Limiting curvature gravity

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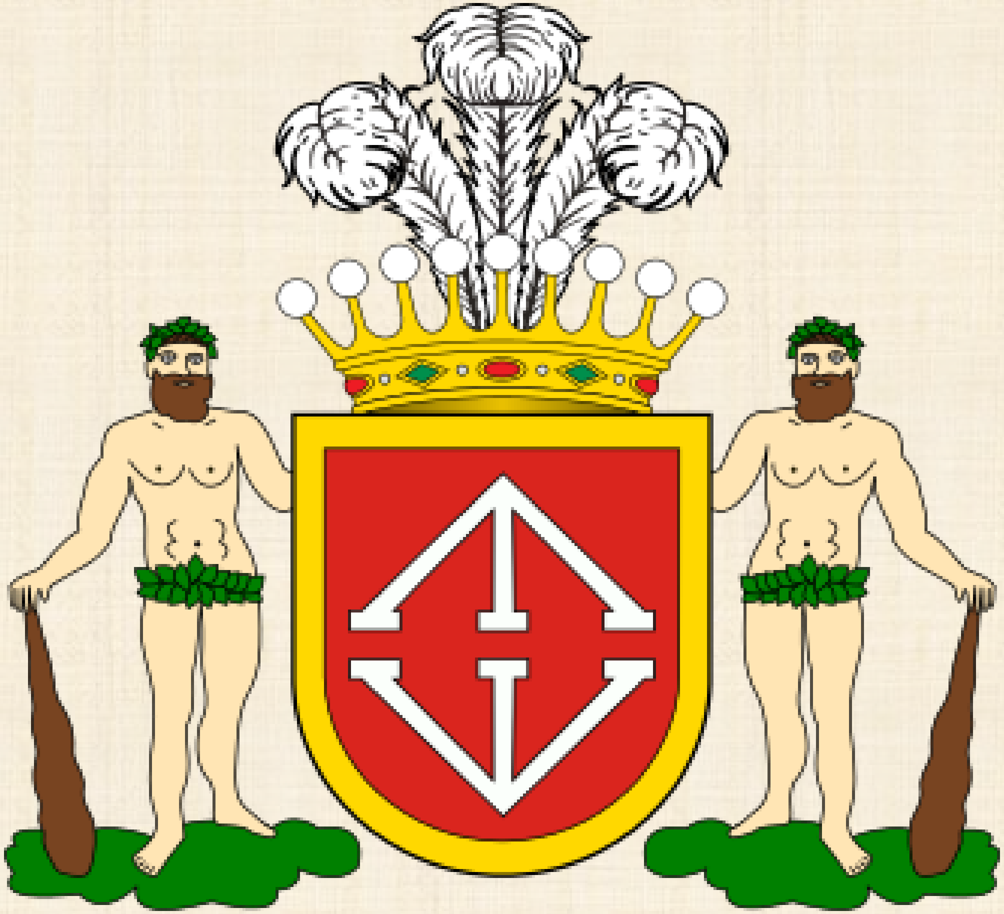
Brief history of Igor Volovich

Origin of the family name "Volovich":

Volovichi - Lithuanian noble family of the coat of arms "Bogoria", who owned the Svyatskaya estate near Grodno. Their ancestor was Grigory Fedorovich Volovich, equerry of the Grand Duke of Lithuania (1459). Of his grandchildren, Grigory Bogdanovich was ambassador to Moscow (1563), and Ostafy Bogdanovich was the chancellor of the Grand Duke of Lithuania (1579-1587)



Coat of arms of Volovichi

Famous representatives

Известные представители

- Волович, Андрей Иванович (ок. 1570—1614) — государственный деятель Великого княжества Литовского, хорунжий великий литовский.
- **Волович, Евстафий**:
  - Волович, Евстафий (около 1520—1587) — государственный деятель Великого княжества Литовского, гуманист и просветитель.
  - Волович, Евстафий (1572—1630) — религиозный и церковный деятель Речи Посполитой, католический епископ виленский.
- Волович, Антоний Эразм (1711—1770) — религиозный и церковный деятель Речи Посполитой, католический епископ луцкий.
- **Волович, Виталий**:
  - Волович, Виталий Георгиевич (1923—2013) — первый человек, совершивший прыжок на парашюте на Северный полюс.
  - Волович, Виталий Михайлович (1928—2018) — советский, российский художник, график. Заслуженный художник РСФСР (1973).
- Волович, Владислав (1615—1668) — государственный и военный деятель Великого княжества Литовского.
- Волович, Григорий Богданович (ум. 1577) — государственный деятель Великого княжества Литовского, дипломат, господарский дворянин.
- Волович, Захар Ильич (1900—1937) — деятель советских спецслужб.
- Волович, Игорь Васильевич (род. 1946) — математик, член-корреспондент РАН.
- Волович, Иероним (?—1636) — государственный деятель Великого княжества Литовского, подканцлер литовский.

Igor Volovoch was born on August 9, 1946 in the village Vokhtomskoye, Konosha district of the Arkhangelsk region. In 1970 he graduated from the Physics Department of Moscow State University.

I first met Igor in 1967 when we were students of the third year.
In that year 7 of us organized a society which we called after the name of *Hieromartyr Abraham of Arvil*, *Bishop*.
Main reason of using his name was because his memorial day, February 17, coincides with the date of our first meeting.

Constructive field theory: Thirring model interaction $(\widetilde{\psi}\gamma^{\mu}\widetilde{\psi})^2_2$. I. Local field IV Volovich, VN Sushko Teoreticheskaya i Matematicheskaya Fizika 9 (2), 211-231	12 *	1971

On the application of field-theory methods in the Ising model II. Calculation of the critical index η IV Volovich, EA Dynin, VA Zagrebnov, VP Frolov Teoreticheskaya i Matematicheskaya Fizika 15 (3), 417-426		1973
Application of field theory methods in the Ising model. I IV Volovich, EA Dynin, VA Zagrebnov, VP Frolov Teoreticheskaya i Matematicheskaya Fizika 14 (2), 272-276	2	1973

"We would like to thank A.S.Arvil'skii for fruitful discussions and support."

Fifty years later…

My warmest congratulations
to Igor with his jubilee.

Limiting curvature gravity

Famous Penrose and Hawking theorems on singularities imply that the General Relativity is UV incomplete. The evidences of this incompleteness are Big Bang cosmological singularities and singularities inside black holes.

Stationary BH solutions of the Einstein equations have a curvature singularity in their interior.

For Schwarzschild BH $\mathcal{K} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = \frac{48M^2}{r^6}.$

In 1982 Markov formulated a limiting curvature condition: There exists a fundamental length scale ℓ such that $|R| \le \Lambda = B\ell^{-2}$. Here R is a scalar invariant constructed from curvatures. We also require that dimensionless constant B is universal: It depends on the theory and on the choice of the invariant, but it is independent of the parameters of a solution.

- One or more new universes formation inside a BH;
- Bouncing solutions in cosmology;
- Solution of the mass inflation problem.

Limiting curvature gravity (LCG) theory.

Main idea: Let $L(q,\dot{q})$ be a Lagrangian and $Q(q,\dot{q}) \le 0$ be an inequality constraint. Denote $L(q,\dot{q},\chi,\zeta) = L(q,\dot{q}) + \chi[Q(q,\dot{q}) + \zeta^2]$.

Variation over the Lagrange multipliers χ and ζ gives $Q(q,\dot{q}) + \zeta^2 = 0, \quad \chi\,\zeta = 0.$

Subcritcal regime: $Q < 0 \rightarrow \zeta^2 = -Q,\ \chi = 0;$

Supercritcal regime: $Q = 0 \rightarrow \zeta = 0,\ \chi \ne 0.$

2D black holes in the LCG theory

"Two-dimensional black holes in the limiting curvature theory of gravity", V.F. and A. Zelnikov, JHEP 08 (2021) 154; e-print: 2105.12808.

$2D$ dilaton gravity model.

$$I_{DG} = \frac{1}{2}\int d^2x\,|g|^{1/2}\,(\psi^2 R + 4(\nabla\psi)^2 + 4\lambda^2\psi^2).$$

C. Callan et al, Nucl. Phys. B262 (1985) 593;

E. Witten, Phys. Rev. D44 (1991) 314;

G. Mandal, A. M. Sengupta and S. R. Wadia, Mod. Phys. Lett. A6 (1991) 1685;

S. Elitzur, A. Forge and E. Rabinovici, Nucl. Phys. B 359 (1991) 581;

V.F., Phys. Rev. D46 (1992) 5383;

C. Callan, Jr., S. Giddings, J. Harvey and A. Strominger,

Phys. Rev. D45 (1992) R1005.

$2D$ limiting curvature gravity (LCG) model

$$I_{LCG} = I_{DG} + I_{\chi},$$

$$I_{\chi} = \frac{1}{2}\int_M d^2x\,|g|^{1/2}\,\chi(R - \Lambda + \zeta^2),$$

Field equations

Constraint equations:

$$\chi\zeta = 0, \quad R - \Lambda + \zeta^2 = 0;$$

Dilaton eqn: $\Box\psi - (\lambda^2 + \frac{1}{4}R)\psi = 0;$

"Gravity" eqns: $\chi_{;\alpha\beta} + \frac{1}{2} g_{\alpha\beta} R\chi = Q_{\alpha\beta},$

$$Q_{\alpha\beta} = 4\psi_{;\alpha}\psi_{;\beta} + \frac{1}{2} g_{\alpha\beta}[-4\psi_{;\epsilon}\psi^{;\epsilon} + (4\lambda^2 + R)\psi^2].$$

Subcritical regime

$$\chi = 0, \quad \zeta^2 = \Lambda - R,$$

$$\Box\psi - (\lambda^2 + \frac{1}{4}R)\psi = 0,$$

$$\psi = \exp(-\phi), \quad \phi_{;\alpha\beta} = -\frac{1}{4}g_{\alpha\beta}R,$$

Killing vector: $\xi^\alpha = -\varepsilon^{\alpha\beta}\phi_{;\beta}$

A static 2D black hole:

Subcritical domain

$$ds^2 = -fdt^2 + f^{-1}dr^2, \quad f = 1 - \frac{m}{\lambda} e^{-2\lambda r},$$

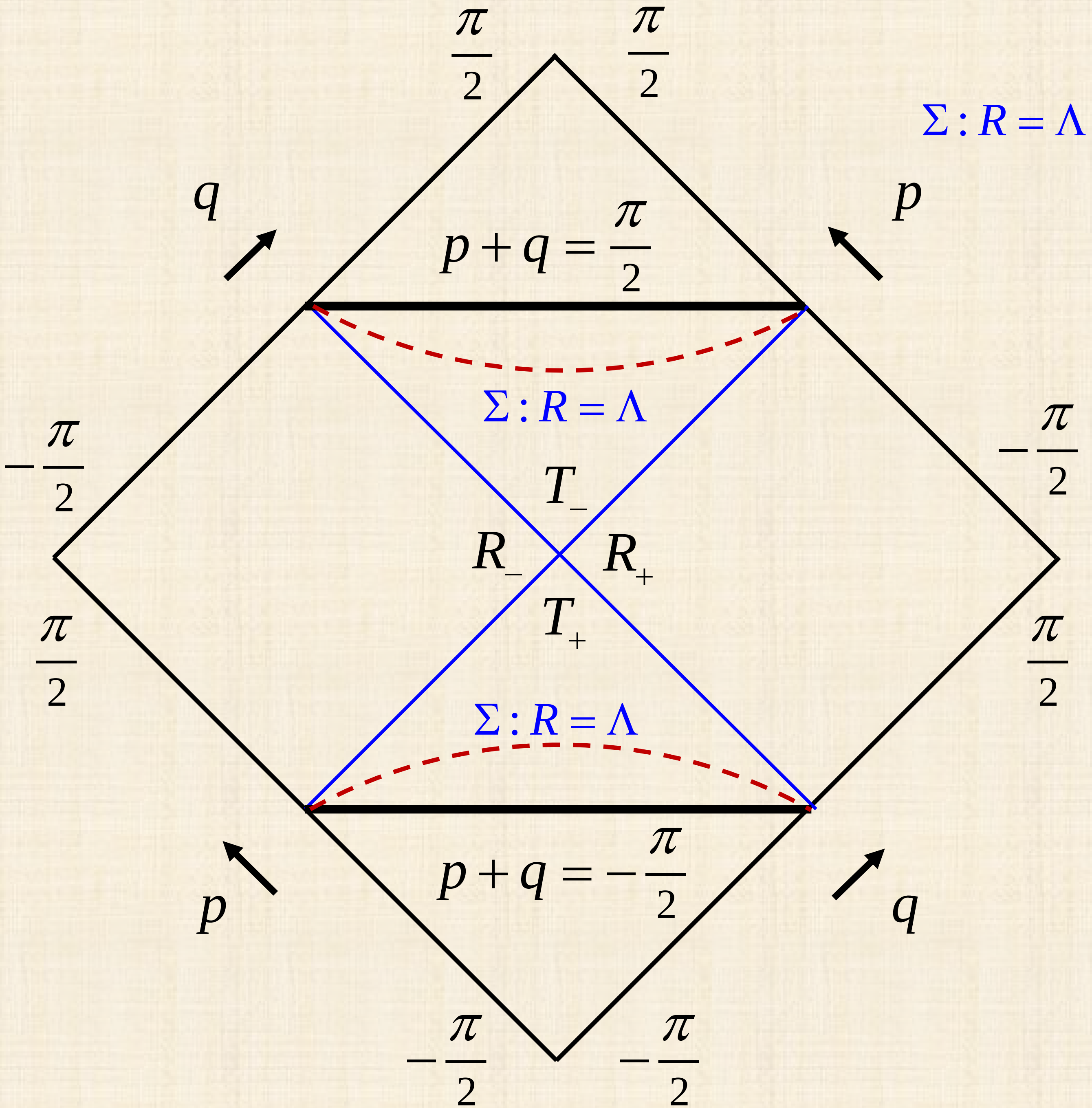
$$\psi = e^{\lambda r}, \quad \phi = -\lambda r; \;\; r_H = \frac{1}{2\lambda} \ln \frac{m}{\lambda}.$$

$$ds^2 = -\Omega\, dp\, dq, \quad -\pi/2 < p, q < \pi/2;$$

$$\Omega = \frac{1}{\lambda^2 \cos p\, \cos q\, \cos(p+q)}.$$

$$\Psi_1 : p \to q, \quad q \to p;$$

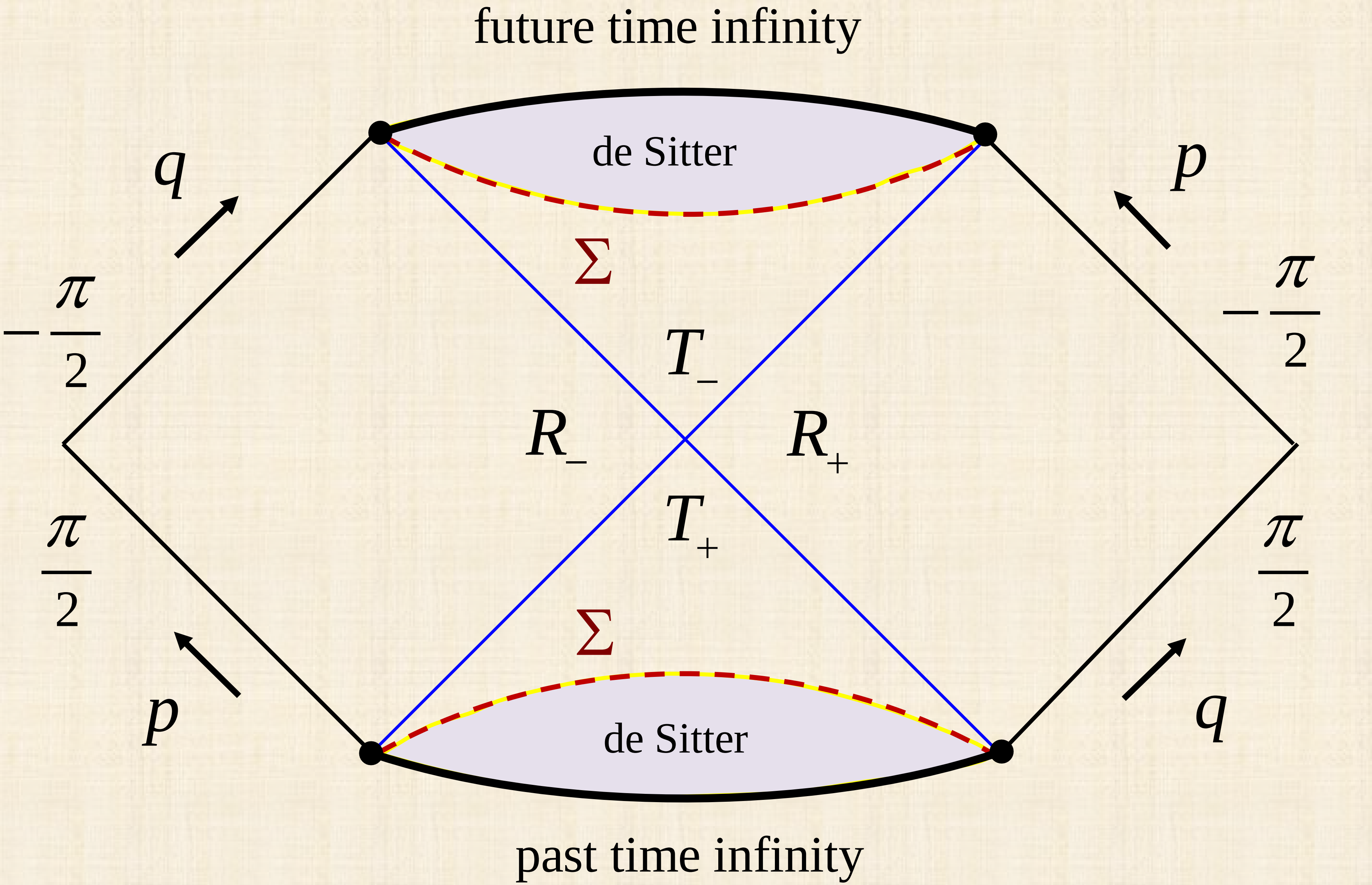
$$\Psi_2 : p \to -q, \quad q \to -p.$$



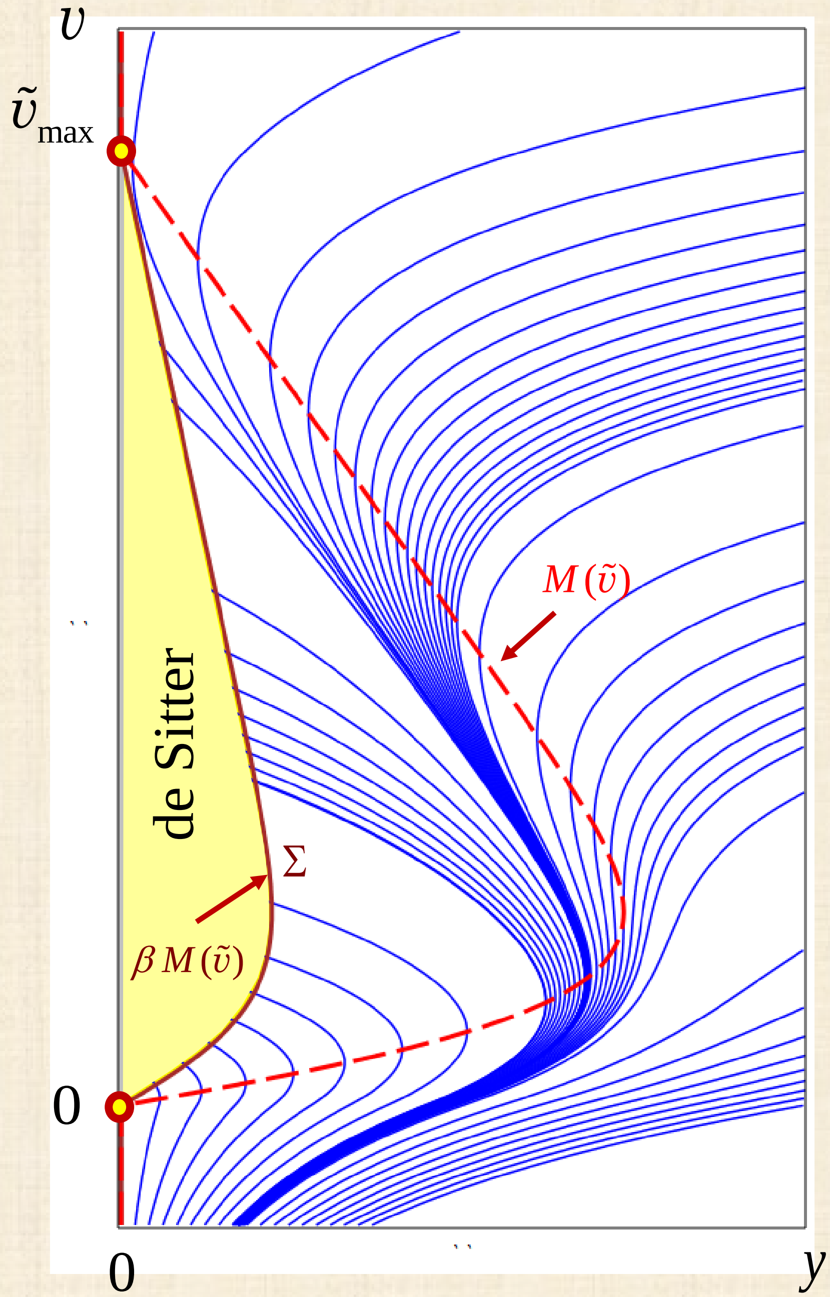
$\frac{\pi}{2}$
$\frac{\pi}{2}$
$\Sigma : R = \Lambda$
q
p
$p + q = \frac{\pi}{2}$
$\Sigma : R = \Lambda$
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$
T_-
R_-
R_+
T_+
$\frac{\pi}{2}$
$\frac{\pi}{2}$
$\Sigma : R = \Lambda$
$p + q = -\frac{\pi}{2}$
p
q
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$

Supercritical solution: 2D deSitter space

1. Matching conditions on Σ;
2. Solving equation for control parameter χ;
3. Glueing sub- and supercritical solutions;
4. Conformal Penrose diagram for a complete ST.



future time infinity
de Sitter
q
p
$-\frac{\pi}{2}$
$-\frac{\pi}{2}$
Σ
T_-
R_-
R_+
T_+
$\frac{\pi}{2}$
$\frac{\pi}{2}$
Σ
p
q
de Sitter
past time infinity



$\tilde{v}$
$\tilde{v}_{\max}$
0
de Sitter
Σ
$\beta M(\tilde{v})$
$M(\tilde{v})$
0
y

Bouncing cosmology in the LCG theory

“Bouncing cosmology in the limiting curvature theory of gravity”, V.F. and Andrei Zelnikov, e-Print: 2108.09927

$$ds^2 = -b^2(t)dt^2 + a^2(t)d\gamma^2,$$

$d\gamma^2$ is a metric on a 3D unit sphere S^3.

Any symmetric tensor $A_{\mu\nu}$ invariant under symmetries of S^3 has a form

$$A_\mu^\nu = \mathrm{diag}(A(t), \hat{A}(t), \hat{A}(t), \hat{A}(t)).$$

Eigenvalues of the Ricci tensor are linear combinations of two invariants

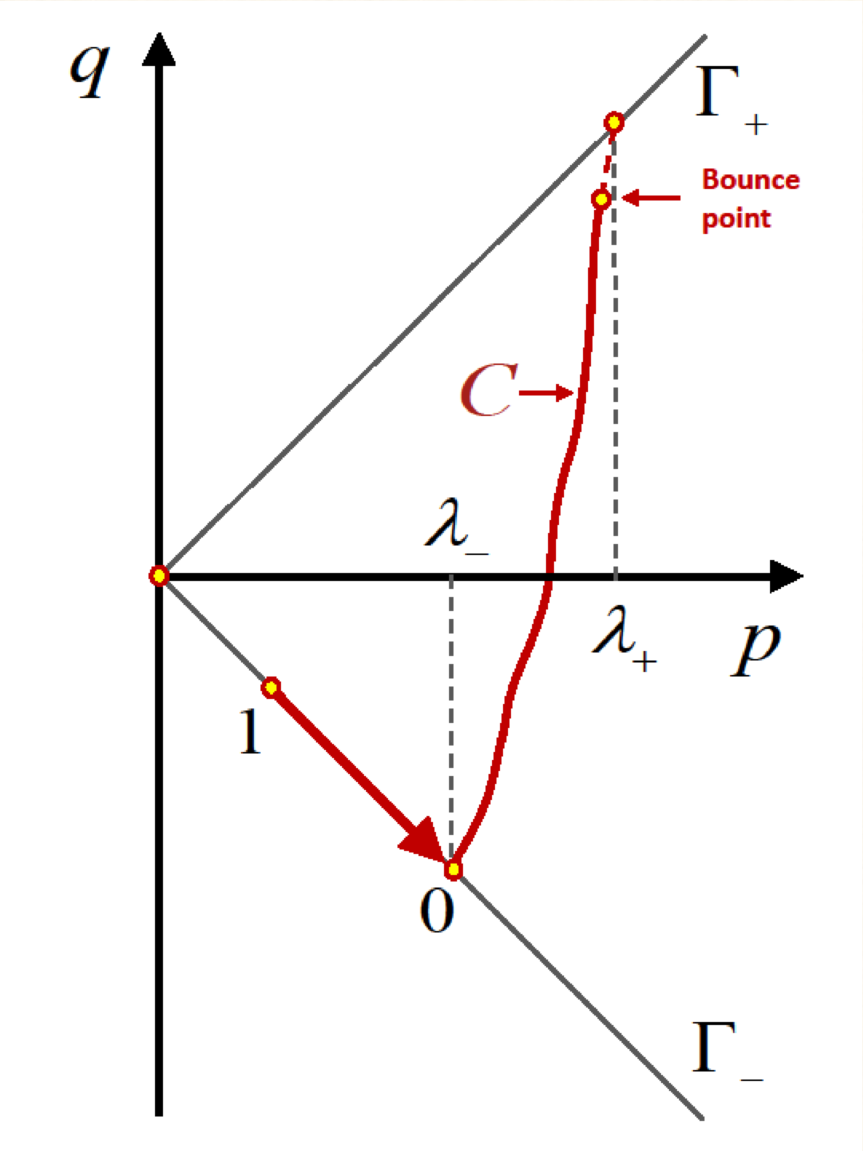
$$p = \frac{\dot{a}^2 + b^2}{a^2 b^2}, \quad q = \frac{1}{b^2}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}\dot{b}}{a^2 b^2} \right).$$

$$R_\mu^\nu = \mathrm{diag}(\mathrm{R}(t), \hat{\mathrm{R}}(t), \hat{\mathrm{R}}(t), \hat{\mathrm{R}}(t)).$$

$$\mathrm{R} = 3q, \quad \hat{\mathrm{R}} = q + 2p; \; R = p + q.$$

Any scalar invariant constructed from the Ricci tensor can be written in the form of a function of its eigenvalues p and q. We write the curvature constraint in the form $f(p, q, \Lambda) = 0$. Λ is a parameter defining the limiting curvature.

Let us consider a collapsing universe filled with thermal radiation. Its scalar curvature vanishes and one has $q = -p$.

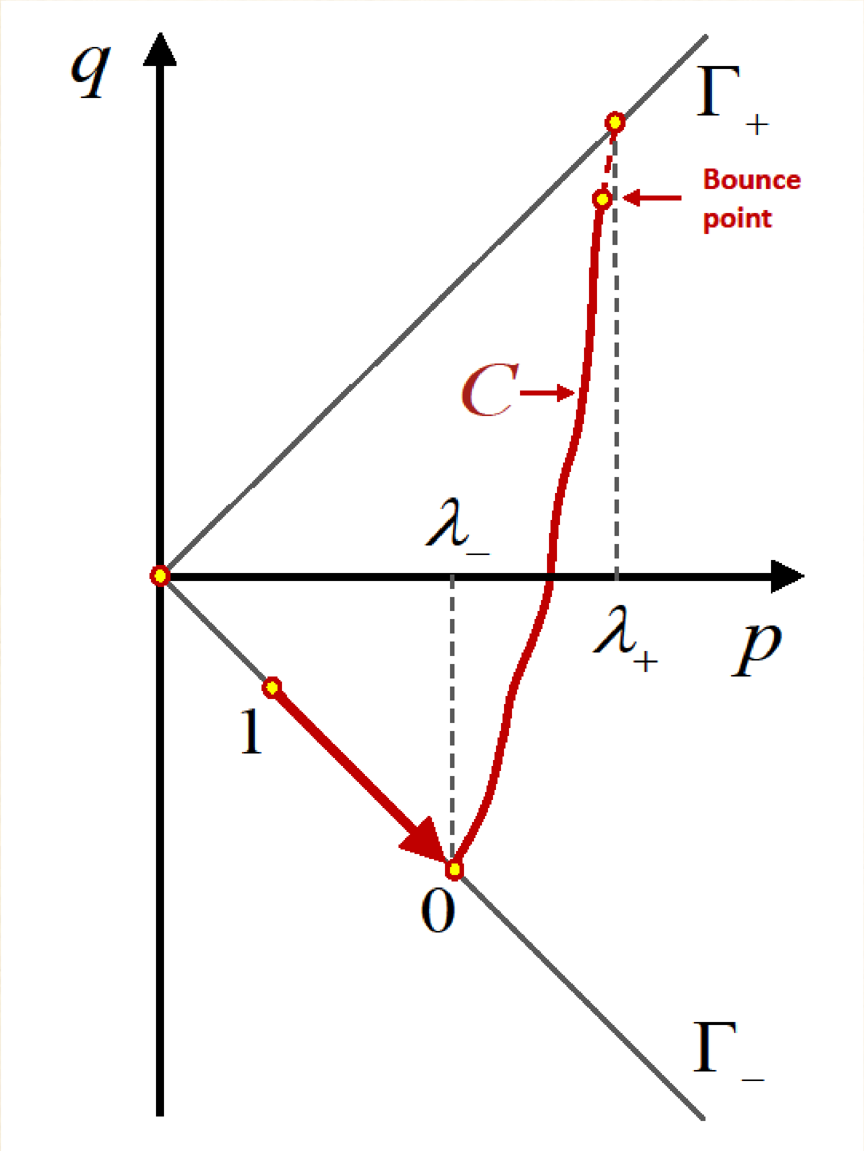


q
Γ+
Bounce
point
C
λ−
λ+
p
1
0
Γ−

1. The constraint curve C intersects lines $\Gamma_{\pm}$ where $q = \mp p$. We denote the coordinate p at the intersection points by $\lambda_{\pm}$.

2. We assume that $\dfrac{\partial f}{\partial q} \neq 0 \;\; \rightarrow q = q(p, \Lambda)$.

3. $\dfrac{dq}{dp} > 0$ on a segment of C between Γ_{-} and Γ_{+}.

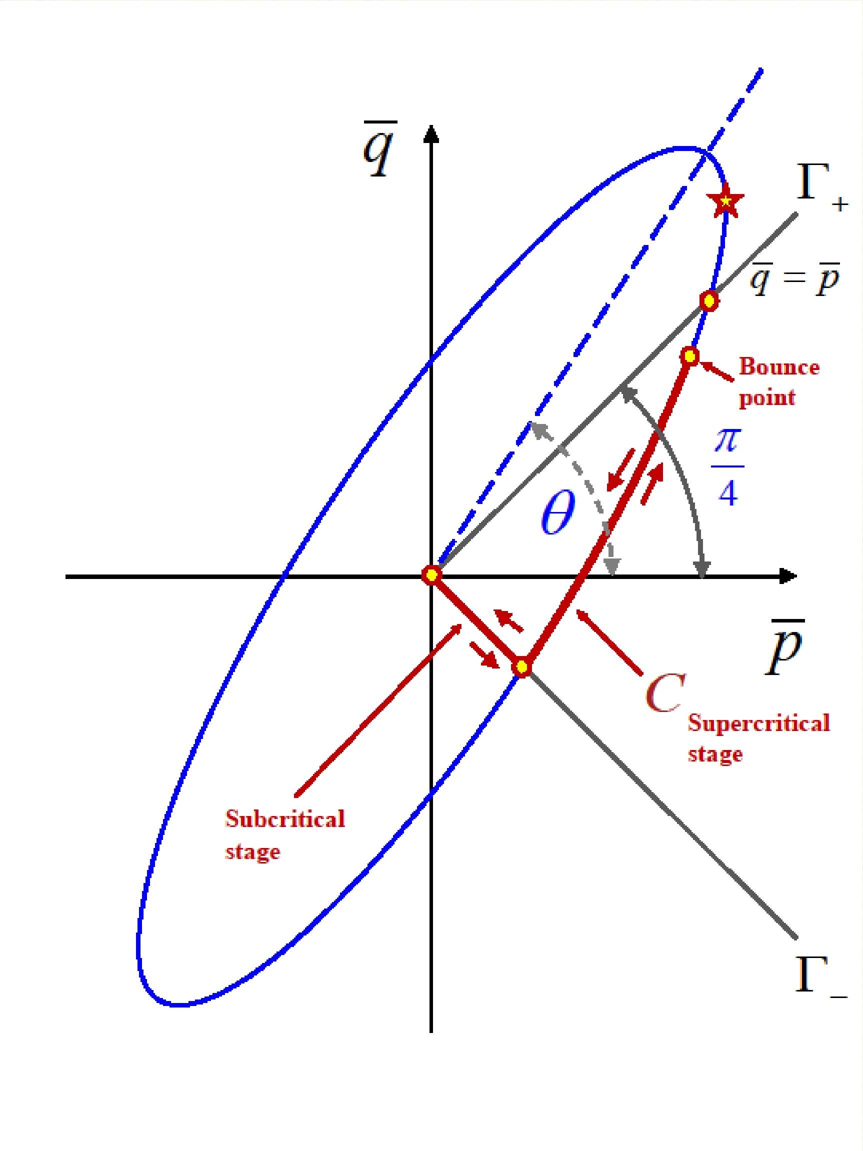

q
Γ+
Bounce point
C
λ−
λ+
p
1
0
Γ−

An example: Quadratic in curvature constraints

$$\rho = \frac{1}{6} R = p + q,$$

$$\sigma^2 = \frac{1}{3} S_{\mu\nu} S^{\mu\nu} = (p - q)^2,$$

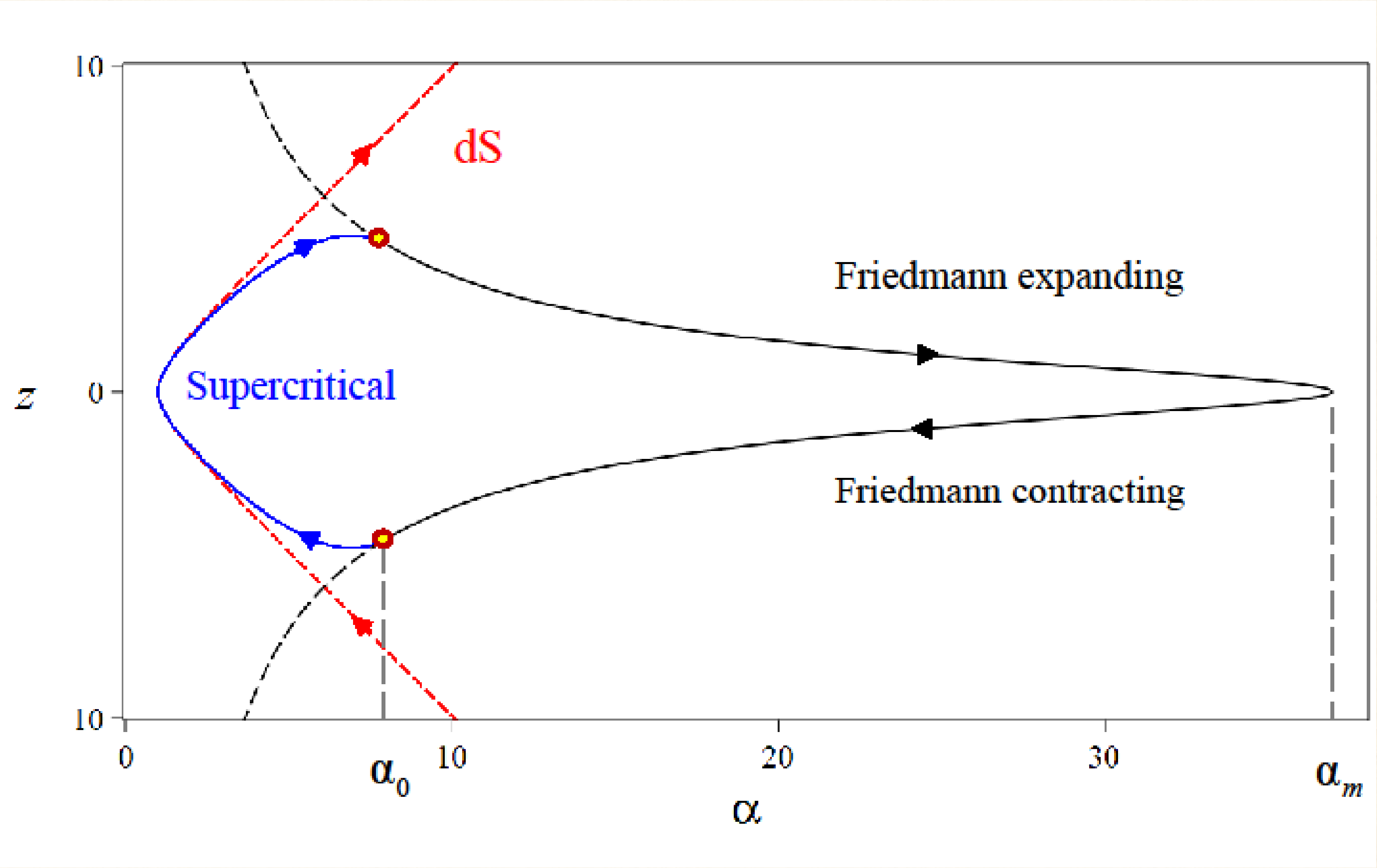
$$f = c_{SS}\sigma^2 + c_{SR}\sigma\rho + c_{RR}\rho^2 - \Lambda^2.$$



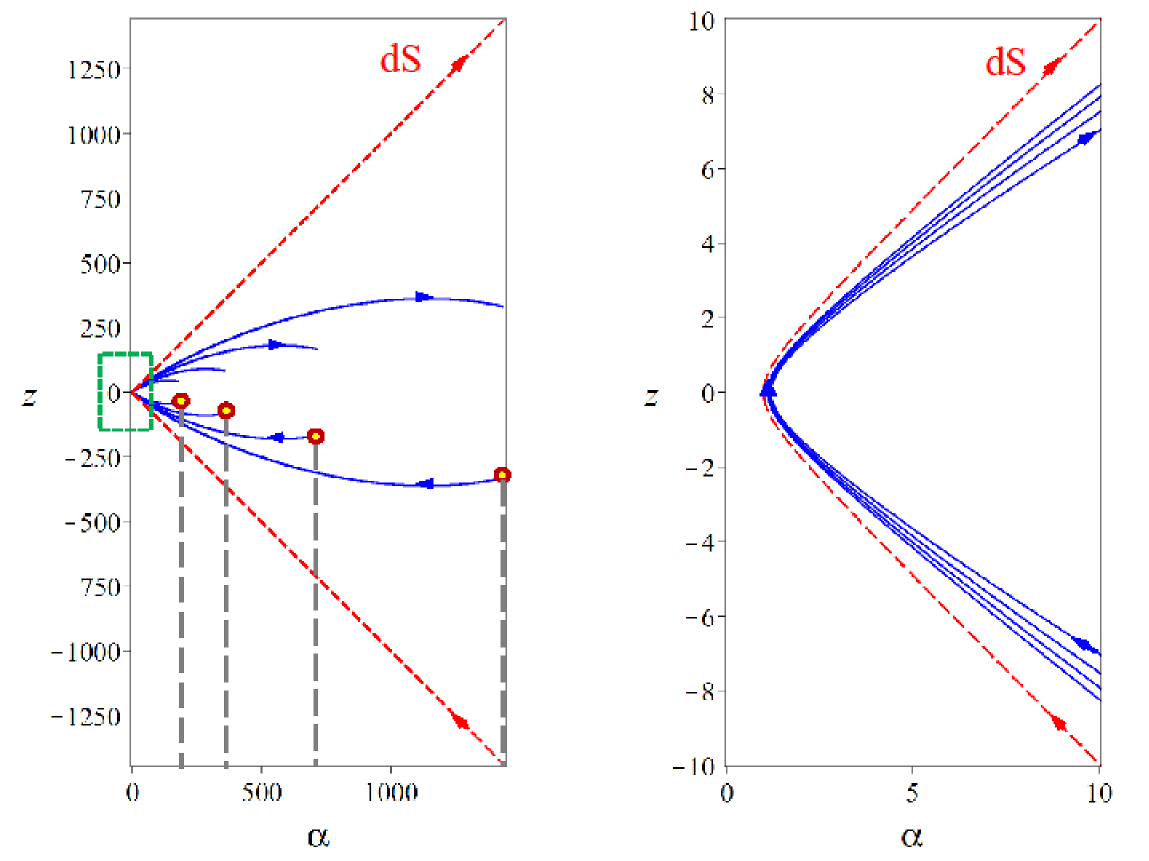
$\bar{q}$
Γ_+
$\bar{q} = \bar{p}$
Bounce point
$\frac{\pi}{4}$
θ
$\bar{p}$
C
Supercritical stage
Subcritical stage
Γ_-

Main results:

1. After the contracting universe reaches the point where its curvature becomes critical, the solution evolves along the constraint.
2. During this supercritical phase it reaches a point of bounce after which the scale function grows.
3. The control function can become zero again at this phase and the universe can leave its supercritical regime.
4. After this one has an expanding universe filled with thermal radiation which follows the corresponding solution of the Einstein equations.



dS
Friedmann expanding
Supercritical
Friedmann contracting
z
10
0
10
0
10
20
30
α0
αm
α



dS
1250
1000
750
500
250
0
-250
-500
750
-1000
-1250
z
0
500
1000
α
dS
10
8
6
4
2
0
-2
-4
-6
-8
-10
z
0
5
10
α

$$\sqrt{R_{\mu\nu}R^{\mu\nu}} = \Lambda \sim \ell^{-2},$$

$$a_0 \sim \ell \sqrt{\frac{l_{Pl}}{\ell}\left(\frac{S}{k_B}\right)^{1/3}} \; ; \frac{S}{k_B} \sim 5.4\times 10^{89},$$

For $a_0 / \ell \gg 1 \qquad a_b \sim \ell.$

Summary

- In the LCG models we use inequality constraints which control the ST curvature behavior and restrict its growing.
- In the 2D black hole model instead of a singularity inside the black hole one has a deSitter-like core.
- In cosmology the LCG theory a contracting universe has a Big Bounce.
- Future work:
  (i) 4D black holes;
  (ii) Anisotopic universes.