Banach's Doctorate: A Case of Mistaken Identity

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In the academic year 2020/2021 we celebrate the hundredth anniversary of Stefan Banach's doctorate. Many books describe the curious way in which Banach supposedly obtained his PhD. The story, which will be presented in the sequel, is eye-catching indeed. But is it true? There are several misleading stories about Banach making the rounds; see [12]. Anyway, before we come to the story about his PhD, let's start from the very beginning.

Banach was born in Kraków in 1892. Sometimes it is mentioned that Banach was not the surname of anyone in his family but that of a washerwoman, Katarzyna Banach, into whose care he was given shortly after his birth. This is not true. Banach was the son of Katarzyna Banach and Stefan Greczek. His parents were unmarried, but that was not unusual. At the end of the nineteenth century, about one in every four children in Kraków was born out of wedlock. Greczek was serving in the Austro-Hungarian army, and he would not have been allowed to marry without permission from the military authorities. Such permission was denied. When the child was several months old, he was placed in the care of a foster mother, a laundry owner named Franciszka Płowa, who together with her daughter Maria took care of the young Stefan until he reached the age of majority.

It is not difficult to find the source of this error. At a conference on functional analysis organized in September 1960, Hugo Steinhaus (1887–1972) presented a talk on Banach (who had died in 1945), and his lecture was later published in the journal *Wiadomości Matematyczne* [33]. Steinhaus's presentation was brilliant, but his article also contained some factual errors. Steinhaus claimed that Banach had been raised by a washerwoman named Katarzyna Banach and confused Banach's biological mother with his foster mother. Moreover, he gave the wrong date for Banach's birth (20 March 1892 instead of 30 March; see [12]).

Steinhaus played a crucial, and unusual, role for Banach at the beginning of his mathematical career. In 1916, during an evening walk in Planty Park in Kraków, Steinhaus heard the words "Lebesgue integral," so he joined the two young men who were conversing. They turned out to be Banach and Otton Nikodym (1887–1974). Steinhaus wrote that Banach and Nikodym told him that they "had a third pal, Wilkosz, on whom they heaped lavish praise" [32, 33]. Witold Wilkosz (1891–1941) would frequently converse with Banach and Nikodym, but he was not with them on that evening. Steinhaus realized that Banach had a superb mathematical talent and began mentoring him. Steinhaus used to say that his best mathematical discovery was Stefan Banach.[1] For details, see [7, 8, 20, 32]. Note that while Steinhaus indeed discovered Banach, he discovered neither Nikodym nor Wilkosz, both of whom later became outstanding mathematicians.

It is not unusual in the history of science that an otherwise trustworthy witness misrepresents events. Wacław Sierpiński (1882–1969) knew both Banach and Steinhaus very well, and he worked closely with Nikodym.[2] In [30], he provides a wildly distorted account of the famous

[1]On the occasion of the hundredth anniversary of that famous talk, a bench with the figures of Banach and Nikodym (designed by Stefan Dousa) was unveiled in the Planty Garden in Kraków (see [7]). The following year, a picture of this bench appeared on the cover of this magazine (*Mathematical Intelligencer* 39:1).

[2]Since we mentioned the cover of the *Mathematical Intelligencer* featuring the bench with Banach and Nikodym, we might mention as well that the cover of the *Mathematical Intelligencer* 17:1 (1975) exhibits a poster with Sierpiński's face formed by his space-filling curve, designed by Fritz Lott.

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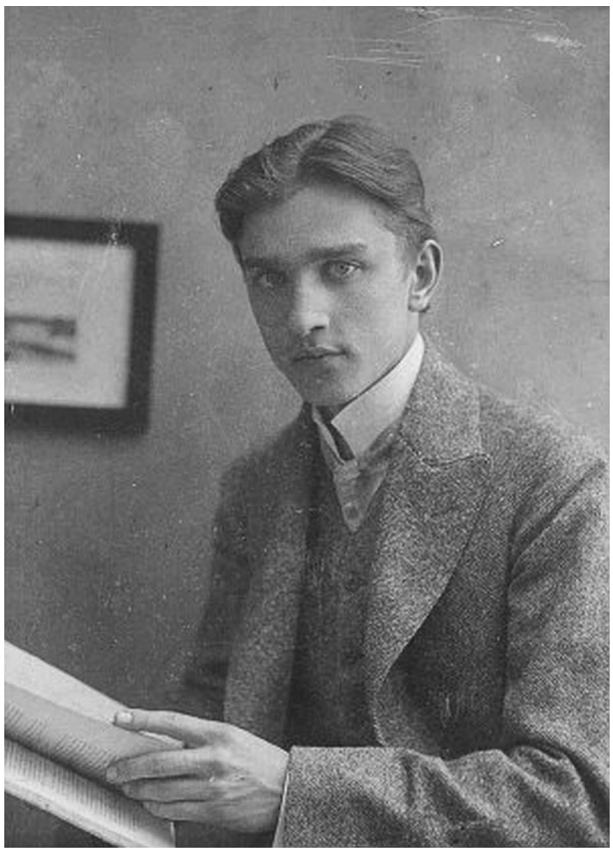

Figure 1. Stefan Banach in 1919. (Courtesy of the Banach family collection [19].)

meeting of Steinhaus and Banach in Planty Park in Kraków in 1916. He reports that on that day, one of the park benches was occupied by three mathematicians: Steinhaus ("then a *docent* at Lwów University"), Nikodym, and Witold Wilkosz. Sierpiński writes, "They were discussing some mathematical problem which had not been solved at that time. Banach eavesdropped, introduced himself into their conversation and offered his opinion as to how it could be solved." As we know from Steinhaus's firsthand account mentioned above, it was the other way round. It was Steinhaus who overheard the mathematical conversation, and Banach and Nikodym were unknown to him at the time (nor did he know Wilkosz, for that matter). Steinhaus was a participant in the event, so his version of the story is undoubtedly correct. The only discrepancy in his accounts is that sometimes he writes that he joined the conversation on unexpectedly hearing the words "Lebesgue integral" [33], while at other times, the words were "Lebesgue measure" [32]. He may have heard both. Note also that contrary to Sierpiński's description, Steinhaus had no connection with Lvov University in 1916 and at the time, did not hold the position of docent anywhere.

This meeting is also recalled in [21]. However, only Steinhaus and Banach are mentioned there (although the name of Nikodym appears elsewhere in that book).

In 1920, Steinhaus was appointed to a chair at Jan Kazimierz University in Lvov.[3] Thanks to Steinhaus's efforts, Banach obtained an assistantship at the Lvov Polytechnic University. Now we come to the central point of our story: Banach's doctorate.

The story goes that Banach could not be bothered with writing a thesis, since he was interested mainly in solving problems not necessarily connected to a possible doctoral dissertation. After some time, the university authorities became impatient. It is said that another university assistant (instructed by Stanisław Ruziewicz) wrote down Banach's theorems and proofs, and those notes were accepted as a superb dissertation. However, an exam was also required, and Banach was unwilling to take it. So one day, Banach was accosted in the corridor by a colleague, who asked him to join him in a meeting with some mathematicians who were visiting the university in order to clarify certain mathematical details, since Banach would certainly be able to answer their questions. Banach agreed and eagerly answered the questions, not realizing that he was being examined by a special commission that had arrived from Warsaw for just this purpose. In some sources [11, 19, 20], this event is described as only a possible version of events. Nevertheless, in several (mainly Polish-language) books, it is presented as fact. There is even a book on the phobias and fears of great Poles that devotes a whole chapter to Banach and this story, claiming to demonstrate that Banach was unable to deal with his own psyche and phobias, although even this story presents Banach simply as someone who did not consider the PhD a very important acquisition.

This is a really attractive story. But is there any chance that it is true? On the one hand, one may recall here a famous anecdote supposedly explaining the nonexistence of the Nobel Prize in mathematics (see [4]): Niels Bohr used to say that there is no Nobel Prize in mathematics because the Swedish mathematician Gösta Mittag-Leffler had seduced Nobel's wife. When Bohr was reminded that this was impossible, because Nobel had never married, Bohr answered, "Never let facts interfere with a good story." Incidentally, the explanation for the lack of a Nobel Prize in mathematics given in [4] is also misleading; for an excellent discussion of this problem, see [17].

On the other hand, good stories aside, the truth about Banach's exam should be known. Nowadays, it is possible to check the facts, since many sources have become more easily available than they were some decades ago. It is enough to look carefully at some dates and university rules to see that the proposed account could not be accurate. Banach moved to Lvov in 1920 to take up his job at the Lvov Polytechnic. On June 24 of that year, he presented his doctoral dissertation to the Philosophy Faculty of Jan Kazimierz University. The time interval of just a couple of months was definitely too short for the university authorities to have become impatient, let alone for someone else to have written a thesis on the basis of Banach's overheard comments. Moreover, in 1920, Banach had already

[3]After World War I, Poland regained independence, and Lvov (now Lviv, Ukraine) reverted from Austria–Hungary to Poland. In 1919, the university was named after Jan II Kazimierz Waza, the Polish king who chartered the university in Lvov in 1661.

published three research papers. Why would he be reluctant to write a doctoral dissertation, which would be a requirement for him to keep the job?

Now let's have a closer look at the exam. According to the university rules, a PhD dissertation had to be refereed and accepted, and then two exams—in the candidate's main scientific disciplines (in Banach's case they were mathematics and physics) and in pure philosophy—were to be taken by the candidate. It turns out that the records of Banach's PhD exams have survived (they are reproduced in [22] and [26]), and we may read that Banach passed his PhD examinations in mathematics and physics. The examining board consisted of four scientists: the dean of the faculty, Zygmunt Weyberg, who was a mineralogist; two mathematicians, Eustachy Żyliński and Hugo Steinhaus; and a physicist, Stanisław Loria. None of them was from Warsaw, and Banach knew all of them.

There is another interesting story concerning Banach's doctoral dissertation. The referees were Żyliński and Steinhaus. In October 1920, Steinhaus, who was mentoring Banach, wrote to the dean to inquire about the date of Banach's doctoral exam, for it had been four months since Banach had delivered his dissertation. The dean replied that everything was ready for the exam, but they were awaiting the referees' report (one of whom was Steinhaus himself!). Indeed, when the joint report from Steinhaus and Żyliński arrived, the exam took place immediately. Banach had submitted his dissertation on June 24, the report is dated October 30, and the exam in mathematics and physics took place on November 3. Bearing in mind that in 1920, October 30 fell on a Saturday, November 3 was therefore a Wednesday, and November 1 (Monday) is a public holiday in Poland, everything must indeed have been prepared for the exam. Banach passed this exam with a unanimous grade of "excellent" from all four examiners.

On December 11, 1920, Banach passed the exam in philosophy (the examining board consisted of the two philosophers Kazimierz Twardowski and Mścisław Wartenberg and the dean, Zygmunt Weyberg). Banach had now fulfilled all the requirements for being granted the PhD degree, and in many sources (including a CV signed by Banach; see [19]), 1920 is given as the year of Banach's doctorate. However, the precise rules for obtaining a PhD from Austro-Hungarian times (see [1]) had been retained by Poland after regaining its independence (see [14]). According to those rules, the candidate was allowed to call himself "doctor" only after the doctoral conferment ceremony, which in the case of Banach took place on January 22, 1921. The official documents state that the academician who conferred the degree on Banach was Kazimierz Twardowski. To a mathematician, that is surprising news indeed. Why Twardowski, who was an eminent Polish philosopher? What was his connection to Banach? Could he have been his dissertation advisor? According to the rules

Figure 2. The main building of Lwów University in the 1920s. (Photo by A. Lenkiewicz, in the public domain.)

then in force, the conferment of a new doctorate had to be celebrated by a professor from the faculty appointed by the dean, and so there is no reason to regard Twardowski as the supervisor of Banach's thesis. By analogy, one might incorrectly claim that Steinhaus's supervisor in Göttingen in 1911 was the German botanist Gustav Albert Peter, who played the same role as Twardowski in Banach's case (for details, see [9]).

It is frequently said that Banach was not a university graduate, so the fact that he obtained a position at the Polytechnic and a university doctorate was exceptional. This is also slightly misleading. According to the rules that were then in effect in Poland [14], four years of study at the university was enough for one to be eligible for a PhD, but even that requirement could be relaxed. The professors of a faculty could, at their discretion, allow someone with outstanding achievements to apply for a PhD. Moreover, in those years, there was no precise definition of who counted as a university graduate. Banach had studied at the Lvov Polytechnic for precisely four years, which was enough.

Let us add some further information on Banach's doctorate. His dissertation was truly impressive. In it, Banach defined a space that came to be known as the Banach space. He proved some results on such spaces, some theorems on linear operators, and the theorem that is now known as the Banach fixed-point theorem. His dissertation was published two years later as [2]. For an excellent analysis of the results contained therein, see [5, 15, 18, 23]. Banach's dissertation was of fundamental importance for the development of mathematics in the twentieth century, which, however, was not evident at the outset.

Independently of Banach, Norbert Wiener presented a very similar axiomatic system for vector spaces. It was published also in 1920, but in the fall, so Banach was a little faster out of the gate (see [35]). The name "Banach space" was first introduced by Maurice Fréchet in [16] as *les espaces de Banach*. Earlier, Banach and his collaborators had used the name "B-spaces." As was noted in [23], this term first appeared in print in Steinhaus's paper [31]. Steinhaus wrote there about *der B-Raum*. For some time, the spaces in question were called "Banach–Wiener spaces." In any case, Wiener wrote later in [35], "For a short while I kept publishing a paper or two on this topic, but I gradually left the field. At present these spaces are quite justly named after Banach alone."

Wiener abandoned this topic, but Banach continued his research. Around 1930, Banach proved three theorems fundamental for functional analysis: the Hahn–Banach theorem, the Banach–Steinhaus theorem, and the Banach open mapping theorem. This marked the real starting point of the development of functional analysis.[4] In 1931, Banach published the monograph *Operacje liniowe* (Linear Operations), and a year later, he published the same book in French [3]. At that point, the mathematical world recognized the importance of Banach's results. Banach was invited to deliver a plenary lecture at ICM 1936, in Oslo.

So, the story of the extraordinary circumstances of Banach's doctorate is completely false. However, it is said that "underneath gossip there's a kernel of truth." Let us then dig further in an attempt to discover it.

This is a good place to recall the illustrious figure of Andrzej Turowicz (1904–1989), a mathematician, priest, and monk active mostly in Kraków, but who also spent some time working in Lvov; see [13]. Turowicz knew many excellent stories, abounding in colorful detail, about mathematics and mathematicians of his time. It was not unusual for participants in various meetings that he attended to ask him to share some of his anecdotes. Whenever Turowicz had himself been a witness of an event, he recounted it with great accuracy, and one could be sure that things had really happened that way, but there were also stories he had heard from others.

On November 17, 1984, the Jagiellonian University Students' Mathematics Society (see [10]) invited several mathematicians to share their memories during a special meeting. Their reminiscences were taped. Turowicz was one of the guests. He contributed the anecdote about Banach's PhD exam, beginning with the words, "This is a story I heard from Nikodym, and I am repeating it here at Nikodym's responsibility." Turowicz recounted this event on several occasions and always credited it to Nikodym. The same attribution is also given in [20].

It was Nikodym whose conversation with Banach was accidentally overheard by Steinhaus in Kraków. Later, Nikodym became a prominent mathematician; after World War II he emigrated to the United States. More information about him can be found in [6] and [34].

And it turns out that it was Nikodym who was reluctant to obtain a PhD. He used to ask, "Will it make me any wiser?" In 1924, Nikodym (aged 35), still without a PhD, and his wife, Stanisława (who also was a mathematician), moved from Kraków to Warsaw. Walerian Piotrowski made a very solid investigation concerning PhDs in mathematics at Warsaw University in the interwar period (see [24, 25]). According to [25], Wacław Sierpiński decided to take the matter of Nikodym's PhD exam into his own hands. He invited Nikodym to a café and began to talk with him. After a while, the dean of the department "accidentally" appeared in the café and joined the conversation, which quickly drifted toward mathematics. More than an hour later, Sierpiński said to Nikodym, "Congratulations. You have just passed your PhD exam."[5]

In our opinion, this is the source of the urban legend about Banach's doctorate. We will never know whether Nikodym gave Turowicz a twisted account of his own PhD exam, changing the main protagonist's name in the process, or whether Turowicz missed something. Our view is that the first explanation is more likely.

We conclude with yet another story about Banach that has been widely circulated (see, for example, [20]). To wit, toward the end of the 1930s (around 1937), Wiener supposedly intended to offer Banach a job in the United States, and he sent an emissary to Lvov to convince Banach to

[4]For an excellent description of the origins of functional analysis, see [29].

[5]Stanisława Nikodym obtained a PhD in Warsaw the same year, without any such stratagem. She was the first women in Poland to be awarded a PhD in mathematics.

Figure 3. Stanisława and Otton Nikodym around 1924. (Courtesy of the Polish Institute of Arts and Sciences of America, Inc.)

Figure 4. Witold Wilkosz with his fultograph. (Courtesy of Narodowe Archiwum Cyfrowe.)

accept the offer. Banach asked, "How much is Professor Wiener willing to pay?" The visitor from the United States answered, "Ah, we anticipated this question" and handed Banach a check signed by Wiener on which only the digit 1 was written. "Please add to it as many zeros as you deem fit!" Banach replied, "Such a sum is too small for me to leave Poland."

This story cannot be true.[6] For one thing, there was no reason for Wiener to invite Banach, as by then he had long ceased to work in Banach's field of interest. Moreover, neither Wiener nor Steinhaus mentions in his memoirs (respectively [35] and [32]) any plans by Wiener to invite Banach to the United States. In his autobiography [35], Wiener does not write anything about Banach personally; he mentions Banach only on the occasion of writing about Banach/Banach–Wiener spaces. Also, Wiener's putative offer appears financially implausible: someone faced with such an offer could well have written in thirty zeros or more. Nevertheless, there exists a more direct argument that definitively falsifies this story. First, Banach spent the academic year 1924/25 in Paris on a fellowship from the Rockefeller Foundation. In the early 1930s, he applied for that fellowship again, but his application was rejected, since "very few of the former fellows received a continuation" (see [28]). Apparently, Banach was not one of those whose fellowships were renewed. Second, in 1937–1938, Banach actually wanted to go to the United States! Not for good, but for some time. Stanisław (Stan) Ulam (1909–1984), who had moved from Lvov to the United States in 1935, was arranging a temporary position for Banach there. In [19] we find a letter from Banach to Ulam dated February 14, 1938, in which he writes, "It seems from your letter that my visit to America is starting to become a reality .... Most important, of course, is the amount they are willing to pay, which should be adequate and suitable for a year's stay for me and my wife and son .... Let me know what you would advise me to prepare for my lectures in America." Banach's visit to the United States failed to materialize, probably because of the outbreak of World War II. Had Ulam managed to arrange it, Banach might have survived the war.

Is the story about the incredible salary completely fictitious? Or as in the case of the story about the doctorate, is there a kernel of truth there?

Yes, there is! Enter the second of Banach's friends from his Kraków years: Witold Wilkosz. Wilkosz and Banach attended the same secondary school in Kraków. Next, Wilkosz took up philological studies at the Jagiellonian University in Kraków. After two years, he switched to mathematics and left to study in Turin. In Italy, he was preparing his doctoral dissertation, but due to the outbreak of the war he returned to Kraków. He received his PhD from the Jagiellonian University in 1918. He had a very broad range of mathematical interests. Apart from obtaining important results in various branches of mathematics, he was a very active teacher and popularizer of science. It is little known that Wilkosz was the author of the first monograph on topology in Poland [36] and the first monograph on topology by a Polish author published abroad [37].[7] Wilkosz was also a pioneer in radio engineering and broadcasting in Poland.[8] He constructed a new type of radio receiver that came to be known as the "Wilkosz radio." Being familiar with Wilkosz's achievements in this area, the famous Dutch electronics company Philips offered Wilkosz a job (see [27]). Philips offered him a salary of $5000 per month, which was then an incredibly large sum. To compare, the monthly salary of a professor at the Jagiellonian University was equivalent to approximately $250. But Philips imposed a condition that everything that Wilkosz invented during his employment by the company

[6]Or it could have originated as a joke made in the Scottish Café.

[7]The book about the topology of the Euclidean plane and its subsets was published by Gauthier-Villars in 1931. It was volume 47 of the series Mémorial des Sciences Mathématiques published under the auspices of l'Académie des Sciences de Paris and ten other national academies. In it, Wilkosz presented an outline of the research in this topic up to the end of the 1920s. Several open problems were also presented.

[8]He also obtained some results concerning mathematical models of generating radio waves, especially connected to the van der Pol equation.

would become the property of Philips. Wilkosz did not accept the offer and remained in Kraków.

Banach, Wilkosz, and Nikodym often discussed mathematics among themselves, but they never published a joint paper. Nevertheless, in his famous dissertation [2], Banach announced that he and Wilkosz were preparing a paper on a similar topic. Such a paper never appeared.

Thus two famous stories attributed to Banach really took place, although not precisely in the form in which they are commonly narrated. In particular, they did not concern Banach, but his two colleagues who used to discuss mathematical problems with him in their younger years: Otton Nikodym and Witold Wilkosz.

A good story is always welcome. But the truth is even more welcome.

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REFERENCES

[1] Austro-Hungarian Ministry of Culture and Education. Verordnung des Ministers für Cultus und Unterricht, betreffend eine abgeänderte Rigorosenordnung für die philosophischen Facultäten der Universitäten der im Reichsrathe vertretenen Königreiche und Länder, 16.03.1899.

[2] S. Banach. Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. *Fund. Math.* 3 (1922), 133–181.

[3] S. Banach. *Théorie des opérations linéaires*. Monografie Matematyczne. Warszawa, 1932.

[4] R. E. Bellman. *Eye of the Hurricane: An Autobiography*. World Scientific, 1984.

[5] M. Bernkopf. The development of function spaces with particular reference to their origins in integral equation theory. *Arch. History Exact Sci.* 3 (1966), 1–96.

[6] D. Ciesielska. Stanisława and Otton Nikodym. In *Against All Odds*, edited by E. Kaufholz-Soldat and N. M. R. Oswald, pp. 151–175. Springer, 2020.

[7] D. Ciesielska and K. Ciesielski. Banach and Nikodym on the bench in Kraków again. *European Mathematical Society Newsletter* 104 (2017), 25–29.

[8] D. Ciesielska and K. Ciesielski. Banach in Kraków: a case reopened. *Math. Intelligencer* 35:3 (2013), 64–68.

[9] D. Ciesielska, L. Maligranda, and J. Zwierzyńska. Doktoraty Polaków w Getyndze. Matematyka. *Analecta* 28:2 (2019), 73–116.

[10] K. Ciesielski. 100th anniversary of the Jagiellonian University students' Maths Society. *Math. Intelligencer* 17:4 (1995), 42–46.

[11] K. Ciesielski. Lost legends of Lvov 2: Banach's grave. *Math. Intelligencer* 10:1 (1988), 50–51.

[12] K. Ciesielski. On some details of Stefan Banach's life. *Opuscula Math.* 13 (1993), 71–74.

[13] K. Ciesielski and Z. Pogoda. Conversation with Andrzej Turowicz. *Math. Intelligencer* 10:4 (1988), 13–20.

[14] T. Czeżowski (editor). *Zbiór ustaw i rozporządzeń o studiach uniwersyteckich oraz innych przepisów ważnych dla studentów uniwersytetu, ze szczególnym uwzględnieniem Uniwersytetu Stefana Batorego w Wilnie*. Wilno, 1926.

[15] R. Duda. *Pearls from a Lost City: The Lvov School of Mathematics*. American Mathematical Society, 2014.

[16] M. Fréchet. Les espaces abstraits topologiquement affines. *Acta Math.* 47 (1926), 25–52.

[17] L. Gårding and L. Hörmander. Why is there no Nobel Prize in mathematics? *Math. Intelligencer* 7:3 (1985), 73–74.

[18] F. Jaëck. Generality and structures in functional analysis: the influence of Stefan Banach. In *The Oxford Handbook of Generality in Mathematics and the Sciences*, edited by K. Chemla, R. Chorlay, and D. Rabouin, pp. 223–254. Oxford University Press, 2016.

[19] E. Jakimowicz and A. Miranowicz (editors). *Stefan Banach. Remarkable Life, Brilliant Mathematics*. Gdańsk University Press, 2010.

[20] R. Kałuża. *Through a Reporter's Eyes: The life of Stefan Banach*. Birkhäuser, 1996.

[21] S. G. Krantz. *Mathematical Apocrypha Redux*. Mathematical Association of America, 2005.

[22] L. Maligranda. 100-lecie doktoratu Stefana Banacha. To appear in *Wiad. Mat.* 56 (2020).

[23] L. Maligranda. ''B''jak przestrzeń Banacha. *Wiad. Mat.* 56 (2020), 15–27.

[24] W. Piotrowski. Doktoraty z matematyki i logiki na Uniwersytecie Warszawskim w latach 1915–1939. In *Dzieje Matematyki Polskiej II*, edited by W. Więsław, pp. 97–131. Instytut Matematyczny Uniwersytetu Wrocławskiego, 2013.

[25] W. Piotrowski. Jeszcze w sprawie biografii Ottona i Stanisławy Nikodymów. *Wiad. Mat.* 50 (2014), 69–74.

[26] J. Prytuła. Doktoraty z matematyki i logiki na Uniwersytecie Jana Kazimierza we Lwowie w latach 1920-1938. In *Dzieje Matematyki Polskiej*, edited by W. Więsław, pp. 137–161. Instytut Matematyczny Uniwersytetu Wrocławskiego, 2012.

[27] K. Rakoczy-Pindor. Przygoda matematyka z radiem. *Wiad. Mat.* 39 (2003), 151–156.

[28] R. Siegmund-Schultze. *Rockefeller and the Internationalization of Mathematics Between the Two World Wars*. Springer, 2001.

[29] R. Siegmund-Schultze. The origins of functional analysis. In *A History of Analysis*, edited by N. Jahnke, pp. 385–407. American Mathematical Society, 2003.

[30] W. Sierpiński. The Warsaw School of Mathematics and the present state of mathematics in Poland. *Polish Review* 4 (1959), 51–63.

[31] H. Steinhaus. Anwendungen der Funktionalanalysis auf einige Fragen der reellen Funktionentheorie. *Studia Math.* 1 (1929), 51–81.

[32] H. Steinhaus. *Mathematician for All Seasons: Recollections and Notes*, Vol. 1 (1887–1945). Birkhäuser, 2015.

[33] H. Steinhaus. Stefan Banach. *Wiad. Mat.* 4 (1961), 251–259.

[34] W. Szymański. Who was Otto Nikodym? *Math. Intelligencer* 12:2 (1990), 27–31.

[35] N. Wiener. *I Am a Mathematician: The Later Life of a Prodigy*. MIT Press, 1956.

[36] W. Wilkosz. *Podstawy ogólnej teorji mnogości*. Trzaska, Evert & Michalski, Warszawa, 1925.

[37] W. Wilkosz. *Les propriétés topologiques du plan euclidien*. Gauthier-Villars, 1931.