V.I. ROMANOVSKIY NOMIDAGI MATEATIKA INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.02/30.12.2019.FM.86.01 RAQAMLI ILMIY KENGASH

MATEMATIKA INSTITUTI

JALILOV ALISHER AKBAROVICH

AYLANA GOMEOMORFIZMLARI BILAN BOG'LIQ DANJUA TENGLIGI VA CHEKSIZ IKKILIK KETMA-KETLIKLARI

01.01.01 – Matematik analiz

FIZIKA-MATEMATIKA FANLARI BO'YICHA FALSAFA DOKTORI (PhD)

DISSERTATISYASI AVTOREFERATI

Toshkent – 2021 yil

UDK: 517.9

Fizika-matematika fanlari bo'yicha falsafa doktori (phd)

dissertatsiyasi avtoreferati mundarijasi

Оглавление автореферата докторской диссертации

доктора философии (PhD) по физико-математическим наукам

Content of dissertation abstract of doctor of philosophy (PhD) on physical-mathematical sciences

Jalilov Alisher Akbarovich

Aylana gomeomorfizmlari bilan bog'liq danjua tengligi va cheksiz ikkilik ketma-ketliklari. 3

Jalilov Alisher Akbarovich

Denjoy equality and infinite binary sequences associated with circle homeomorphisms 19

Жалилов Алишер Акбарович

Равенство Данжуа и бесконечные двоичный последовательности ассоциированные с гомеоморфизмами окружности . . . . ……………………………………………. 35

E'lon qilingan ishlar ro'yxati

List of published works

Список опубликованных работ 38

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Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Vazirlar Mahkamasi huzuridagi Oliy attestatsiya komissiyasida B2019.4.PhD/FM430 raqam bilan ro'yxatga olingan.

Dissertatisya V.I.Romanovskiy nomidagi Matematika institutida bajarilgan.

Dissertatsiya avtoreferati uch tilda (o'zbek, ingliz, rus(rezyume)) Ilmiy kengash veb-sahifasi (http://kengash.mathinst.uz) va «ZiyoNet» Axborot ta'lim portalida (http://www.ziyonet.uz) joylashtirilgan.

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Yetakchi tashkilot:

Dissertatsiya himoyasi V.I. Romanovskiy nomidagi Matematika instituti huzuridagi DSc.02/30.12.2019.FM.86.01 raqarmli Ilmiy kengashning 2021 yil « ___ »_________ soat ____ dagi majlisida bo'lib o'tadi. (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+99871)-207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz).

Dissertatsiya bilan V.I. Romanovskiy nomidagi Matematika institutining Axborot-resurs markazida tanishish mumkin (_____-raqami bilan ro'yxatga olingan). (Manzil: 100174, Toshkent sh., Olmazor tumani, Universitet ko'chasi, 9-uy. Tel.: (+99871)-207-91-40.

Dissertatsiya avtoreferati 2021 yil « ___ » __________ kuni tarqatildi.
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KIRISH (falsafa doktori (PhD) dissertatsiyasi annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurati. Dunyoda ko'plab ilmiy va amaliy tadqiqotlar bir o'lchovli va simvolik dinamik sistemalarni o'rganishga keltiriladi. Aylana gomeomorfizmlari nazariyasi bir o'lchovli dinamik sistemalarning muhim qismidir. Aylana gomeomorfizmlari birinchi marta A. Puankarening fundamental asarlarida osmon mexanikasi muammosi bilan bog'liq holda o'rganilgan.

Hozirgi vaqtda dunyoda dinamik sistemalar nazariyasining qiziqarli muammolaridan biri aylananing akslantirishlari bilan bog'liq muammolardir. Aylana akslantirishlarining muhim sinfi - aylana diffeomorfizmlaridir. Umuman olganda, aylana diffeomorfizmlari hozirgacha yaxshi o'rganilgan. Aylana diffeomorfizmlarining tabiiy umumlashmasi - bu kritik akslantirishlar va bo'lakli silliq akslantirishlaridir. So'nggi 20 yil ichida maxsuslikka ega aylana akslantirishlari nazariyasida fundamental natijalar A. Puankare, A. Denjoy, A.N. Kolmogorov, V.I. Arnold, J. Moser, M. Erman, J.C. Yokkoz, J.G. Sinay, K.M. Xanin, D. Ornshteyn, J. Katsnelson va boshqa taniqli matematiklarning asarlarida qo'lga kiritilgan.

Dinamik sistemalar, ma’lumotlar nazariyasi, geometriya, yurak muammolari, biologiya va amaliyotning ko'plab muammolari simvolik dinamik sistemalarni o'rganishga keltiriladi. Cheksiz simvolik ketma-ketliklarning murakkablik funksiyasini o'rganish simvolik dinamik sistemalarning klassik muammolaridan biridir. Aylana akslantirishlarini irratsional burish natijasida ma’lumotlar nazariyasida juda muhim bo'lgan Shturm so'zlari paydo bo'ldi.

Mamlakatimizda amaliy va fundamental fanlarda qo'llaniladigan bir o'lchovli dinamik sistemalar va simvolik dinamikaning muhim yo'nalishlarini ishlab chiqishga katta e'tibor qaratildi. Funksional tahlil, matematik fizika, ehtimollik nazariyasi va dinamik sistemalar nazariyasi kabi muhim sohalarda xalqaro miqyosda olib borilgan tadqiqotlar fundamental tadqiqotlarning asosiy vazifasi hisoblanadi. Aylana akslantirishlari va simvolik dinamik sistemalar nafaqat tabiiy fanlar uchun, balki iqtisodiyot, ma’lumotlar nazariyasi, biologiya, yurakning turli kasalliklarini o'rganish, qon tekshiruvi va boshqa soxalar uchun ham muhim ahamiyatga ega.

O’zbekiston Respublikasi Prezidentining 2017 yil 7 fevraldagi «O’zbekiston Respublikasini yanada rivojlantirish bo’yicha xarakatlar strategiyasi to’g’risida»gi PF-4947-son Farmoni, 2019 yil 9 iyuldagi «Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo’llab-quvvatlash, shuningdek, O’zbekiston Respublikasi Fanlar Akademiyasining V.I.Romanovskiy nomidagi Matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to’g’risida»gi PQ-4387-son Qarori va 2020 yil 7 maydagi PQ-4708-son «Matematika sohasidagi ta’lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora-tadbirlari to’g’risida»gi PQ-4708-sonli Qarori hamda mazkur faoliyatga tegishli boshqa normativ–huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishda ushbu dissertatsiya tadqiqoti muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalar rivojlanishining ustuvor yo’nalishlariga bog’liqligi. Mazkur tadqiqot respublika fan va texnologiyalar rivojlanishining IV. «Matematika, mexanika va informatika» ustuvor yo’nalishi doirasida bajarilgan.

Muammoning o’rganilganlik darajasi. Simvolik dinamika sohasi umumiy dinamik sistemalarni tahlil qilish vositasi sifatida rivojlandi. Hadamard 1898 yilda salbiy egilish yuzalaridagi geodezik oqimlarni tahlil qilishda simvolik dinamika usullaridan birinchi marta muvaffaqiyatli foydalangan. Qirq yil o'tgach, bu mavzu Marsten Morse va Gustav Hedlund asosli maqolasida birinchi sistemali tadqiqot qilingan va unga nom berilgan. Kelgusida C.E.Shennon tomonidan ma’lumotlar nazariyasida simvolik dinamika qo'llaniladigan.

Ya. Sinay o'zining asosiy ishida Anosov oqimlarini tadqiq qilish uchun simvolik dinamikadan foydalangan. E. Vul, Ya. Sinay va K. Xanin Feigenbaum asklantirishining simvolik dinamikasini tuzdi va termodinamik formalizmni yaratdi. Simvolik dinamika aylana akslantirishlari nazariyasida muhim rol o'ynaydi. Ma'lumki, aylananing har bir irratsional burilishi natijasida ikkilik ketma -ketlik fazosi yaratiladi. Ikkilik ketma-ketligining murakkablik funksiyalarini o'rganish simvolik dinamik sistemalarning asosiy muammolaridan biridir. Coelho, De Faria irratsional burishlar uchun qayta urilgan zarbalarning xatti-harakatlarini o'rgangan. A. Djalilov kritik aylana akslantirishlari uchun termodinamik formalizmni qurgan.

Danjua tengsizligi aylana dinamikasida muhim o’rin egallaydi. Sinay va Xanin o`z maqolasida Danjua tengsizligini rivojlantirgan va bunga asosan conjugation akslantirishining siliqligini isbotlagan.

Tadqiqotning vazifalari:

irratsional aylana akslantirishlari bilan bog'liq cheksiz ikkilik ketma -ketliklarning murakkab funksiyalarini o'rganish;

Erman akslantirishlari va uning yuqori itaratsiyalari tomonidan hosil qilingan cheksiz ketma -ketliklarning murakkablik funksiyalari o'rganish;

Erman akslatirishlari uchun sinish nuqtalari orbitasining tuzilishini o'rganish;

Erman akslatirishlarining Danjua ko’paytmasi uchun sinish nuqtalarining sinish kattaliklari va inavriant o’lcho orqali aniq ifodasini topish.

Tadqiqotning ob’ekti. Cheksiz ketma-ketliklar, aylanani irratsiona burish, Erman akslantirishi, kritik aylana akslanirishlari, tushish sonlari.

Tadqiqotning predmeti. Simvolik dinamik sistemalar, aylana akslantirishlar nazariyasi, inavariant o'lchov

Tadqiqotning usullari. Dissertatsiyada matematik analiz, , funksional analiz, simvolik dinamika, ergodik nazariya va ehtimollar nazariyalari usullaridan foydalaridan foydalanilgan.

Tadqiqotning ilmiy yangiligi қуйидагилардан иборат:

Irratsional aylana burishi orqali bog`langan cheksiz ikkilik ketma-ketliklarning murakkablik funksiyalari topilgan.

Bitta kubik kritik nuqtaga va “oltin kesim” burish soniga ega bo`lgan aylana akslantirishlarining nomallangan umumlashgan tushish vaqtlari taqsimot funksiyalari bo`yicha yaqinlashadi va limit taqsimot funksiyasi singular, ya’ni u uzluksiz, qat’iy o`suvchi va Lebeg o`lchovi bo`yicha deyarli barcha yerda hosilasi 0 ga teng.

Ikkita sinish nuqtasiga ega bo'lakli-chiziqli aylana gomeomorfizmlarining sinish nuqtalari joylashuvi topilgan.

h^{q_n} Erman akslantirishlari uchun Danjua tengligi isbotlangan.

h^{q_n} ning normallangan hosilalari bog`langan ikkilik ketma-ketliklar Shturm ekanligi ko`rsatilgan.

Tadqiqotning amaliy natijasi matematik biologiyadagi populyatsiya jarayonlarining matematik modellari bo'lakli silliq funksiyalar orqali ifodalashni taklif etilganligi va ushbu funksiyalar yordamida hosil bo'lgan dinamik sistemada traektoriya limit nuqtalari to'plamini aniqlash usullarini bayon qilinganligidan iborat.

Tadqiqot natijalarining ishonchliligi funksional analiz, stoxastik jarayonlar va diskret vaqtli dinamik sistemalar nazariyasi usullaridan foydalanilgani hamda matematik mulohazalarning qat'iyligi bilan asoslanadi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati bo'lakli silliq funksiyalar dinamikasida traektoriya limit nuqtalar to'plamining tavsifidan nochiziqli diskret dinamik sistemalar nazariyasida foydalanish mumkinligi bilan izohlanadi.

Tadqiqotning amaliy ahamiyati bo'lakli silliq akslantirishlar yordamida hosil bo'lgan dinamik sistemada traektoriya limit nuqtalari to'plamini aniqlash orqali matematik biologiyadagi populyatsiya jarayonlariga tatbiq etish bilan belgilanadi.

Tadqiqot natijalarining joriy qilinishi. Diskret vaqtli bo'lakli silliq dinamik sistemalar bo'yicha olingan natijalar asosida:

bir o'lchovli simpleksda aniqlangan uzilishga ega kvadratik stoxastik operatorning diskret vaqtli dinamikasi parametrlarning noldan farqli qiymatlarida xaotikligidan OT-F-4-03 raqamli «Uzluksiz hamda diskret vaqtli aniq dinamik sistemalar, qismiy integral operatorlar spektrlari» mavzusidagi fundamental ilmiy loyihada stoxastik operatorlar traektoriyalarining limit nuqtalari to'plamini tavsiflashda foydalanilgan (Qarshi davlat universitetining 2021 yil 12 iyundagi № 04/1900-sonli ma'lumotnomasi). Ilmiy natijaning qo'llanilishi Volterra hamda novolterra tipidagi kvadratik va kubik stoxastik operatorlarning limit nuqtalarini tavsiflash imkonini bergan;

dissertatsiyada olingan natijalardan o'lchovlik funksiyalar sinfida indekslangan integral empirik protsesslarning asimptotik xossalarini tadqiq qilish jarayonida ko'plab "kuchli" bog'langan diskret tasodifiy miqdorlar uchun limit teoremalarni- ya'ni ularning asimptotik holatini o'rganishga va limitik ehtimollik taqsimotlarini absolyut uzluksizligi masalasini o'rganishda O'zbekiston Milliy Universitetida Abdushukurov A. rahbarligida 2017-2021 yilllarda bajarilgan OT-F-4-40 raqamli «O'lchovlik funksiyalar sinfida indekslangan integral empirik protsesslarning asimptotik xossalarini tadqiq qilish» mavzusidagi fundamental loyihada foydalanilgan

Erman akslantirishlari uchun Danjua tipidagi tengliklarda olingan natijalar "Umumlashgan qo'zg'almas nuqta metodi va Homotopiya nazariyasi metodlari orqali Nochiziqli kasrli differensial tenglamalar yechimlari" (Utara univeristeti, Malayziya, qaynoma raqami: FRGS/1/2018/STG06/UUM/02/13) mavzusidagi ilmiy loyihada qo'llanilgan va Riman-Luvil kasrli integral operatorlari iteratsiyalarining universal baholarini olishda muhim o'rin egallagan. Undan tashqari, olingan natijalar kars darajali differensial tenglamalarning mavjudligini isbotlashda foydalanilgan.

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqot natijalari 3 ta xalqaro va 3 ta respublika ilmiy-amaliy anjumanlarida muhokamadan o'tkazilgan.

Tadqiqot natijalarining e'lon qilinganligi. Dissertatsiya mavzusi bo'yicha jami 11 ta ilmiy ish chop etilgan, shulardan, O'zbekiston Respublikasi Oliy attestatsiya komissiyasining falsafa doktori dissertatsiyalari asosiy ilmiy natijalarini chop etish tavsiya etilgan ilmiy nashrlarda 5 ta maqola, jumladan, 2 tasi xorijiy va 3 tasi respublika jurnallarida nashr etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish qismi, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan tashkil topgan. Dissertatsiyaning hajmi 92 betni tashkil etgan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zaruriyati asoslangan, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi ko'rsatilgan, muammoning o'rganilganlik darajasi keltirilgan, tadqiqot maqsadi, vazifalari, ob'ekti va predmeti tavsiflangan, tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan, olingan natijalarning nazariy va amaliy ahamiyati ochib berilgan, tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo'yicha ma'lumotlar keltirilgan.

Dissertatsiyaning "Simvolik dinamika va aylana akslantirishlari" deb nomlangan birinchi bobida biz simbolik dinamik sistemalardan, xususan, cheksiz ikkilik so'zlar va Shturm so'zlaridan kerakli ta'riflar va teoremalari bayon qilingan.

Shuningdek, aylana gomomorfizmlari va simvolik dinamika o'rtasidagi bog'liqlik o`rganilgan. Bundan tashqari, aylana dinamikasidan asosiy tushunchalar va tasdiqlar keltirilgan: yo`nalishni saqlovchi aylana gomeomorfizmlarining burish soni, ehtimollik invariant o'lchovi, kritik aylana akslantirishlari, chekli sondagi sinish nuqtalariga ega bo`lgan bo`lakli-silliq aylana gomeomorfizmlari va tushish vaqti.

1.1 – bo`limda cheksiz ikkilik so`zlar doir kerakli faktlar, xususan, ikkilik so'zlarning tili, davriyligi va murakkabligi, takrorlanishi keltirilgan. Shuningdek, cheksiz ikkilik so'zlarning alohida turlari o`rganilgan.

Agar biror u chekli so`z va $y$ cheksiz so`z uchun $\omega = uwy$ bo`lsa, u holda w chekli son ω cheksiz sonning **qismso'zi** deyiladi.

1-ta'rif. *Cheksiz so'z ω ning* ***murakkabligi*** *deb ω ning uzunligi n ga teng barcha qismso'zlari sonini aniqlaydigan funksiyaga aytiladi va bu funksiya quyidagi xossalarga ega:*

• *kamaymaydigan funksiya, ya'ni.,* $p_\omega(n+1) \ge p_\omega(n)$

• $p(1) = \# A$

• *ω davriy so`zlar uchun barcha $n\in\mathbb{N}$ larda shunday $p_\omega(n)\le C$ shartni qanoatlantiruvchi $C\in\mathbb{N}$ topiladi.*

2-ta’rif. Agar barcha n natural sonlar uchun $p_s(n)=n+1$ bajarilsa, u holda *s cheksiz so`z Shturm so’z deyiladi.*

Irratsioanal burish bilan bog`langan $\omega(x)=\{\omega_1,\omega_2,...\}$ cheksiz so`z **irratsional mexanik so`z** deyiladi.

Xususan, Shturm so'zlarining fundamental ekvivalentlik teoremasi keltirib o`tilgan va bu 3.3 -bo'limda asosiy tasdiqni isbotlash uchun juda muhim.

1-teorema. *Aytaylik ω cheksiz ikkilik so'z bo'lsin. Quyidagilar ekvivalentdir:*

(1) ω Shturm.

(2) ω irratsional mexanik.

(3) ω muvozanatli va aperiodik.

1.2 - bo'limda aylana gomeomorfizmlari nazariyasi, kritik aylana akslantirishlari, dinamik bo'linishlar va simvolik dinamika haqidagi dastlabki zarur tushunchalar o`rganilgan.

3-ta’rif. *Agar biror ε- atrof $U_\varepsilon(x_{cr})=(x_{cr}-\varepsilon,x_{cr}+\varepsilon)$ uchun f funksiya $C^{2d+1}(U_\varepsilon(x_{cr}))$ ga tegishli bo`lsa va quyidagi shart bajarilsa*

$$\frac{df}{dx}(x_{cr})=\frac{d^2f}{dx^2}(x_{cr})=...=\frac{d^{2d}f}{dx^{2d}}(x_{cr})=0,\ \ \frac{d^{2d+1}f}{dx^{2d+1}}(x_{cr})\neq 0,$$

u holda $x_{cr}\in S^1$ nuqta f gomeomorfizmning $(2d+1), d\ge 1$ tartibdagi kritik nuqtasi deyiladi..

4-ta’rif. *Agar f akslantirish yagona toq tartibli kritik nuqtaga ega bo`lsa, u holda f akslantirish kritik akslantirish deyiladi.*

So'nggi 20-25 yil mobaynida ko'plab mualliflar kritik akslantirishlar uchun qattiqlik muammosini o'rgandilar.

Ostlund va boshqa olimlar kritik aylana akslantirishlari sinfidagi renormgruppa almashtirishini o'rganganlar. Ostlund va boshqalarning ishidan foydalanib, uchinchi tartibli haqiqiy-analitik kritik aylana gomeomorfizmlar to'plamiga mos keladigan haqiqiy-analitik juftliklarni aniqlaymiz. Haqiqiy sonlar to'plamida qat'iy o`suvchi va quyidagi shartlani qanoatlantiradigan haqiqiy analitik juftliklar (ξ,η) to'plami $\mathcal{X}_{cr}$ ni qaraymiz:

(c_1) $0<\xi(0)<1, \xi(0)=\eta(0)+1;$

(c_2) $\xi(\eta(0))=\eta(\xi(0))>0;$

(c_3) $\xi'(0)=\eta'(0)=\xi''(0)=\eta''(0)=0,$ ammo $\xi'''(0)\neq 0$ va $\eta'''(0)\neq 0$

(c_4) $(\xi\circ\eta)'''(0)=(\eta\circ\xi)'''(0).$

(c_1) va (c_2) shartlar birlik aylanada $f=f_{\xi,\eta}$ gomeomorfizmni har bir $(\xi,\eta)\in\mathcal{X}_{cr}$ bilan bog`lash imkonini beradi. $[\eta(0),0]$ da $f=\xi$ va $[0,\xi(0)]$ da $f=\eta$ deb aniqlaymiz va $[\eta(0),\xi(0)]$ birlik intervalni chekka nuqtalari bilan ayniylanuvchi aylana bilan bog`laymiz. $f_{\xi,\eta}$ akslantirishning $\rho=\rho(f_{\xi,\eta})$ burish soni odatiy yo`l bilan aniqlanadi.

Burish soni “oltin kesimga”, ya’ni $\rho(f_{\xi,\eta})=\bar{\rho}=\dfrac{\sqrt{5}-1}{2}$ ga teng $\mathcal{X}_{cr}$ ning qism to`plami bo`lgan (ξ,η) juftliklarni $\mathcal{X}_{cr}(\bar{\rho})$ bilan belgilaymiz.

Quyidagi renormgruppa almashtirishini aniqlaymiz:

$$\Re:\mathcal{X}_{cr}(\bar{\rho})\to\mathcal{X}_{cr}(\bar{\rho}) \text{ da } \Re(\xi,\eta)=(\alpha\eta(\alpha^{-1}x),\alpha\eta(\xi(\alpha^{-1}x)),$$

bu yerda $\alpha:=\alpha_{\xi,\eta}=[\eta(0)-\eta(\xi(0))]^{-1}$.

(c_1), (c_2) shartlaridan $\alpha<-1$ ekanligi kelib chiqadi. $\Re$ renormgruppa almashtirishi $\mathcal{X}_{cr}(\bar{\rho})$ qism fazoda yagona giperbolik fiksirlangan (ξ_0,η_0) nuqtaga ega. Shuningdek, $\xi_0(x)$, $\eta_0(x)$ funksiyalar x^3 ning haqiqiy analitik funksiyalari va $\alpha_0:=\alpha_{\xi_0,\eta_0}\approx-1,2886$. (ξ_0,η_0) bilan bog`langan aylana akslantirishlarini $f_{cr}:=f_{\xi_0,\eta_0}$ orqali belgilaymiz.

f_{cr} ga C^1-qo`shma bo`lgan barcha aylana gomemomorfizmlari to`plamini $Cr(\bar{\rho})$ orqali belgilaymiz. Ma’lumki, ixtiyoriy ikkita topologik qo`shma gomeomorfizmlar bir hil burish soniga ega. Bundan tashqari, $Cr(\bar{\rho})$ ga tegishli gomeomorfizmlarning burish sonlari bir hil va $\bar{\rho}$ ga teng.

Axtam Djalilov $Cr(\bar{\rho})$ sinfdan bo`lgan kritik aylana akslantirishlari uchun termodinamik formalizm qurgan.

1.3 - bo'limda kritik aylana akslantirishlarining tushish vaqtlari uchun zarur bo'lgan ta'riflar va teoremalar mavjud.

Aytaylik f irratsional $\rho=\rho(f)$ burish soniga ega bo`lgan $S^1=R^1/Z\cong[0,1)$ aylanadagi yo`nalishni saqlovchi gomeomorfizm bo`lsin. $\mu=\mu(f)$ o`lchov f ning yagona invariant ehtimollik o`lchovi bo`lsin. $z\in S^1$ nuqtani fiksirlaymiz va $V_\varepsilon(z)=[z,z+\varepsilon]\subset S^1$ intervalni qaraymiz. Quyidagi formula yordamida V_ε intervalga birinchi tushish vaqtini aniqlaymiz: $E_\varepsilon(t)=\inf\{i\geq1:T^i(t)\in V_\varepsilon(z)\}$.

Endi esa normallangan tushish vaqtini $\bar{E}_\varepsilon(t)=\mu(V_\varepsilon(z))\mu(E_\varepsilon(t))$ orqali aniqlaymiz. $\bar{E}_\varepsilon(t)$ tasodifiy miqdorning taqsimot funksiyasi yaqinlashishi masalasini qaraymiz, ya’ni quyidagi taqsimot funksiyasining $\varepsilon\to0$ da, limit funksiyasining uzluksizlik nuqtalariga tegishli bo'lgan har bir t uchun:

$$F_\varepsilon(t)=\mu\left(x\in S^1:E_\varepsilon^{(1)}(x)\leq t\right),\forall t\in\mathbb{R}^1$$

yaqinlashishi o’rganamiz.

Coelho va de Faria $f_\rho(x)=x+\rho \; mod 1$ chiziqli irratsional burishlar uchun $\bar{E}_\varepsilon(t)$ tasodifiy miqdorning yaqinlashish masalasini o`rgangan. Ma’lumki, $f_\rho$ chiziqli irratsional burishlar uchun yagona invariant o`lchov bu ℓ Lebeg o`lchovidir. Agar $\varepsilon=\varepsilon_n$ ni Coelho va de Faria ishlaridagi kabi tanlab $V_{\varepsilon_n}(z)$ interval $f_\rho$ ning renorm intervallari ketma-ketligiga mos bo`lsa, u holda Lebeg o`lchovida deyarli barcha $\rho$ burish sonlari uchun $E_\varepsilon(t)=\mu(V_\varepsilon(z))E_\varepsilon(t)$ normallangan tushish vaqti $\varepsilon$ nolga intilganda qonuniyat bo`yicha yaqinlashmasligi isbotlangan hamda ε_n qismiy ketma-ketlikning barcha mumkin bo'lgan limit qonunlari olingan. Shuningdek, agar $F_{\varepsilon_n}(t)$ har biri nolga yaqinlashadigan biror ε_n uchun yaqinlashsa, $F(t)$ ehtimollik taqsimotining limiti ikkita uzilish nuqtasiga ega zinapoya funksiyasi yoki [0,1] oraliqda tekis taqsimot bo`ladi.

Aytaylik q_n, $n \geq 1$ da f_ρ ning birinchi qaytish vaqti bo`lsin. Biror $\theta \in (0,1)$ ni fiksirlaymiz. Quyidagi formula yordamida barcha $n \geq 1$ lar uchun $c_n(\theta)$ nuqtalarni aniqlaymiz:

$$\mu([x_0, c_n(\theta)]) = \theta \cdot \mu([x_0, f_\rho^{q_n}(x_0)]).$$

$\Delta_{n,c}$ orqali $[x_0, c_n(\theta))$ intervalni belgilaymiz. $\Delta_{n,c}$ intervalga tushish vaqti $E_{n,c}$ ni qaraymiz. Normallangan tushish vaqtini quyidagi ko`rinishda aniqlaymiz:

$$\bar{E}_{n,c}(x) = \mu(\Delta_{n,c}) E_{n,c}(x).$$

$F_{n,c}(t)$ orqali $\bar{E}_{n,c}(x)$ ning taqsimot funksiyasini belgilaymiz. Ixtiyoriy ρ irratsional son uchun $\bar{E}_{n,c}(x)$ normallangan tushish vaqti ε nolga intilganda qonuniyat bo`yicha yaqinlashmasligi isbotlangan.

Ushbu dissertatsiyada, yagona kritik nuqtaga ega bo`lgan aylana gomeomorfizmlari uchun normallangan tushish vaqtlarini tadqiq qilamiz. Aytaylik, $f \in Cr(\bar{\rho})$ kritik aylana gomeomorfizmi bo`lsin. Aylanada ikkita tabiiy o`lchovlar mavjud: $\mu$ invariant ehtimollik o`lchovi va ℓ Lebeg o`lchovi. $\bar{E}_{n,c}(x)$ normallangan tushish vaqtining mos ravishda μ va ℓ ga nisbatan $F_{n,c}(t)$ va $\Phi_{n,c}(t)$ taqsimot funksiyalarini qaraymiz. f akslantirishning $F_{n,c}(t)$ taqsimot funksiyasi f_ρ chiziqli burishning taqsimot funksiyasi bilan ustma-ust tushadi. Shu bois, Coelho va de Faria ishidagi barcha tasdiqlar f invariant μ o`lchovlar uchun ham o`rinli. μ invariant o`lchov Lebegga nisbatan singular. E'tiborli jihati chekli sondagi sinish nuqtalari va irratsional $\rho$ burish soniga ega bo`lakli silliq f aylana gomeomorfizmlari Lebeg o`lchoviga nisbatan ham ergodik bo`ladi. Ta'rifga ko`ra

$$\Phi_\varepsilon(t) = l\left(x \in S^1 : E_\varepsilon^{(1)}(x) \leq t\right), \ \forall t \in \mathbb{R}^1.$$

A.Djalilov "oltin kesim" irratsional burish soniga ega kritik aylana akslantirishlari va kritik nuqtaning renorm atrofi uchun taqsimot funksiyalarinig Legeb o`lchoviga nisbatan limit holatlarini o`rgangan. A. Djalilov va J. Karimov bitta sinish nuqtasi va "oltin kesim" irratsional burish soniga ega aylana gomeomorfizmlari uchun taqsimot funsiyasining limit holatini tadqiq qilgan.

1.4-bo`limda sinishga ega bo`lgan aylana gomeomorfizmlariga tegishli zarur tasdiqlar keltirilgan.

Sinish nuqtalariga ega bo'lgan aylana akslantirishlari so'nggi 20 yil ichida K. Xanin, D. Xmelev, A. Teplinskiy, D. Smania, K. Kunya, D. Mayer, A. Djalilov, I. Liousse va boshqalar tomonidan muttasil ravishda o`rganilgan. Sinish nuqtasiga ega bo'lakli silliq aylana gomeomorfizmlari bu - aylana diffeomorfizmlarining tabiiy umumlashmasidir. Agar aylaning biror nuqtasida birinchi tartibli hosilasida sakrash mavjud bo'lsa, bu nuqta **sinish nuqtasi** deb ataladi. Sinishlarga ega eng oddiy gomeomorfizmlar – bu bo`lakli chiziqli aylana gomeomorfizmlaridir. M. Erman ikkita sinish nuqtasiga va irratsional burish soniga ega bo`lgan bo`lakli chiziqli gomeomorfizmlarning invariant o'lchovi absolyut uzluksiz bo`lishi uchun ikkala sinish nuqtasi bir orbitada yotishi zarur va yetarliliginini isbotlagan.

Sinishga ega nochiziqli bo`lakli silliq aylana gomeomorfizmlarining invariant o`lchovlari K. Xanin, A. Djalilov, A.Teplinskiy, D. Mayer, I. Liousse, D. Smania, va boshqalar tomonidan o`rganilgan.

Diffeomorfizmlardan farqli o'laroq, bunday gomeomorfizmlarning o'zgarmas o'lchovlari Lebeg o'lchoviga nisbatan singulyardir (K. Xanin, A. Djalilov, D. Mayer, I. Liousse). Sinishga ega yetarlicha silliq (har bir oraliqda silliq) gomeomorfizmlar uchun ularning renormlari chiziqli kasrli akslantirishlar bilan yaqinlashadi (K. Xanin, D. Xmelev, A. Teplinskiy, D. Smaniya, K. Kunya, S. Kosich).

"**Aylana akslantirishlari bilan bog`langan murakkablik funksiyalari va tushish vaqtlari**" deb nomlangan ikkinchi bobda aylananing irratsional burishlari va kritik aylana akslantirishlarining tushish vaqtlari asimptotik holatlari bilan bog'liq bo'lgan cheksiz ikkilik so'zlar tadqiq qilingan.

2 - bobning birinchi bo`limida birlik aylanada irratsional burish bilan bog'liq bo'lgan cheksiz ikkilik ketma-ketliklarning asosiy xossalari o`rganilgan.

$[0,1)$ intervalda $f_\rho(x) = x + \rho \; mod 1$ irratsional burishni qaraymiz va $b \in S^1$ nuqtani fiksirlaymiz. Aylanada $P = \{[0,b),[b,1)\}$ bo`linishni qaraylik. Har bir $n \geq 1$ uchun $S_n(0,b)$ to`plamni aniqlaymiz:

$$S_n(0,b) := \{0, f_\rho^{-1}(0),..., f_\rho^{-n+1}(0)\} \bigcup \{b, f_\rho^{-1}(b),..., f_\rho^{-n+1}(b)\}.$$

$S_n(0,b)$ to`plamning nuqtalari $P_n$ bilan belgilangan bo`linishni tashkil qiladi. $P_1 := P$ bo`lsin. Aslini olib qaraganda,

$$P_n := P \vee f_\rho^{-1}(P) \vee ... \vee f_\rho^{-n+1}(P)\}$$

Bu yerda $P \vee Q := \{A \bigcap B : A \subset P, B \subset Q\}$.

P_n va P_{n+1} bo'linishlarning elementlarini hech qanday indekslarsiz I^n va I^{n+1} bilan belgilaymiz.

P_n, $n \geq 0$ bo`linish ketma-ketligidan foydalangan holda simvolik dinamikaning ba'zi maxsus turlarini hosil qilamiz. Quyidagi $I_1 := [0,b)$ va $I_0 := [b,1)$ belgilashlarni kiritamiz. $\nu_b : S^1 \to \{0,1\}$ kodlash funksiyasini qaraymiz: barcha $i \geq 0$ lar uchun

$$\nu_b(f_\rho^i(x)) := \begin{cases} 0, \text{ agar } x \in f_\rho^{-i}(I_0), \\ 1, \text{ agar } x \in f_\rho^{-i}(I_1). \end{cases}$$

Ixtiyoriy $x \in S^1$ ni olaylik.

$$\underline{\omega} = (\omega_0 \omega_1 ... \omega_n ...) := (\nu_b(x) \nu_b(f_\rho(x)) ... \nu_b(f_\rho^n(x)) ...) \qquad (1)$$

orqali nol va birlardan iborat $\underline{\omega} := \underline{\omega}(x)$ cheksiz ketma-ketlikni aniqlaymiz.

Shunday maqbul cheksiz sonlar to`plamini $L_\omega(\rho,b)$ bilan belgilaymiz, ya'ni $L_\omega(\rho,b) = \{\underline{\omega}(x), x \in S^1\}$. Quyidagi belgilashlarni kiritamiz:

$\omega_m^{m+s} = (\omega_m \omega_{m+1} ... \omega_{m+s}),\ m,\ s \geq 0,$ $\underline{\omega}$ cheksiz so'zning qismso'zi, $\omega_m^{\infty} = (\omega_m \omega_{m+1} ...),\ m \geq 0,$ $\underline{\omega}$ ning dum qismi.

4-ta'rif. *Agar barcha $n \geq N$ lar uchun $\omega_{n+T} = \omega_n$ shatrni qanoatlantiruvchi shunday $N, T \in N$ topilsa, u holda $\underline{\omega}$ cheksiz so`z shartli davriy deyiladi. Aks holda $\underline{\omega}$ davriy emas deyiladi.*

Simvolik kodlash ta'rifidan quyidagi foydali faktlar osongina kelib chiqadi.

a) *Aytaylik, I^n interval P_n bo`linishning ixtiyoriy intervali bo`lsin. U holda ixtiyoriy ikkita $x, y \in I^n$ nuqtalar uchun ularning $n+1$ uzunlikdagi prefikslari ustma-ust tushadi, ya'ni $\omega_0^n(x) = \omega_0^n(y)$. Shu sababli, $\bigcap_{i=0}^{m-1} f_\rho^{-i}(P)$ ning har bir intervaliga uzunligi m bo`lgan bitta so`z mos keladi.*

b) *$u_0 u_1 ... u_{m-1}$ so`zi biror maqbul $\underline{\omega} = (\omega_0 \omega_1 ... \omega_n ...)$ cheksiz so`zning qismso'zsi bo`lishi uchun, ya'ni $i = 0, ..., (m-1)$ larda $\omega_{n+i} = u_i$ bajarilishi uchun $f_\rho^n(x) \in I(u_0 u_1 ... u_{m-1})$ bo`lishi zarur va yetarli, bu yerda $I(u_0 u_1 ... u_{m-1}) = \bigcap_{i=0}^{m-1} f_\rho^{-i}(I_{b_i}) \in P_m$.*

c) *Xususan, $I(u_0 u_1 ... u_{m-1})$ interval bo`sh bo`lmasligi uchun $u_0 u_1 ... u_{m-1}$ ning $\underline{\omega}$ ning qismso'zi bo`lishi zarur va yetarli.*

Quyidagi belgilashni kiritamiz: $d_n = \max_{I^n \in P_n} |I^n|$.

1-lemma. *Barcha $n \geq 1$ lar uchun $d_n \geq d_{n+1}$ va $\lim_{n \to \infty} d_n = 0$ o`rinli.*

$I^n(x)$ orqali x nuqtani o'z ichiga olgan $n-$bo`linish $P_n$ ning intervalini va $W_n(\underline{\omega}(x))$ orqali $\underline{\omega}(x)$ ikkilik cheksiz so`zning uzunligi n ga teng qism so`zlari to`plamini belgilaymiz.

2-teorema. *Aytaylik $x, y \in S^1$ va $\underline{\omega}(x), \underline{\omega}(y)$ cheksiz so'zlar (1) bo`yicha aniqlangan ularning cheksiz kodlash ketma-ketliklari bo`lsin. U holda quyidagilar o`rinli:*

1. *Barcha $n \geq 0$ lar uchun $W_n(\underline{\omega}(x)) = W_n(\underline{\omega}(y))$ bo'ladi.*
2. *$\underline{\omega}(x)$ tekis takrorlanuvchi.*
3. *Ixtiyoriy $x \in S^1$ uchun $\underline{\omega}(x)$ davriy emas.*

2.2-bo'limda birlik aylanada irratsional burish bilan bog'liq bo'lgan cheksiz ikkilik ketma-ketliklarning murakkablik funksiyalari o`rganilgan.

Eslatib o'tamiz, $\underline{\omega}$ cheksiz so'zning murakkabligi - bu uning n uzunlikdagi qismso'zlari sonini aniqlaydigan $p_{\underline{\omega}}(n)$ funksiya.

Bundan buyog'iga, soddalik uchun qismso'z murakkabligi o'rniga murakkablik funksiyasi deb yozamiz.

Barcha $b \in [0,1)$ uchun $L_\omega(\rho, b)$ ning cheksiz so`zlari murakkablik funsiyalarini o`rganamiz. Ma'lumki,

i) Agar $b=\{\rho,1-\rho\}$ bo`lsa, u holda barcha $n\geq 1$ lar uchun $p_{\underline{\omega}}(n)=n+1$ o`rinli,

ii) Agar $b\neq\{\rho,1-\rho\}$ bo`lsa, u holda barcha $n\geq 1$ lar uchun $p_{\underline{\omega}}(n)\geq n+1$ o`rinli bo’ladi.

Biz ikkinchi holatni o’rganamiz.

$r(n)$ orqali n uzunlikdagi maxsus o`ng qismso’zlar (right special factors (**r.s.f**)) sonini belgilaymiz va aytaylik $k_0=min\{n\geq 1: r(n)=2\}$ bo`lsin.

A ikkilik alifboda $r(n)$ maxsus o`ng qismso’zlar soni quyidagi formula yordamida murakkablik funksiyalarini aniqlashda foydalaniladi:

$$p_{\underline{\omega}}(n+1)=p_{\underline{\omega}}(n)+r(n).$$

Endi esa 2.2-bo'limning ikkita asosiy teoremasini keltiramiz. Birinchi teoremada b nuqta 0 nuqtaning orbitasida bo'lmagan holat o’rganilgan.

3-teorema. *Aytaylik, f_ρ akslantirish $\rho\in(0,1)$ irratsional burchakka chiziqli burish bo`lsin va $b+n\rho\notin Z$ bajarilsin. U holda quyidagilar o`rinli:*

1. Agar $0<\rho<b$ bo`lsa, u holda barcha $n\geq 1$ uchun $p_{\underline{\omega}}(n)=2n$.

2. Agar $0<b<\rho$ bo`lsa, u holda

$$p_{\underline{\omega}}(n):=\begin{cases} n+1, \ \ agar \ \ n<k_0, \\ 2n-k_0+1, \ \ agar \ \ n\geq k_0. \end{cases}$$

Quyidagi teoremada b nuqta 0 nuqtaning orbitasida yotadigan holat o’rganilgan.

4-teorema. *Aytaylik, f_ρ 3-teoremadagi shartlarni qanoatlantirsin va ayrim $d\in Z\diagdown\{0\}$ uchun $b+d\rho\in Z$ bo`lsin. U holda quyidagilar o`rinli:*

1. Agar $0<\rho<b$ bo`lsa, u holda $p_{\underline{\omega}}(n):=\begin{cases} 2n, \ \ \ agar \ \ n<|d|, \\ n+d, \ agar \ \ n\geq |d|. \end{cases}$

2. Agar $0<b<\rho$ bo`lsa. u holda $p_{\underline{\omega}}(n):=\begin{cases} n+1, \ agar \ n<k_0, \\ 2n-k_0+1, \ agar \ k_0\leq n<|d| \\ n+d-k_0-1, \ agar \ n\geq |d|. \end{cases}$

2.3-bo`limda yagona kubik kritik nuqtaga ega bo`lgan $f\in Cr(\bar{\rho})$ kritik aylana akslantirishlari uchun normallangan tushish vaqtlarining asimptotik holatlari tadqiq qilingan.

Quyidagi 5-teorema isbotlangan, unda $E_{n,\theta}^{(1)}(x)$ tasodifiy miqdor uchun $\Phi_{n,\theta}(t)=0$ taqsimot funksiyasining aniq formulasi topilgan.

2.3 -bo'limning asosiy natijasini keltiramiz.

5-teorema. *Aytaylik $\bar{\rho}=\dfrac{\sqrt{5}-1}{2}$ va $f\in Cr(\bar{\rho})$ kritik akslantirish bo`lsin. $\theta\in(0,1)$ da $[x_c,c_n(\theta)]$ intervalga normallangan birinchi tushish vaqti $E_{n,\theta}^{(1)}(x)$ ga* *aylanada Lebeg o`lchovi bo`yicha mos keluvchi* $\left\{\Phi_{n,\theta}(t)\right\}_{n=1}^{\infty}$ *taqsimot funksiyalari ketma-ketligini qaraymiz. U holda*

1) Barcha $t \in \mathbb{R}^1$*lar uchun quyidagi chekli limit* $\lim\limits_{n\to\infty} \Phi_{n,\theta}(t)\ (t) = \Phi_{\theta}(t)$ *mavjud, bundan tashqari* $t \le 0$ *da* $\Phi_{\theta}(t) = 0$ *va* $t > 1$ *da* $\Phi_{\theta}(t) = 1$*;*

2) $\Phi_{\theta}(t)$ *limit funksiya [0,1] da qat'iy o`suvchi va* $\mathbb{R}^1$ *uzluksiz taqsimot funksiyasi;*

3) $\Phi_{\theta}(t)$ *limit funksiya [0,1] da singulyar, ya'ni deyarli barcha yerda (aylanada* ℓ *Lebeg o`lchovi bo`yicha)* $\dfrac{\Phi_{\theta}(t)}{dt} = 0$.

"**DANJUA TENGLIGI VA P-GOMEOMORFIZMLAR UCHUN SHTURM SO`ZLARI**" deb nomlangan 3-bobda ikkita sinish nuqtasiga ega bo`lgan bo`lakli-silliq aylana gomeomorfizmlarining sinish nuqtalari joylashuvi hamda $h^{q_n}$ Erman akslantirishlarining normallangan hosilalari holati o`rganilgan. Shuningdek, h^{q_n} akslantirishining normallangan hosilalariga bog`liq cheksiz ikkilik ketma-ketliklari murakkablik funksiyalari tadqiq qilingan.

$x_0 \in S^1$ nuqta va uning $O(x_0) = \{x_i = f^{\,i}(x_0),\ i \in \mathbb{Z}\}$ orbitasini qaraymiz. Eslatib o`tamiz, $\mathcal{P}_n(x_0) = \{I_i^{(n-1)}(x_0), 0 \le i < q_n\} \cup \{I_j^{(n)}(x_0), 0 \le j < q_{n-1}\}$ intervallar sistemasi ixtiyoriy $n$ uchun aylananing bo`linishini aniqlaydi, bu x_0 nuqtaning n -dinamik bo`linishi deyiladi.

ρ_f irratsional burish soni va bitta orbitada yotmagan ikkita a_0 va c_0 sinish nuqtalariga ega bo`lgan f P-gomeomorfizmni qaraymiz. $\dfrac{p_n}{q_n}$ orqali ρ_f ning yaqinlashuvchi kasrlarini belgilaymiz. S^1 aylanada $f^{\,q_n}$ ning sinish nuqtalari joylashuvini va $Df^{\,q_n}$ hosilani aniqlaymiz. Ravshanki, $f^{\,q_n}$ akslantirish $BP_f^n := BP_f^n(a_0) \cup BP_f^n(c_0)$ orqali ifodalangan $2q_n$ ta sinish nuqtasiga ega, bu yerda mos ravishda $BP_f^n(a_0) := \{a_0^*, a_{-1}^*, ..., a_{-q_n+1}^*\}$ va $BP_f^n(c_0) := \{c_0^*, c_{-1}^*, ..., c_{-q_n+1}^*\}$ hamda mos ravishda $a_{-i}^* = f^{-i}(a_0)$ va $c_{-i}^* = f^{-i}(c_0)$, $0 \le i \le q_n - 1$. Ko'rish mumkinki, $f^{\,q_n}$ akslantirishning ushbu sinish nuqtalari S^1 aylanada o`zaro kesishmaydigan $2q_n$ ta intervallarga bo`linadigan $B_n(f)$ bo`linishni aniqlaydi.

Ikkinchi sinish nuqtasi c_0 ning $B_n(f)$ bo`linish intervallaridagi joylashuvini 4 ta farqli holatlarga bo`lamiz: I hol, II hol, III hol va IV hol.

Aytaylik, $\mathcal{P}_n(a_0^*)$ bo`linish $f$ akslantirishga nisbatan $a_0^* = a_0$ sinish nuqtasi orqali aniqlangan $n$-dinamik bo`linish bo`lsin. Faraz qilaylik, a_0^* va c_0^* sinish nuqtalari bitta orbitada yotmasin, u holda ikkinchi sinish nuqtasi uchun quyidagi 3ta holat o'rinli bo'ladi:

I hol. $c_0^* \in I_{i_0}^{(n)}(a_0)$, ba'zi $0 \le i_0 < q_{n-1}$ lar uchun;

II hol. $c_0^* \in f^{j_0}((a_0, a_{-q_n}])$, ba'zi $0 \le j_0 < q_n$ uchun;

III hol. $c_0^* \in f^{j_0}((a_{-q_n}, a_{q_{n-1}}))$, ba'zi $0 \le j_0 < q_n$ uchun.

Undan tashqari, a_0^* va c_0^* sinish nuqtalari bitta orbitada yotishi ham mumkin, ya'ni

IV hol. $c_0^* = f^{i_0}(a_0^*)$, ba'zi $0 \le i_0 < q_n$ lar uchun.

3.1-bo`limda $f^{q_n}$ akslantirish sinish nuqtalari joylashuvini I va III hollarda o`rganganmiz. Va biz, Erman akslantirishlari uchun Danjua tengligini I va III holatlarda isbotlaganmiz.

Biz h orqali Erman akslantirishlarini belgilaymiz, ya'ni ikkita sinish nuqtasiga ega bo`lakli-chiziqli aylana akslantirishlari. $\rho$ irratsional burish soniga ega bo`lgan ixtiyoriy Erman akslantirishlari Danjua teoremasi shartlarini qanoatlantiradi. Demak h akslantirish f_ρ chiziqli burish bilan topologik ekvivalent.

Biz, h^{q_n} akslantirishning normallangan hosilalarini $\sigma_h(a_0)$ sinish kattaligi hamda h^{q_n} ning sinish nuqtalaridan hosil qilingan $B_n(h)$ bo`linish intervallarining $\mu_h$-o`lchovi yordamida ifodalaganmiz.

I va III hollarda $a_0^* = 0$ dan kelib chiqadigan $PB_h(a_0^*)$ sinish nuqtalari va $c_0^* = c_0$ dan kelib chiqadigan $PB_h(c_0^*)$ sinish nuqtalari S^1 aylanada navbatma-navbat joylashadilar.

Aytaylik n toq bo`lsin. h^{q_n} akslantirishning sinish nuqtalaridan foydalanib quyidagi subintervallarni aniqlaymiz:

$$A_n := \bigcup_{s=1}^{i_0} [c^*_{-i_0+s},\, a^*_{-q_n+s}], \quad B_n := \bigcup_{s=i_0+1}^{q_n} [c^*_{-i_0-q_n+s},\, a^*_{-q_n+s}].$$

III holda keltirilgan farazimizda subintervallar mos ravishda $[a^*_{-q_n+s}, c^*_{-i_0+s}]$, $1 \le s \le i_0$ va $[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}]$, $i_0+1 \le s \le q_n$ ko`rinishida berilgan. Bu subintervallarni quyidagi qism to`plamlarga birlashtiramiz:

$$A_n := \bigcup_{s=1}^{i_0} [a^*_{-q_n+s}, c^*_{-i_0+s}], \quad B_n := \bigcup_{s=i_0+1}^{q_n} [a^*_{-q_n+s}, c^*_{-i_0-q_n+s}].$$

Juft n uchun yuqoridagi intervallarning yo'nalishini o'zgartirish kerak. Shunday qilib, I holda bizda mos ravishda quyidagi kesishmaydigan intervallar sistemasi mavjud $[a^*_{-q_n+s}, c^*_{-i_0+s}]$, $1 \le s \le i_0$, va $[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}]$, $i_0+1 \le s \le q_n$.

III holda mos ravishda $[c^*_{-i_0+s}, a^*_{-q_n+s}]$, $1 \le s \le i_0$ va $[c^*_{-i_0-q_n+s}, a^*_{-q_n+s}]$, $i_0+1 \le s \le q_n$ topiladi. I hol va n juft bo`lganda va mos ravishda III hol va $n$ toq bo`lganda A_n va B_n qism to`plamlarini avvalgidek aniqlash mumkin. Yuqoridagi konstruktsiyalar shuni ko'rsatadiki, A_n va B_n qism to'plamlaridagi har bir intervalning chegaralari mos ravishda $PB_h(a_0^*)$ va $PB_h(c_0^*)$ dan olingan sinish nuqtalaridan iborat intervaldir.

$\sigma = \sigma_h(a_0)$ orqali h akslantirishning $x = 0$ nuqtadagi sinish kattaligini belgilaymiz va 3.1-bo`limning asosiy natijasini keltiramiz.

6-teorema. *Aytaylik, h akslantirish ρ_h irratsional burish soniga, turli orbitalarda yotuvchi ikkita $a_0^* = 0$ va $c_0^* := c_0$ sinish nuqtalariga ega va umumiy sinish kattaligi $\sigma_h = 1$ bo`lgan bo`lakli-chiziqli aylana gomeomorfizmi bo`lsin. Faraz qilaylik, c_0^* biror i_0, $0 \le i_0 < q_{n-1}$ uchun mos ravishda I va III hollardagi shartlarni bajarsin. U holda I holda*

$$(Dh^{q_n}(x))^{(-1)^n} = \begin{cases} \sigma^{\mu_h(A_n \cup B_n)-1}, & agar\ x \in A_n \cup B_n \\ \sigma^{\mu_h(A_n \cup B_n)}, & agar\ x \in S^1 \setminus (A_n \cup B_n); \end{cases}$$

va mos ravishda III holda

$$(Dh^{q_n}(x))^{(-1)^{n+1}} = \begin{cases} \sigma^{\mu_h(A_n \cup B_n)-1}, & agar\ x \in A_n \cup B_n \\ \sigma^{\mu_h(A_n \cup B_n)}, & agar\ x \in S^1 \setminus (A_n \cup B_n). \end{cases}$$

o`rinli.

3.2-bo`limda II va IV hollarda h^{q_n} sinish nuqtalarining joylashuvi tadqiq qilingan. II va IV hollarda Erman akslantirishlarida h^{q_n} ning normallangan hosilalari uchun Danjua tengligi isbotlangan.

3.3-bo`limda $h$ Erman akslantirishlari bilan bog`liq cheksiz ketma-ketliklarning murakkablik funksiyalari o`rganilgan. $h^{q_n}$, $n > 0$ ning normallangan hosilalari bilan bog'liq cheksiz ikkilik ketma-ketliklari Shturm bo`lishi isbotlangan.

XULOSA

Ushbu dissertatsiya aylanada irratsional burish bilan bog`liq cheksiz ikkilik ketma-ketliklar murakkablik funksiyalarini, kritik aylana akslatirishlari uchun tushish vaqtlari holatlarini va bo`lakli-chiziqli akslantirishlar uchun Danjua funksiyasini baholashni tadqiq qilishga bag`ishlangan.

Tadqiqot ishining asosiy natijalari quyidagilar:

Irratsional aylana burishi bilan bog'liq cheksiz ikkilik ketma-ketliklarning murakkablik funksiyalari topilgan.

Bitta kubik kritik nuqtaga va "oltin kesim" burish soniga ega bo`lgan aylana akslantirishlarining nomallangan umumlashgan tushish vaqtlari taqsimot funksiyalari bo`yicha yaqinlashishi va limit taqsimot funksiyasi singular, ya'ni u uzluksiz, qat'iy o`suvchi va Lebeg o`lchovi bo`yicha deyarli barcha yerda hosilasi 0 ga teng ekanligi isbotlangan.

Ikkita sinish nuqtasiga ega bo`lakli-chiziqli aylana gomeomorfizmlarining sinish nuqtalari joylashuvi topilgan.

h^{q_n} Erman akslantirishlari uchun Danjua tengligi isbotlangan.

h^{q_n} ning normallangan hosilalari bilan bog'liq cheksiz ikkilik ketma-ketliklar Shturm ekanligi ko`rsatilgan.

SCIENTIFIC COUNCIL AWARDING OF THE SCIENTIFIC DEGREES DSc.02/30.12.2019.FM.86.01 INSTITUTE OF MATHEMATICS NAMED AFTER V.I. ROMANOVSKIY

INSTITUTE OF MATHEMATICS

JALILOV ALISHER AKBAROVICH

DENJOY EQUALITY AND INFINITE BINARY SEQUENCES ASSOCIATED WITH CIRCLE HOMEOMORPHISMS

01.01.01-Mathematical analysis

ABSTRACT OF THESIS OF THE DOCTOR OF PHILOSOPHY (PHD) ON PHYSICAL AND MATHEMATICAL SCIENCES

TASHKENT-2021

The theme of thesis of doctor of philosophy (PhD) on physical and mathematical sciences was registered at the Supreme Attestation Commission at the Cabinet of Ministers of the Republic of Uzbekistan under number B2019.4.PhD/FM430.

Thesis has been prepared at Institute of Mathematics named after V.I.Romanovskiy.

The abstract of the thesis is posted in three languages (Uzbek, English, Russian (summary)) on the website http://kengash.mathinst.uz and in the website of "ZiyoNet" Information and educational portal http://www.ziyonet.uz/.

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Defense will take place "___" _________ 2021 at______at the meeting of Scientific Council number DSc.02/30.12.2019.FM.86.01 at Institute of Mathematics named after V.I. Romanovskiy. (Address: University str. 9, Almazar area, Tashkent city, 100174, Uzbekistan, Ph.: (99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz)

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INTRODUCTION

Actuality and demand of the theme of the thesis. In the world, many scientific and applied researches are reduced to the study of one dimensional and symbolic dynamical systems. The theory of circle homeomorphisms is important part of one-dimensional dynamical systems. Circle homeomorphisms were first studied in the pioneering works of A. Poincaré in connection with the problem of celestial mechanics.

At the present time in the world one of the interesting problems of the theory of dynamical systems are the problems associated with the maps of the circle. An important class of circle maps is circle diffeomorphisms. In a sense, circle diffeomorphisms have been well studied by now. A natural generalization of circle diffeomorphisms are critical maps and piecewise smooth homeomorphisms with break points. In the last 20 years such kind of circle maps with singularity points were obtained in the works of such outstanding mathematicians as A. Poincaré, A. Denjoy, A.N. Kolmogorov, V.I. Arnold, J. Moser, M. Erman, J.C. Yokkoz, J.G. Sinai, K.M. Khanin, D. Ornstein, J. Katznelson and others. Nowadays, intensively studied by many authors (M. Herman, J. Yoccoz, G. Swiatek, A.Avila, K.Khanin, D. Mayer, A. Dzhalilov and others).

Many problems of dynamical systems, information theory, geometry, cardiac problems, biology and practice reduced to study symbolic dynamical systems. To study the complexity of infinity symbolic sequences is one of classical problems of symbolic dynamical systems. The irrational rotation of the circle maps produced such called Sturmanian words which are very important in information theory.

In our country, much attention has been paid to develop important directions of one-dimensional dynamical systems and symbolic dynamics which have applications to the applied and fundamental sciences. Investigations on the international level in such important areas as the functional analysis, mathematical physics, theory of probability and theory of dynamical systems considered as the main task of fundamental research. Circle maps and symbolic dynamical systems are important not only for natural sciences, but also for economics, information theory, biology, for the study of various heart diseases, for blood tests, etc.

The subject and object of research of this dissertation are in line with tasks identified in the Decrees and Resolutions of the President of the Republic of Uzbekistan of February 7, 2017, PF-4947, «On the strategy of action for the further development of the Republic of Uzbekistan», PQ-4387 dated July 9, 2019 «On state support for the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovskiy of the Academy of Sciences of the Republic of Uzbekistan», PQ-4708 of May 7, 2020 «On measures to improve the quality of education and research in the field of mathematics» as well as in other regulations related to basic sciences.

Connection of research to priority directions of development of science and technologies of the Republic. This study was performed in accordance with the priority areas of science and technology of the Republic of Uzbekistan IV, «Mathematics, Mechanics and Computer Science».

The degree of scrutiny of the problem. The field of symbolic dynamics evolved as a tool for analyzing general dynamical systems. Hadamard first successfully used the symbolic dynamics techniques in his analysis of geodesic flows on surfaces of negative curvature in 1898. Forty years later the subject got its first systematic study, and its name, in the foundational paper of Marsten Morse and Gustav Hedlund. In the future the symbolic dynamics used in information theory by C.E. Shannon.

Ya. Sinai in his fundamental work used the symbolic dynamis for investigating Anosov's flows. E. Vul, Ya. Sinai and k. Khanin constructed the symbolic dynamics for Feigenbaum's map and build the thermodynamic formalism. The symbolic dynamics plays important role in the theory circle maps. It is well known that every irrational rotation of the circle produced the space of dual sequences X. To study the complexity functions of dual sequences is one of main problem of symbolic dynamical systems. Coelho, De Faria studied the behaviour of rescaled hitting times for irrational rotations. A. Dzhalilov builds the thermodynamic formalism for critical circle maps.

Denjoe inequalities play an important role in circle dynamics. Sinai and Khanin obtained the improved version of Denjoe inequality and on its base proved the smothness of conjugationg map.

The connection of the theme of the thesis with the research plans of the higher education institute, where the research on the thesis is carried out. The dissertation research is done in accordance with the planned theme of scientific research OT-F4-82 + OT-F4-87 «Local derivations and automorphisms of operator and nonassociative algebras, phase transitions and chaos in nonlinear dynamical systems» + «Investigation the asymptotic properties indexed imperic processes in the class of measurable functions» (2017–2021) at the National University of Uzbekistan.

The aim of the research work is to study the complexity functions for binary infinite sequences associated by irrational rotations of the circle, the hitting times for critical circle maps and Denjoy equality for Herman's maps.

Research problems:

to investigate the complexity functions of infinite binary sequences associated by irrational circle rotations;

to study the complexity functions of infinite sequences produced by Denjoy functions and its high iterations;

to study the structure of orbits of break points for Herman's map;

to find an explicit expression for Denjoy products of Herman's map by the size of break point and invariant measure;

The research object: the complexity function of infinite sequences, irrational rotations of the circle, Herman's map, critical circle maps, hitting times for circle maps.

The research subject: symbolic dynamical systems, the theory of circle maps, invariant measures.

Research methods. In the work used the methods of mathematical analysis, functional analysis, symbolic dynamics, ergodic theory and probability theory.

Scientific novelty of the research work is as follows:

It is found the complexity functions of infinite binary sequences associated by irrational circle rotations;

It is proved that the rescailed generalized hitting times of critical circle maps with one cubic critical point and golden mean rotation number converges in distribution and the limit distribution is singular function i.e. it is continuous, strictly increasing and has zero derivative almost everywhere w.r.t. Lebesgue measure.

It is found the location of break points of piecewise-smooth circle homeomorphisms with two break points.

It is proved the Denjoy equality for Herman's map h^{q_n}.

It is showed that the binary sequence associated by rescaled derivatives of h^{q_n} is Sturmian.

Practical results of the research are that mathematical models of population processes in mathematical biology are proposed to be expressed by piecewise-smooth functions and methods for determining the set of trajectory limit points in a dynamical system generated by these functions are described.

The reliability of the results of the study. The results have been obtained by using the methods of functional analysis, theory of discrete time dynamical systems and stochastic processes. The obtained results are mathematically strongly proved.

Scientific and practical significance of the research results. The scientific significance of the research results is explained by the fact that the description of the set of trajectory limit points in the dynamical systems of piecewise-smooth functions can be used in the theory of nonlinear discrete-time dynamical systems.

The practical significance of the research is determined by applications to population processes in mathematical biology by determining the set of limit points of trajectory in a dynamical system generated by piecewise-smooth maps.

Implementation of the research results. Results related to piecewise-smooth dynamical systems with discrete-time were used in the following research projects:

the results on the chaoticity of the discrete-time dynamics of a discontinuous quadratic stochastic operator on the one-dimensional simplex for non-zero values of parameters have been used for describing the sets of limit points of trajectories of stochastic operators in the research project No. OT-Ф-4-03 (Reference No. 04/1900 of Karshi State University dated June 12, 2021). The application of the scientific result allowed to classify the limit points of quadratic and cubic stochastic operators of Volterra and non-Volterra;

the results on Denjoy's-type equality for Hermans maps played an important role to get universal estimates for the iterations of the Riemann-Liouville fractional integral operators in many functional spaces which were used in the research project University Utara Malaysia, with identification number: FRGS/1/2018/STG06/UUM/02/13 titled "Solutions of Nonlinear Fractional Differential Equations via a Generalized Fixed Point Method and Homotopy Analysis Method" approved by Ministry of Education of Malaysia in 2018 .

Moreover, these results were used to prove the existence of solutions of fractional order differential equations.

Approbation of the research results. The main results of the research have been discussed at 1 international and 3 national scientific conferences.

Publications of the research results. On the topic of the dissertation 5 research papers have been published in the scientific journals, 3 of them are included in the list of journals proposed by the Higher Attestation Commission of the Republic of Uzbekistan for defending the PhD thesis; in addition 2 of them were published in international journals.

The structure and volume of the thesis. The dissertation consists of an introduction, three chapters, conclusion and bibliography. The general volume of the thesis is 92 pages.

THE MAIN CONTENT OF THE THESIS

The the introduction we gave the motivation of research theme an correspondence to the priority research areas of science and technology of the Republic. We showed the degree of study of the problem, formulates goals and objectives, identifies the object and subject of research, sets out scientific novelty and practical the results of the research, the theoretical and practical significance of the results obtained is disclosed, information is given on the implementation of the research results, on the published works and on the structure of the dissertation

In the first chapter of the thesis, titled "**SYMBOLIC DYNAMICS AND CIRCLE MAPS**" we gave necessary definitions and theorems from the symbolic dynamical systems, especially, infinite binary words and Sturmian words.

Also, we considered relation between circle homeomorphisms and symbolic dynamics. Moreover, we gave the main notions and facts from circle dynamics: rotation number of the orientation preserving circle homeomorphism, probability invariant measure, critical circle maps, piecewise-smooth circle homeomorphisms with finite number of break points and hitting times.

In section 1.1, we gave the necessary definitions and facts on infinite binary words such as, language, periodicity and complexity, recurrence of a binary word. We have learned some special types of the infinite binary words.

A finite word w is a **factor** of an infinite word ω if $\omega = uwy$ for some finite word u and infinite word y.

Definition 1. *The* ***factor complexity*** *of an infinite word* ω *is the function* $p_\omega(n)$ *counting the number of its factors of length* n *which satisfies the following properties:*

- *non-decreasing function, i.e.,* $p_\omega(n+1) \geq p_\omega(n)$
- $p(1) = \# A$
- *for a periodic word* ω *there exists* $C \in \mathbb{N}$ *such that* $p_\omega(n) \leq C$ *for all* $n \in \mathbb{N}$.

Definition 2. *An infinite word s is called* ***Sturmian*** *if* $p_s(n) = n+1$ *for every natural* n.

The infinite word $\omega(x)=\{\omega_1,\omega_2,...\}$ associated by irrational rotation is called **irrational mechanical word**.

In particular, we provided fundamental equivalence theorem of Sturmian words which is very important to prove main fact in section 3.3.

Theorem 1. *Let ω be an infinite binary word. The followings are equivalent:*

(1) ω is Sturmian.

(2) ω is irrational mechanical.

(3) ω is balanced and aperiodic.

In section 1.2 we studied prelimianry informations on the theory of circle homeomorphisms, critical circle maps, dynamical partitions and symbolic dynamics.

Definition 3. *A point $x_{cr}\in S^1$ is called **critical point** of order $(2d+1), d\geq 1$ for homeomorphism f, if for some ε- neighborhood $U_\varepsilon(x_{cr})=(x_{cr}-\varepsilon,x_{cr}+\varepsilon)$, the function f belongs to $C^{2d+1}(U_\varepsilon(x_{cr}))$ and*

$$\frac{df}{dx}(x_{cr})=\frac{d^2 f}{dx^2}(x_{cr})=...=\frac{d^{2d} f}{dx^{2d}}(x_{cr})=0,\ \ \frac{d^{2d+1} f}{dx^{2d+1}}(x_{cr})\neq 0.$$

Definition 4. *The map f is called **critical map**, if it has unique critical point of the odd order.*

During the last 20-25 years the rigidity problem for critical maps was studied by many authors.

Ostlund and others investigated the renormalization transformation in the class of critical circle maps. Following to the work of Ostlund and others we define a set of real-analytic commuting pairs that corresponds to a set of real-analytic critical circle homeomorphisms the order of three. Consider the set $\mathcal{X}_{cr}$ of pairs (ξ,η) of real-analytic, strictly increasing on real line and satisfying the following conditions:

(c_1) $0<\xi(0)<1, \xi(0)=\eta(0)+1;$

(c_2) $\xi(\eta(0))=\eta(\xi(0))>0;$

(c_3) $\xi'(0)=\eta'(0)=\xi''(0)=\eta''(0)=0,$ but $\xi'''(0)\neq 0$

(c_4) $(\xi\circ\eta)'''(0)=(\eta\circ\xi)'''(0).$

Conditions (c_1) and (c_2) permit us to associate a homeomorphism $f=f_{\xi,\eta}$ on the unit circle with each $(\xi,\eta)\in\mathcal{X}_{cr}$. Define $f=\xi$ on $[\eta(0),0]$ and $f=\eta$ on $[0,\xi(0)]$ and associate the unit interval $[\eta(0),\xi(0)]$ with the circle by identifying endpoints. A rotation number $\rho=\rho(f_{\xi,\eta})$ can be defined for $f_{\xi,\eta}$ in the usual way.

We denote by $\mathcal{X}_{cr}(\bar{\rho})$ the subset $\mathcal{X}_{cr}$ of pairs (ξ,η) for which the rotation number $\rho(f_{\xi,\eta})=\bar{\rho}=\dfrac{\sqrt{5}-1}{2}$ i.e. it is equal to the golden mean.

Next we define the renormalization group transformation

$$\Re:\mathcal{X}_{cr}(\bar{\rho})\to\mathcal{X}_{cr}(\bar{\rho}) \text{ by } \Re(\xi,\eta)=(\alpha\eta(\alpha^{-1}x),\alpha\eta(\xi(\alpha^{-1}x)),$$

where $\alpha:=\alpha_{\xi,\eta}=[\eta(0)-\eta(\xi(0))]^{-1}.$

The conditions $(c_1),(c_2)$ imply that $\alpha<-1$.The renormalization group transformation $\Re$ has a single hyperbolic fixed point (ξ_0,η_0) in the subspace $\mathcal{X}_{cr}(\bar{\rho})$. Notice that $\xi_0(x),\eta_0(x)$ are real analytic functions of x^3 and the constant $\alpha_0:=\alpha_{\xi_0,\eta_0}\approx-1{,}2886$. Denote by $f_{cr}:=f_{\xi_0,\eta_0}$ the circle map associated to (ξ_0,η_0).

Denote by $Cr(\bar{\rho})$ the set of all circle homeomorphisms whose are C^1- conjugated to f_{cr}. It is well known that any two topological conjugated homeomorphisms have the same rotation number. Therefore, the rotation numbers of homeomorphisms of $Cr(\bar{\rho})$ are the same and equal to $\bar{\rho}$.

A. Dzhalilov build the thermodynamic formalism for the critical circle maps from $Cr(\bar{\rho})$.

Section 1.3 cointains the necessary definitions and theorems on hitting times for critical circle maps.

Let f be an orientation preserving homeomorphism of the circle $S^1=R^1/Z\cong[0,1)$ with irrational rotation number $\rho=\rho(f)$. Let $\mu=\mu(f)$ be the unique invariant probability measure of f. Fix a point $z\in S^1$ and consider the interval $V_\varepsilon(z)=[z,z+\varepsilon]\subset S^1$. Consider the first entrance time to the interval V_ε by $E_\varepsilon(t)=\inf\{i\ge1:T^i(t)\in V_\varepsilon(z)\}$.

Next, we define rescaled entrance time by $\bar{E}_\varepsilon(t)=\mu(V_\varepsilon(z))\mu(E_\varepsilon(t))$. We are interested in the converges of the distribution function of the random variable $\bar{E}_\varepsilon(t)$ i.e. in the convergence of the distribution function

$$F_\varepsilon(t)=\mu\Big(x\in S^1:E_\varepsilon^{(1)}(x)\le t\Big),\ \forall t\in\mathbb{R}^1,$$

as $\varepsilon\to0$, for every t belonging to the continuity points of the limit function.

Coelho and de Faria investigated the problem of convergence of random variables $\bar{E}_\varepsilon(t)$ for linear irrational rotations $f_\rho(x)=x+\rho\ mod1$. It is known that for linear irrational rotation f_ρ unique invariant measure is Lebesgue measure ℓ. If $\varepsilon=\varepsilon_n$ is chosen such that $V_{\varepsilon_n}(z)$ corresponds to a sequence of renormalisation intervals for f_ρ as done in Coelho and de Faria's work, it is proved that for Lebesgue almost every rotation number ρ, the rescaled entrance times $E_\varepsilon(t)=\mu(V_\varepsilon(z))E_\varepsilon(t)$ do not converge in law as ε tends to zero, and all possible limit laws under a subsequence of ε_n are obtained. Notice that if $F_{\varepsilon_n}(t)$ converges for some ε_n converging to zero every, the limit probability distribution $F(t)$ either step function with two discontinuity points, or uniform distribution on interval [0,1].

Let $q_n,\ n\ge1$be the first return times for f_ρ. Fix $\theta\in(0,1)$. For every $n\ge1$, we define the points $c_n(\theta)$ by:

$$\mu([x_0,c_n(\theta)])=\theta\cdot\mu([x_0,{f_\rho}^{q_n}(x_0)]).$$

We denote by $\Delta_{n,c}$ the interval $[x_0, c_n(\theta))$. Consider the entrance time $E_{n,c}$ to the interval $\Delta_{n,c}$. Define rescaled entrance time by

$$\bar{E}_{n,c}(x) = \mu(\Delta_{n,c})E_{n,c}(x).$$

Denote by $F_{n,c}(t)$ the distribution function of $\bar{E}_{n,c}(x)$. It is proved that for any irrational number ρ the rescaled hitting times $\bar{E}_{n,c}(x)$ do not converge in law as ε tends to zero.

In this paper we investigate the rescaled entrance times for circle homeomorphisms with a single critical point. Let $f \in Cr(\bar{\rho})$ be critical circle homeomorphism. There are two natural measures on the circle: invariant probability measure μ and Lebesgue measure ℓ. Now, we consider two distribution functions $F_{n,c}(t)$ and $\Phi_{n,c}(t)$ of rescaled entrance time $\bar{E}_{n,c}(x)$ with respect to measures μ and ℓ, respectively. The distribution function $F_{n,c}(t)$ of f coincide with distribution function of linear rotation f_ρ. Therefore all statements of Coelho and de Faria's work are true for f invariant measure μ. The invariant measure μ is singular with respect to Lebesgue. Notice that for piecewise smooth circle homeomorphisms f with finite number of break points and irrational rotation number the map ρ is ergodic w.r.t. Lebesgue measure also. By definition

$$\Phi_\varepsilon(t) = l\left(x \in S^1 : E_\varepsilon^{(1)}(x) \le t\right), \forall t \in \mathbb{R}^1,$$

A.Dzhalilov studied the limit behaviour of distribution function with respect to Lebesgue measure for critical circle maps with ``golden'' irrational rotation number and for renormalized neighborhood of critical point. A. Dzhalilov and J. Karimov investigated the limit behavior of distribution function with respect to Lebesgue measure for circle homeomorphism with single break point and with golden mean irrational rotation number.

Section 1.4 cointains the necessary fact on circle homeomorphisms with break points.

Circle maps with break points have been intensively studied over the past 20 years in the works of K. Khanin, D. Khmelev, A. Teplinsky, D. Smania, K. Cunha, D. Mayer, A. Dzhalilov, I. Liousse and others. Piecewise smooth homeomorphisms with breaks are a natural generalization of circle diffeomorphisms. The point is called a breakpoint if there is a jump of the first derivative in it. The simplest homeomorphisms with breaks are piecewise linear homeomorphisms of the circle. M. Herman showed that the invariant measure of a piecewise linear homeomorphism with two breaks and an irrational number of rotations is absolutely continuous if and only if both break points lie on the same orbit.

Invariant measures of nonlinear piecewise smooth homeomorphisms of a circle with breaks were studied by K. Khanin, A. Dzhalilov, A.Teplinsky, D. Mayer, I. Liousse, D. Smania, and others.

In contrary to diffeomorphisms, the invariant measures of such homeomorphisms are singular with respect to the Lebesgue measure (K. Khanin, A. Dzhalilov, D. Mayer, I. Liousse). For sufficiently smooth (smoothness on each interval) homeomorphisms with breaks, their renormalizations are approximated by linear fractional maps (K. Khanin, D. Khmelev, A. Teplinsky, D. Smania, k. Cunha, S. Kosich).

In chapter two, titled "**COMPLEXITY FUNCTIONS AND HITTING TIMES ASSOCIATED BY CIRCLE MAPS**" we investigated the infinite binary words associated by irrational rotations of the circle and the asymptotic behavior of hitting times of the critical circle maps.

In the first section of chapter 2, we study main properties of infinite binary sequences associated by irrational rotation on the unit circle.

We consider an irrational rotation $f_\rho(x) = x + \rho \ mod 1$ on the interval $[0,1)$. Fix a point $b \in S^1$. Consider the partition $P = \{[0,b),[b,1)\}$ of the circle. For each $n \geq 1$ we define the set $S_n(0,b)$:

$$S_n(0,b) := \{0, f_\rho^{-1}(0),..., f_\rho^{-n+1}(0)\} \bigcup \{b, f_\rho^{-1}(b),..., f_\rho^{-n+1}(b)\}.$$

The points of the set $S_n(0,b)$ determine a partition denoted by P_n. Put $P_1 := P$.

In fact,

$$P_n := P \vee f_\rho^{-1}(P) \vee ... \vee f_\rho^{-n+1}(P)\}$$

where $P \vee Q := \{A \bigcap B : A \subset P, B \subset Q\}$.

Here we denoted elements of P_n and P_{n+1} by just I^n and I^{n+1} without any indexes.

Next using the sequence of partitions P_n, $n \geq 0$ we construct some special kind of symbolic dynamics. Put $I_1 := [0,b)$ and $I_0 := [b,1)$. Define the coding function $\nu_b : S^1 \to \{0,1\}$: for all $i \geq 0$

$$\nu_b(f_\rho^i(x)) := \begin{cases} 0, \text{ if } & x \in f_\rho^{-i}(I_0), \\ 1, \text{ if } & x \in f_\rho^{-i}(I_1). \end{cases}$$

Take any $x \in S^1$. The corresponding infinite sequence $\underline{\omega} := \underline{\omega}(x)$ of zeros and ones we define as $\underline{\omega} = (\omega_0\omega_1...\omega_n...) := (\nu_b(x)\nu_b(f_\rho(x))...\nu_b(f_\rho^n(x))...)$.

Denote the collection of such admissible infinite words $L_\omega(\rho,b)$ i.e. $L_\omega(\rho,b) = \{\underline{\omega}(x), x \in S^1\}$. We introduce the following notations: $\omega_m^{m+s} = (\omega_m\omega_{m+1}...\omega_{m+s}), m,s \geq 0,$ the factor of $\underline{\omega}$, $\omega_m^\infty = (\omega_m\omega_{m+1}...), m \geq 0,$ the tale of $\underline{\omega}$.

Definition 4. *An infinite word* $\underline{\omega}$ *is ultimately periodic if there exist* $N,T \in N$ *such that* $\omega_{n+T} = \omega_n$ *for each* $n \geq N$. *Otherwise* $\underline{\omega}$ *is called aperiodic. From the definition of symbolic coding easily follows the following useful* ***facts:***

a) *Let* I^n *be any interval of the partition* P_n. *Then for any two points* $x,y\in I^n$ *their prefixes of length* $n+1$ *are coincides i.e.* $\omega_0^n(x)=\omega_0^n(y)$. *Hence, for each interval of* $\bigcap_{i=0}^{m-1} f_\rho^{-i}(P)$ *corresponds one word of length* m.

b) *The word* $u_0u_1...u_{m-1}$ *is a factor of some admissible infinite word* $\underline{\omega}=(\omega_0\omega_1...\omega_n...)$ *i.e.* $\omega_{n+i}=u_i$ *for* $i=0,...,(m-1)$ *if and only if* $f_\rho^n(x)\in I(u_0u_1...u_{m-1})$, *where* $I(u_0u_1...u_{m-1})=\bigcap_{i=0}^{m-1} f_\rho^{-i}(I_{b_i})\in P_m$.

c) *In particular, the interval* $I(u_0u_1...u_{m-1})$ *is nonempty if and only if* $u_0u_1...u_{m-1}$ *is a factor of* $\underline{\omega}$.

Denote by $d_n=\max_{I^n\in P_n}|I^n|$.

Lemma 1. *For all* $n\geq 1$, $d_n\geq d_{n+1}$ *and* $\lim_{n\to\infty} d_n=0$.

Denote by $I^n(x)$ the interval of $n-$th partition P_n containing x and $W_n(\underline{\omega}(x))$ be the set of subwords of the length n of the infinite binary word $\underline{\omega}(x)$.

Theorem 2. *Let* $x,y\in S^1$ *and* $\underline{\omega}(x)$, $\underline{\omega}(y)$ *are their infinite coding sequences defined by (2.1.3). Then followings are hold*

1. *for all* $n\geq 0$, *we have* $W_n(\underline{\omega}(x))=W_n(\underline{\omega}(y))$.
2. $\underline{\omega}(x)$ *is uniformly recurrent.*
3. $\underline{\omega}(x)$ *is aperiodic for any* $x\in S^1$.

In section 2.2, we studied the complexity functions of infinite binary sequences associated by irrational rotation on the unit circle.

Recall that, the complexity of an infinite word $\underline{\omega}$ is the function $p_{\underline{\omega}}(n)$ counting the number of its factors of length n.

Further, for simplicity, instead of factor complexity we write complexity function.

We study the complexity functions of infinite words of $L_\omega(\rho,b)$ for all $b\in[0,1)$. We know that,

i) If $b=\{\rho,1-\rho\}$, then $p_{\underline{\omega}}(n)=n+1$ for all $n\geq 1$,

ii) If $b\neq\{\rho,1-\rho\}$, then $p_{\underline{\omega}}(n)\geq n+1$ for all $n\geq 1$.

We consider the latter case.

Denote by $r(n)$ the number of right special factors (**r.s.f**) of length n and let

$$k_0=min\{n\geq 1: r(n)=2\}.$$

On a binary alphabet A the number of right special factors $r(n)$ is used to determine the complexity function by following formula:

$$p_{\underline{\omega}}(n+1)=p_{\underline{\omega}}(n)+r(n).$$

Now we formulate two main theorems of the section 2.2.

The first theorem corresponds to the case when the point b is not in the orbit of 0.

Theorem 3. *Let f_ρ be the linear rotation to the irrational angle ρ, $\rho \in (0,1)$ and let $b + n\rho \notin Z$. Then the followings are hold*

1. If $0 < \rho < b$, then $p_{\underline{\omega}}(n) = 2n$ for all $n \geq 1$

2. If $0 < b < \rho$, then

$$p_{\underline{\omega}}(n) := \begin{cases} n+1, \text{ if } \ n < k_0, \\ 2n - k_0 + 1, \text{ if } \ n \geq k_0. \end{cases}$$

The following theorem corresponds to the case when b lies in the orbit of 0.

Theorem 4. *Let f_ρ be same as theorem 3. and $b + d\rho \in Z$, for some $d \in Z \setminus \{0\}$. Then the followings are hold*

1. If $0 < \rho < b$ then $p_{\underline{\omega}}(n) := \begin{cases} 2n, & \text{if } \ n < |d|, \\ n + d, & \text{if } \ n \geq |d|. \end{cases}$

2. If $0 < b < \rho$, then $p_{\underline{\omega}}(n) := \begin{cases} n+1, \text{ if } \ n < k_0, \\ 2n - k_0 + 1, \text{ if } \ k_0 \leq n < |d| \\ n + d - k_0 - 1, \text{ if } \ n \geq |d|. \end{cases}$

In the section 2.3, we investigate asymptotic behavior of the rescaled hitting times for critical circle maps $f \in Cr(\bar{\rho})$ with a single cubic critical point.

It is proved the Theorem 5., where found the explicit formula for distribution functions $\Phi_{n,\theta}(t) = 0$ of random variables $E_{n,\theta}^{(1)}(x)$.

We formulate the main result of the section 2.3.

Theorem 5. *Let $\bar{\rho} = \dfrac{\sqrt{5}-1}{2}$ and let $f \in Cr(\bar{\rho})$ be critical circle map. Consider for $\theta \in (0,1)$ the sequence of distribution functions $\left\{\Phi_{n,\theta}(t)\right\}_{n=1}^{\infty}$ with respect to Lebesgue measure on circle corresponding to the first rescaled hitting times $E_{n,\theta}^{(1)}(x)$ to interval $[x_c, c_n(\theta)]$. Then*

1) for all $t \in \mathbb{R}^1$ there exists the finite limit $\lim\limits_{n\to\infty} \Phi_{n,\theta}(t)\ (t) = \Phi_\theta(t)$, where $\Phi_\theta(t) = 0$, if $t \leq 0$, and $\Phi_\theta(t) = 1$, if $t > 1$;

2) the limit function $\Phi_\theta(t)$ is a strictly increasing on [0,1] and continuous distribution function on $\mathbb{R}^1$;

3) $\Phi_\theta(t)$ is singular on [0,1] i.e. $\dfrac{\Phi_\theta(t)}{dt} = 0$ a.e. with respect to Lebesgue measure ℓ on the circle.

In chapter three, titled “**DENJOY EQUALITY AND STURMIAN WORDS FOR P-HOMEOMORPHISMS**” we studied the location of break points of piecewise-smooth circle homeomorphisms with two break points and the behavior of rescaled derivatives of Herman maps h^{q_n}. Also, we investigated the complexity function of infinite binary sequences associated by rescaled derivatives of h^{q_n}.

Take some $x_0 \in S^1$ and consider its orbit $O(x_0) = \{x_i = f^i(x_0), i \in \mathbb{Z}\}$. Recall that the system of intervals $\mathcal{P}_n(x_0) = \{I_i^{(n-1)}(x_0), 0 \le i < q_n\} \cup \{I_j^{(n)}(x_0), 0 \le j < q_{n-1}\}$, determines for any n a partition of the circle, which is called the n-th dynamic partition of the point x_0.

Consider, an arbitrary P-homeomorphism f with irrational rotation number ρ_f and two break points a_0 and c_0, which are not on the same orbit. Denote by $\frac{p_n}{q_n}$ the partial convergent of ρ_f. We will next determine the location of the break points of f^{q_n} and the derivative Df^{q_n} on S^1. Obviously, the map f^{q_n} has $2q_n$ break points denoted by $BP_f^n := BP_f^n(a_0) \cup BP_f^n(c_0)$ with $BP_f^n(a_0) := \{a_0^*, a_{-1}^*, ..., a_{-q_n+1}^*\}$, respectively $BP_f^n(c_0) := \{c_0^*, c_{-1}^*, ..., c_{-q_n+1}^*\}$, where $a_{-i}^* = f^{-i}(a_0)$, respectively $c_{-i}^* = f^{-i}(c_0)$, $0 \le i \le q_n - 1$. It is clear, that these break points of the map f^{q_n} define a partition $B_n(f)$ of the circle S^1 into $2q_n$ intervals with pairwise non-intersecting interior.

We divided the location of the second break point c_0 in intervals of partition $B_n(f)$ to four nonintersecting cases: Case I, Case II, Case III and Case IV.

Let $\mathcal{P}_n(a_0^*)$ be the n-th dynamical partition determined by the break point $a_0^* = a_0$ with respect to the map f. Assume that breaks points c_0^* and a_0^* are in different orbits, then the following three cases hold for the second break point c_0^*:

Case I. $c_0^* \in I_{i_0}^{(n)}(a_0)$, for some $0 \le i_0 < q_{n-1}$;

Case II. $c_0^* \in f^{j_0}((a_0, a_{-q_n}])$, for some $0 \le j_0 < q_n$;

Case III. $c_0^* \in f^{j_0}((a_{-q_n}, a_{q_{n-1}}))$, for some $0 \le j_0 < q_n$;

Also it is possible the case when both breaks points c_0^* and a_0^* are in the same orbit i.e.

Case IV. $c_0^* = f^{i_0}(a_0^*)$, for some $0 \le i_0 < q_n$.

In section 3.1 we studied the location of break points of f^{q_n} in Case I and Case III. We also proved Denjoy equality for Herman's map in the Cases I and III.

We denote by h Herman's maps i.e. PL circle homeomorphisms with two breaks. Each Herman's map with irrational rotation number ρ satisfies the conditions of Denjoy's theorem. Hence h topologically conjugated by linear rotation f_ρ.

We expressed the rescaled derivative of h^{q_n} by the jump ratio $\sigma_h(a_0)$ and the μ_h-measures of intervals of the partition $B_n(h)$ produced by break points of h^{q_n}.

In Case I and Case III the break points $PB_h(a_0^*)$ originating from $a_0^*=0$ and the break points $PB_h(c_0^*)$ originating from $c_0^*=c_0$ alternate in their order along the circle S^1.

Let n be odd. Using the break points of h^{q_n}, we define the following subintervals:

$$A_n := \bigcup_{s=1}^{i_0}[c^*_{-i_0+s},\, a^*_{-q_n+s}],\quad B_n := \bigcup_{s=i_0+1}^{q_n}[c^*_{-i_0-q_n+s},\, a^*_{-q_n+s}].$$

In Case III the subintervals are given by $[a^*_{-q_n+s}, c^*_{-i_0+s}], 1\le s\le i_0,$ respectively $[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}], i_0+1\le s\le q_n,$ which we combine to the subsets

$$A_n := \bigcup_{s=1}^{i_0}[a^*_{-q_n+s}, c^*_{-i_0+s}],\quad B_n := \bigcup_{s=i_0+1}^{q_n}[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}].$$

For n even, the orientation of the above intervals has to be reversed. Therefore in Case I we have the following system of disjoint intervals $[a^*_{-q_n+s}, c^*_{-i_0+s}],\ 1\le s\le i_0,$ respectively, $[a^*_{-q_n+s}, c^*_{-i_0-q_n+s}],\ i_0+1\le s\le q_n.$

In Case III one finds $[c^*_{-i_0+s}, a^*_{-q_n+s}],\ 1\le s\le i_0,$ respectively $[c^*_{-i_0-q_n+s}, a^*_{-q_n+s}], i_0+1\le s\le q_n.$ In Case I and n even, respectively in Case II and n odd, the subsets A_n and B_n can be defined as before. The above constructions show, that the boundaries of every interval in the subsets A_n and B_n is an interval whose boundaries consist of break points from $PB_n(a_0^*)$ respectively, $PB_n(c_0^*)$.

We put $\sigma=\sigma_h(a_0)$ and formulate the main result of the section 3.1.

Theorem 6. *Let h be a PL circle homeomorphism with irrational rotation number ρ_h and two break points $a_0^*=0$ and $c_0^*:=c_0$, whose total jump ratio $\sigma_h=1$, and which lie on different orbits. Assume c_0^* fulfills the assumptions of Case I respectively Case II for some i_0 with $0\le i_0<q_{n-1}$. Then in Case I*

$$(Dh^{q_n}(x))^{(-1)^n} = \begin{cases} \sigma^{\mu_h(A_n\cup B_n)-1}, & if\ x\in A_n\cup B_n \\ \sigma^{\mu_h(A_n\cup B_n)}, & if\ x\in S^1\setminus(A_n\cup B_n); \end{cases}$$

respectively, in Case III,

$$(Dh^{q_n}(x))^{(-1)^{n+1}} = \begin{cases} \sigma^{\mu_h(A_n\cup B_n)-1}, & if\ x\in A_n\cup B_n \\ \sigma^{\mu_h(A_n\cup B_n)}, & if x\in S^1\setminus(A_n\cup B_n). \end{cases}$$

In section 3.2 we investigated the location of break points of h^{q_n} in the Cases II and IV. It was proved Denjoy equality for rescaled derivative of h^{q_n} for Herman's map in the Cases II and IV.

In section 3.3 we studied the complexity functions of infinite sequences associated by Herman's map h. We showed that the binary sequence associated by rescaled derivatives of h^{q_n} , $n > 0$ is Sturmian.

CONCLUSION

The thesis is devoted to investigation of the complexity functions for infinite binary sequences associated by irrational rotations of the circle, the behavour of hitting times for critical circle maps and the estimate of Denjoy function for piecewise-linear maps.

The basic results of the research work are follows:

It is found the complexity functions of infinite binary sequences associated by irrational circle rotations;

It is proved that the rescaled generalized hitting times of critical circle maps with one cubic critical point and golden mean rotation number converges in distribution and the limit distribution is singular function i.e. it is continuous, strictly increasing and has zero derivative almost everywhere w.r.t. Lebesgue measure.

It is found the location of break points of piecewise-smooth circle homeomorphisms with two break points.

It is proved the Denjoe equality for Herman's map h^{q_n} .

It is showed that the binary sequence associated by rescaled derivatives of h^{q_n} is Sturmian.

НАУЧНЫЙ СОВЕТ DSc.02/30.12.2019.FM.86.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ
ИНСТИТУТЕ МАТЕМАТИКИ ИМЕНИ В.И.РОМАНОВСКОГО
ИНСТИТУТ МАТЕМАТИКИ

ЖАЛИЛОВ АЛИШЕР АКБАРОВИЧ

РАВЕНСТВО ДАНЖУА И БЕСКОНЕЧНЫЕ ДВОИЧНЫЕ ПОСЛЕДОВАТЕЛЬНОСТИ СВЯЗАННЫЕ С ГОМЕОМОРФИЗМАМИ ОКРУЖНОСТИ

01.01.01 –Математический анализ

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

ТАШКЕНТ-2021

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Кабинете Министров Республики Узбекистан за № B2019.4.PhD/FM430.

Диссертация выполнена в Институте Математики имени В.И.Романовкого.

Автореферат диссертации на трех языках (узбекский, английский, русский, (резюме)) размещен на веб-странице по адресу http://kengash.mathinst.uz и на Информационно-образовательном портале «ZiyoNet» по адресу http://www.ziyonet.uz.

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Защита диссертации состоится «____»___________ 2021 года в _______ на заседании Научного совета DSc.02/30.12.2019.FM.86.01 при Институте Математики имени В.И.Романовского. (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9.Тел.: (+99871) 207-91-40, e-mail: uzbmath@umail.uz, Website: www.mathinst.uz)

С диссертацией можно ознакомиться в Информационно-ресурсном центре Института Математики имени В.И.Романовского (зарегистрирована за № _____). (Адрес: 100174, г. Ташкент, Алмазарский район, ул. Университетская, 9.Тел.: (+99871) 207-91-40).

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ВВЕДЕНИЕ (аннотация диссертации доктора философии(PhD))

Целью исследования является изучение функций сложности для двоичных бесконечных последовательностей, ассоцированных с иррациональными вращениями окружности, времени попадания для критических отображений окружности и равенства Данжуа для отображения Эрмана.

Объект исследования: функция сложности бесконечных последовательностей, иррациональные повороты окружности, отображение Эрмана, критические отображения окружностей, времена попадания для отобра-жения окружности.

Научная новизна исследования состоит в следующем:

Полностью найдены функции сложности бесконечных двоичных последовательностей ассоциированная с иррациональными вращениями окружности;

Доказано, что нормированные время попадания критического отображения окружности с одной кубической критической точкой и с числом вращения равным золотому сечению сходится по распределению и предельное распределение является сингулярной функцией, т.е. оно непрерывная, строго возрастающая и почти всюду имеет нулевую производную по отношению к мера Лебега.

Полностью найдено расположение точек излома кусочно-гладких гомеоморфизмов окружности с двумя точками излома.

Доказано равенство Данжуа для отображения Эрмана. Показано, что двоичная последовательность ассоциированная с перенормированными производными кусочно-гладких отображений окружности с двумя изломами является штурмовской.

Внедрение результатов исследования. Результаты, связанные с кусочно-гладкими динамическими системами с дискретным временем, были использованы в следующих исследовательских проектах:

результаты о хаотичности дискретной динамики разрывного квадратичного стохастического оператора на одномерном симплексе при ненулевых значениях параметров были использованы для описания множеств предельных точек траекторий стохастических операторов в исследовательском проекте № ОТ-Ф-4-03 (Справка Каршинского государственного университета № 04/1900 от 12 июня 2021 г.). Применение научного результата позволило классифицировать предельные точки квадратичных и кубических стохастических операторов Вольтерра и не Вольтерра;

результаты отом, что множество предельных точек при заданных значениях параметров для динамики разрывного квадратичного стохастического оператора на двумерном симплексе представляет собой либо единственную точку, либо бесконечное множество, были использованы для описания динамики моделей химических реакций в исследовательском проекте № ЁФА-Атех-2018-182 (Справка Академии наук Республики

Узбекистан № 2/1255-1825 от 23 июня 2021 г.). Использование научных результатов обеспечило определение фиксированных точек и периода динамических систем в соответствии с моделями химических реакций, а также качественный анализ траекторий.

Структура и объем диссертации. Диссертация состоит из введения, трёх глав, заключения и списка использованной литературы. Объем диссертации составляет 92 страниц.

E’LON QILINGAN ISHLAR RO’YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
LIST OF PUBLISHED WORKS

I bo’lim (part I;I часть)

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Avtoreferat «O’zbekiston matematika jurnali tahririyatida
2021 yil ______ tahrirdan o’tkazildi.

Бичими: 84x60 $^{1}/_{16}$. «Times New Roman» гарнитураси.
Рақамли босма усулда босилди.
Шартли босма табоғи: 2,5. Адади 100. Буюртма № 26/21.

Гувоҳнома № 851684.
«Tipograff» МЧЖ босмахонасида чоп этилган.
Босмахона манзили: 100011, Тошкент ш., Беруний кўчаси, 83-уй.