_1) = S\Big($  $\Big) = S\Big($  $\Big) = S\Big($  $\Big) = S\Big($  $\Big) = S(\rho_2)$.

Concurrence — мера перепутанности двухкубитных состояний

Wootters '98

$C(\psi) =$



символ Л.-Ч. для $n=2$

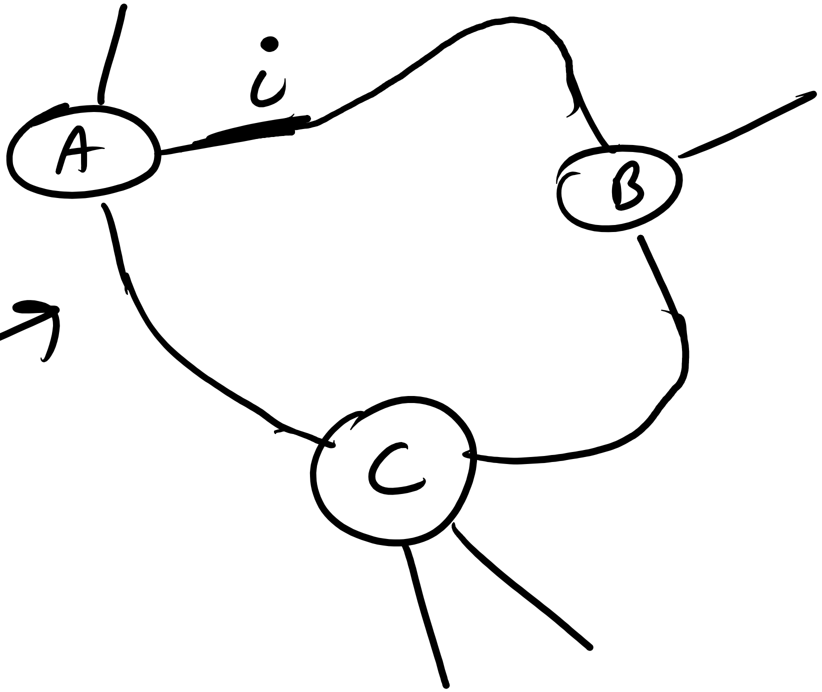
$$C\left(U_1 \otimes U_2 |\psi\rangle\right) = C\left(|\psi\rangle\right)$$

$\forall$ унит. U_1 и U_2.

$e^{12} = 1$ $\quad e^{11} = e^{22} = 0$

$e^{21} = -1$

~~Дерев~~



Не тенз. дерево.

Тензорная сеть — тензор, полученный из тензоров A, B, C, ... путём взятия свёрток между ними.

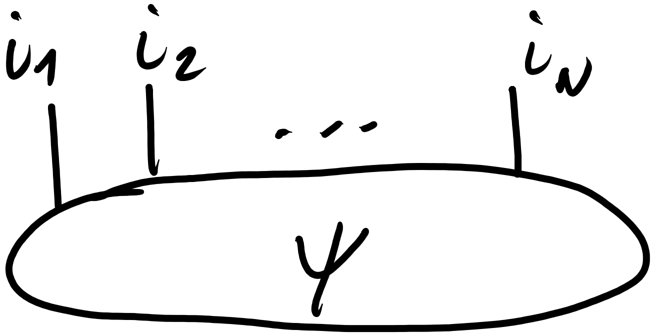
Архитектура тензорной сети = "~~зада~~ формы тензоров" + "граф свёрток".

Тензорные деревья — тенз. сети без циклов.

Чистое состояние из N d-уровневых систем

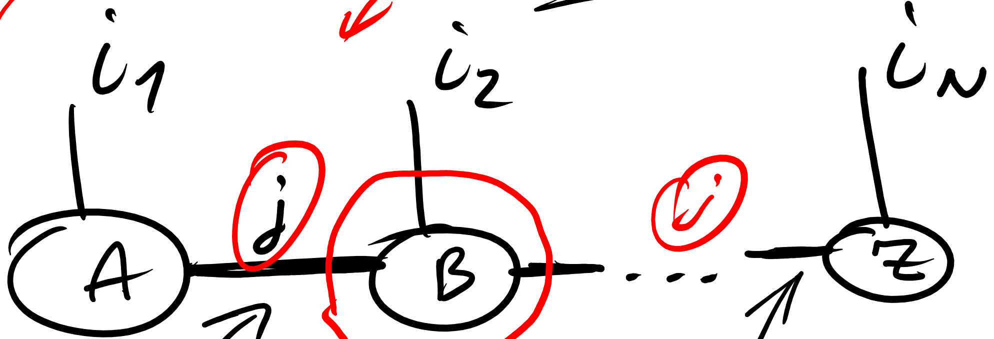
$$|\psi\rangle = \sum_{i_1,\dots,i_N=1}^{d} \psi^{i_1\dots i_N}\, |i_1\rangle \otimes \dots \otimes |i_N\rangle$$

$O(d^N)$



d^N компл. чисел задают ψ

$O(Nd)$



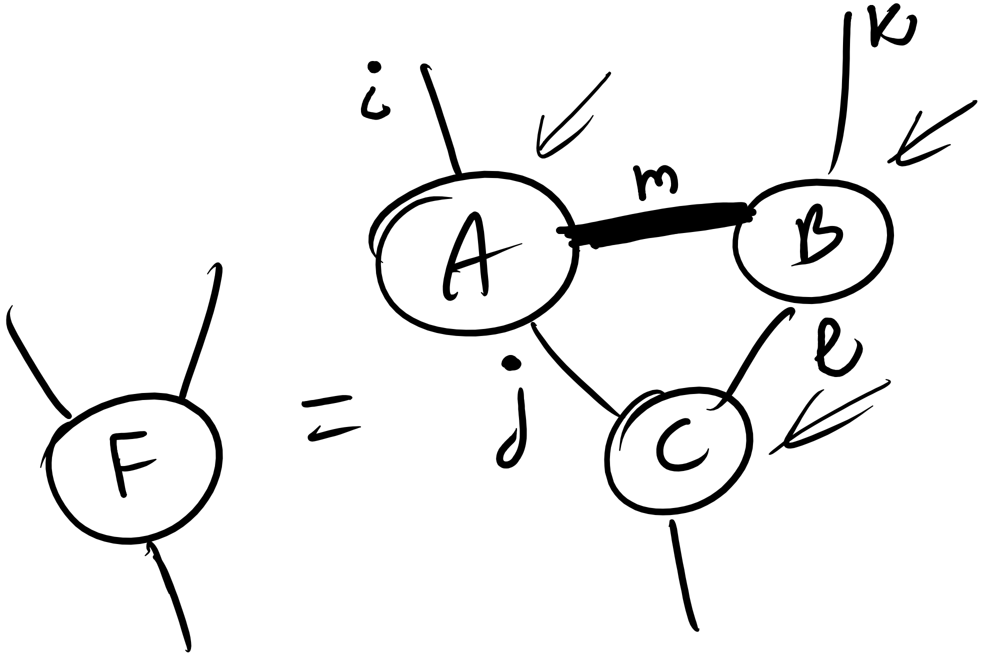
$j = 1, \dots, r$

$|\{j\}_j| = r$

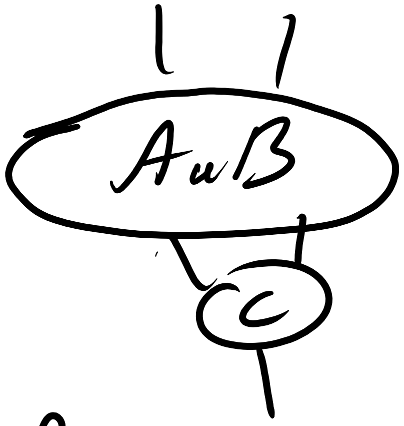
изм. $= r$

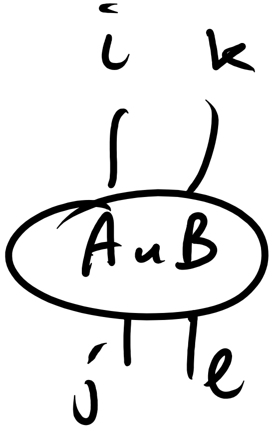
$(N-2)\, r^2 d + 2 \cdot r \cdot d$ компл. чисел.

Вычислительная сложность свёртки



в памяти компьютера





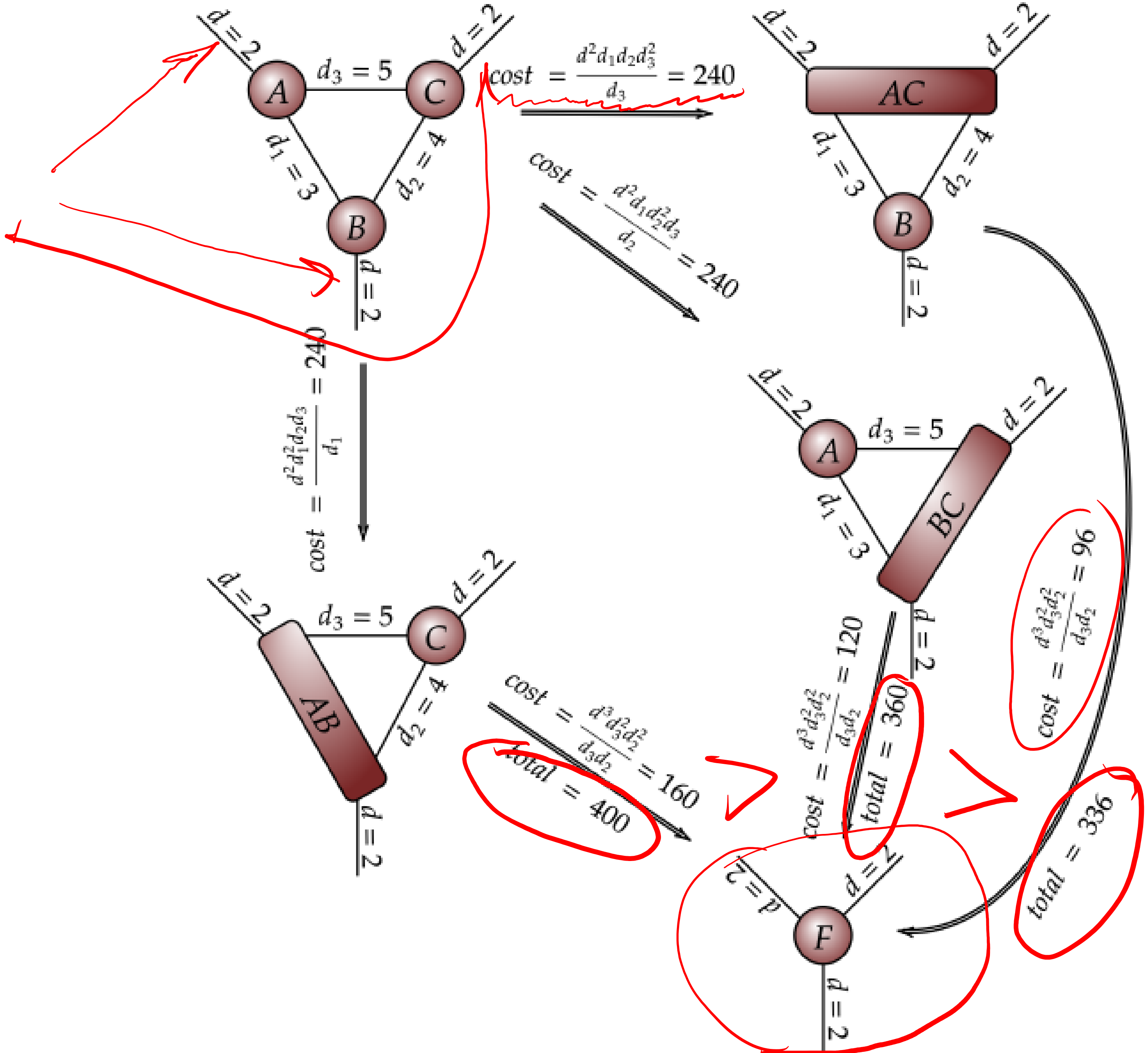
$$\sum_m A_{ijm} B_{mkl}$$

$|A \cap B|$

Сложность:

$$|A| = |\{i\}_i| \cdot |\{j\}_j| \cdot |\{m\}_m| \cdot |\{k\}_k| \cdot |\{l\}_l| = |B|$$

$$\mathrm{cost}(A;B) = \frac{|A| \cdot |B|}{|A \cap B|}$$



A
B
C
d = 2
d_3 = 5
d_1 = 3
d_2 = 4
cost = d^2 d_1 d_2 d_3^2 / d_3 = 240
AC
cost = d^2 d_1 d_2^2 d_3 / d_2 = 240
cost = d^2 d_1^2 d_2 d_3 / d_1 = 240
BC
AB
cost = d^3 d_3^2 d_2^2 / d_3 d_2 = 160
total = 400
cost = d^3 d_3^2 d_2^2 / d_3 d_2 = 120
total = 360
cost = d^3 d_3^2 d_2^2 / d_3 d_2 = 96
total = 336
F