Missed Opportunities or Overcoming Deadlocks?

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Steklov Mathematical Institute, RAS

International Conference "Selected Topics in Mathematical Physics"
Dedicated to 75-th Anniversary of I. V. Volovich

September 27–30, 2021, online, Moscow

Few remarks about the title of my talk.

The meaning of «Missed Opportunities» is double-valued

Our «Missed Opportunities» vs
«Missed Opportunities» by others :

i) We could have missed opportunities

ii) Or, other authors may have missed some opportunities
that provide a way out of the impasse

I will present two examples of these two different types of missed opportunities.
Since this is a jubilee conference, examples will be related to hero of the day.

1-st example.

Our «Missed Opportunities»

- As is commonly believed, at anniversary conferences, people talk about the most known articles and the main achievements of the hero of the day.

- I'll follow a different way. I will talk about works not finished, or even about not started works

- I cannot say that I was privy to all the scientific ideas of Igor, so I can speak about common started papers that were not finished or were not pushed enough.

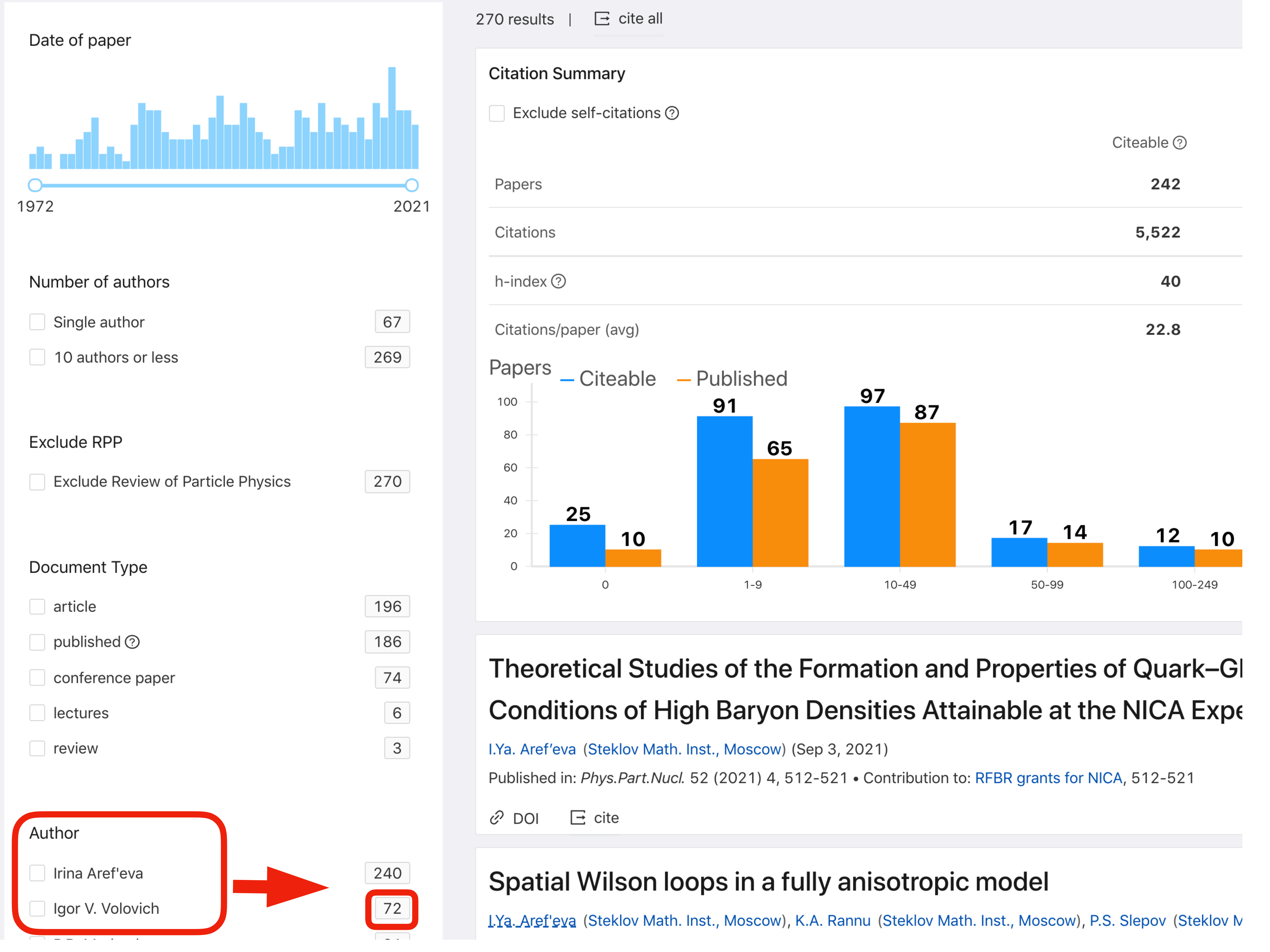
1-st example.
Our «Missed Opportunities»

- I have checked on INSPIRE database and found that we have written about 70 articles together over the past 40 years

- Now I'll show what I have found there.
  Here is a screenshot from INSPIRE.

1-st example. Our «Missed Opportunities»

Screenshot from INSPIRES


Date of paper
1972
2021
Number of authors
Single author 67
10 authors or less 269
Exclude RPP
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Document Type
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100-249
Theoretical Studies of the Formation and Properties of Quark–Gl
Conditions of High Baryon Densities Attainable at the NICA Expe
I.Ya. Aref'eva (Steklov Math. Inst., Moscow) (Sep 3, 2021)
Published in: Phys.Part.Nucl. 52 (2021) 4, 512-521 • Contribution to: RFBR grants for NICA, 512-521
DOI
cite
Spatial Wilson loops in a fully anisotropic model
I.Ya. Aref'eva (Steklov Math. Inst., Moscow), K.A. Rannu (Steklov Math. Inst., Moscow), P.S. Slepov (Steklov M

- I have checked on INSPIRE database, that is OK for theoretical physics related to our common interests.

- But theoretical physics does not cover all Igor's interests.

- For mathematical physics, it would be better to use google scholar.

- I have found the following.

Igor Volovich

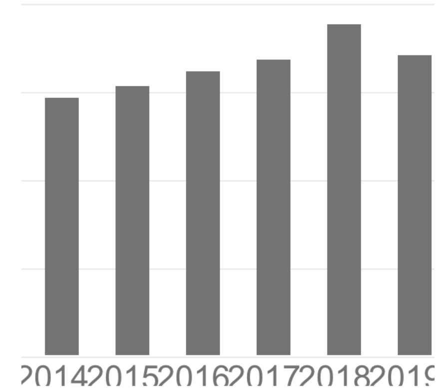
Steklov Mathematical Institute
Подтвержден адрес электронной почты в домене mi.ras.ru
mathematical physics

ПОДПИСКА ОФОРМЛЕНА

НАЗВАНИЕ	ПРОЦИТИРОВАНО	ГОД
p-adic Analysis and Mathematical Physics VS Vladimirov, IV Volovich, EI Zelenov	~~1251~~	1994
p-adic Analysis and Mathematical Physics VS Vladimirov, IV Volovich, EI Zelenov	1251	1994
Quantum theory and its stochastic limit L Accardi, YG Lu, I Volovich Springer Science & Business Media	536	2013
Theory of defects in solids and three-dimensional gravity MO Katanaev, IV Volovich Annals of Physics 216 (1), 1-28	495	1992
p-Adic string IV Volovich Classical and Quantum Gravity 4 (4), L83	438	1987
On p-adic mathematical physics B Dragovich, AY Khrennikov, SV Kozyrev, IV Volovich P-Adic Numbers, Ultrametric Analysis, and Applications 1 (1), 1-17	288	2009
p-Adic quantum mechanics VS Vladimirov, IV Volovich Communications in mathematical physics 123 (4), 659-676	275	1989

Процитировано ПРОС

	Все
Статистика цитирования	11739
h-индекс	54
i10-индекс	194



Общий доступ ПРОС

2 статьи

недоступно

На основе финансирован

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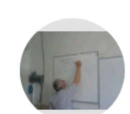
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Our «Missed Opportunities»: started but not finished /not done

I.A, I.V. Volovich,

TETRAD FORMALISM IN STRING FIELD THEORY,

Theor. Math. Phys. 71 (1987) 562-563

1. The basic entity in string field theory is the scalar string field $\Phi[X(\sigma), c(\sigma), \bar{c}(\sigma)]$, which was introduced by Siegel [1]. The field Φ is a functional of the string coordinates $X^{\mu}(\sigma)$ and the Faddeev–Popov ghosts $c(\sigma)$, $\bar{c}(\sigma)$. The expansion of Φ in a series with respect to the ghosts leads to the string differential forms of Banks and Peskin [2]. On the basis of the field Φ one can construct the interacting theory in either a fixed gauge [3] or in a gauge-invariant manner [4-7]. The mode expansion for the string field Φ for open strings contains the Yang–Mills field, and for closed strings a metric tensor $g_{\mu\nu}$, i.e., the gravitational field [1].

$$\Phi[X(\sigma), c(\sigma), \bar{(}\sigma)] = c_0 \int dx \phi(x) + c_0 \int dx A_{\mu}(x) \partial X^{\mu}(x) + \ldots$$

$$\mathcal{A}[X^R(\sigma), X^L(\sigma), c^R(\sigma)\bar{c}^L(\sigma), \ldots] = c_0\bar{c}_0 \int dx \phi(x) + c_0\bar{c}_0 \int dx g_{\mu\nu}(x) \partial X^{\mu}_R(x) \partial X^{\mu}_L(x) + \ldots$$

TETRAD FORMALISM IN STRING FIELD THEORY

We note however that in the low-energy limit superstring theory goes over into supergravity, and it is well known [8] that in supergravity the basic entity is not the metric but the Cartan variables: the field of tetrads e^a_μ, which is the root of the metric in the sense that $e_\mu{}^a e_{a\nu} = g_{\mu\nu}$, and the Lorentz connection $\omega^a_{b\mu}$. Therefore, it appears to us that the field theories of closed superstrings and the heterotic string [9] must contain the tetrad string field E^a and the string connection Ω^a_b, their mode expansions leading to the appearance of e^a_μ and $\omega^a_{b\mu}$, respectively. In this paper, we discuss their possible transformation properties and a mechanism for constructing a gauge-invariant theory.

The string fields E^a and Ω^a_b must be the elements of a certain algebra with differential Q with, in general, nonassociative multiplication $\circ$ (see the description of the $\mathscr{B}$ algebra in [10] and also, for the associative case, [11] of Connes and [4] of Witten). By analogy with (1), it is natural to postulate the transformation law

$$\delta_L E^a = \Lambda_b{}^a \circ E^b, \quad \delta_L E_a = E_b \circ \Lambda_a{}^b, \quad \delta\Omega_b{}^a = Q\Lambda_b{}^a + (\Lambda\circ\Omega)_b{}^a, \tag{3}$$

where Λ^a_b is the string gauge parameter.

$$E_a[X(\sigma), c(\sigma), \bar{c}(\sigma)] = c_0 \int dx \phi_a(x) + c_0 \int dx e_{a\mu}(x) \partial X^\mu(x) + \ldots$$

$$\Omega^b_a[X(\sigma), c(\sigma), \bar{c}(\sigma)] = c_0 \int dx \phi^b_a(x) + c_0 \int dx \omega^b_{a\mu}(x) \partial X^\mu(x) + \ldots$$

$$\delta E_a = \Lambda^b_a \star E_b, \qquad \delta\Omega^b_a = Q\Lambda^b_a + \Lambda^c_a \star E_c$$

TETRAD FORMALISM IN STRING FIELD THEORY

To obtain the string generalization of the general coordinate transformations, one must introduce the string Lie derivative, which can be chosen in the form

$$\delta_G E^a = \mathscr{L}_\Xi E^a = \Xi \circ E^a, \quad \delta \Omega_b{}^a = \mathscr{L}_\Xi \Omega_b{}^a = \Xi \circ \Omega_b{}^a, \tag{4}$$

where Ξ is a string gauge parameter.

Further, by analogy with the curvature tensor $R = d\omega + \omega \wedge \omega$ and the Hilbert-Einstein-Cartan Lagrangian $L = R_{\mu\nu}{}^{ab} e_a{}^\mu e_b{}^\nu$ it is natural in string theory to define the curvature

$$\mathscr{R}_b{}^a = Q\Omega_b{}^a + {}^1/_2 (\Omega \circ \Omega)_b{}^a \tag{5}$$

and the action

$$S = (\mathscr{R}_b{}^a, E_a \circ E^b), \tag{6}$$

$$\mathcal{R}_a^b = Q\Omega_a^b + \frac{1}{2}\Omega_a^c \star \Omega_c^b, \qquad S = (\mathcal{R}, E \star E)$$

NOT realized

I.A. and I.Volovich,
`Knots and matrix models,"

Inf. Dim. Anal. Quant. Probab. Rel. Top. 1, 167-173 (1998) [arXiv:hep-th/9706146 [hep-th]].

We consider a matrix model with d matrices $N \times N$ and show that in the limit $N \to \infty$ and $d \to 0$ the model describes the knot diagrams.

Let be given positive integers N and d and let $A_\mu = (A_\mu^{ij})$ and $B_\mu = (B_\mu^{ij})$, $i, j = 1, ..., N$ are $N \times N$ Hermitian matrices, $A_\mu^* = A_\mu$, $B_\mu^* = B_\mu$. Here $\mu = 1, ..., d$. The matrix model has the following partition function

$$Z = Z(N, d, g) = \int e^{iS} dAdB \tag{2.1}$$

where the Lagrangian is

$$S = Tr(A_\mu B_\mu) + \frac{g}{2N} Tr(A_\mu B_\nu A_\mu B_\nu) \tag{2.2}$$

Knots and matrix models

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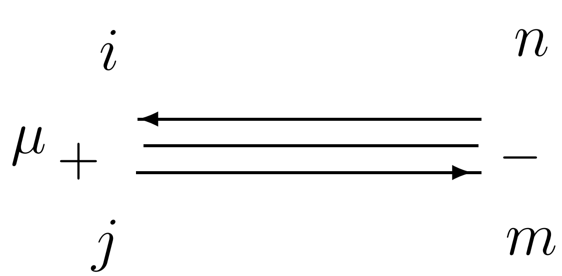
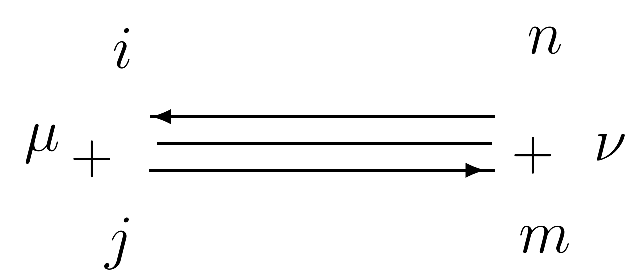
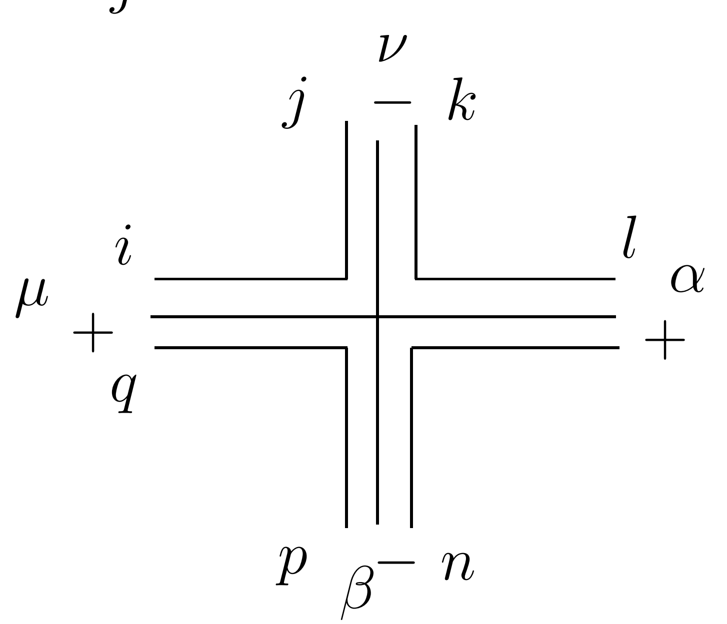


$= i\delta_{\mu\nu}\delta_{in}\delta_{jm}$



$= 0$



$= \frac{ig}{N}\delta_{\mu\alpha}\delta_{\nu\beta}\delta_{ij}\delta_{kl}\delta_{mn}\delta_{pq}$

Knots and matrix models

Theorem. The set of connected vacuum Feynman diagrams for the model (2.1), (2.2) in the limit $N \to \infty$ and $d \to 0$ is in one-to-one correspondence with the set of alternating knot diagrams. The generating function for the alternating knot diagrams is given by the expression

$$F(g) = \lim_{d\to 0} \lim_{N\to\infty} \frac{1}{dN^2} \ln Z(N,d,g) \qquad (2.5)$$

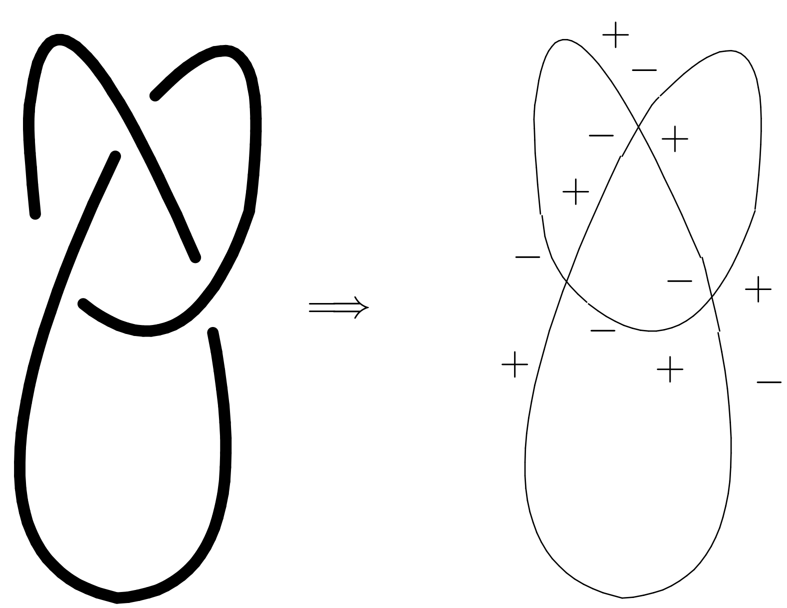


Figure 1: Trefoil

2-nd example. «Missed Opportunities by others»

It is related to information paradox that occurs in black hole evaporation

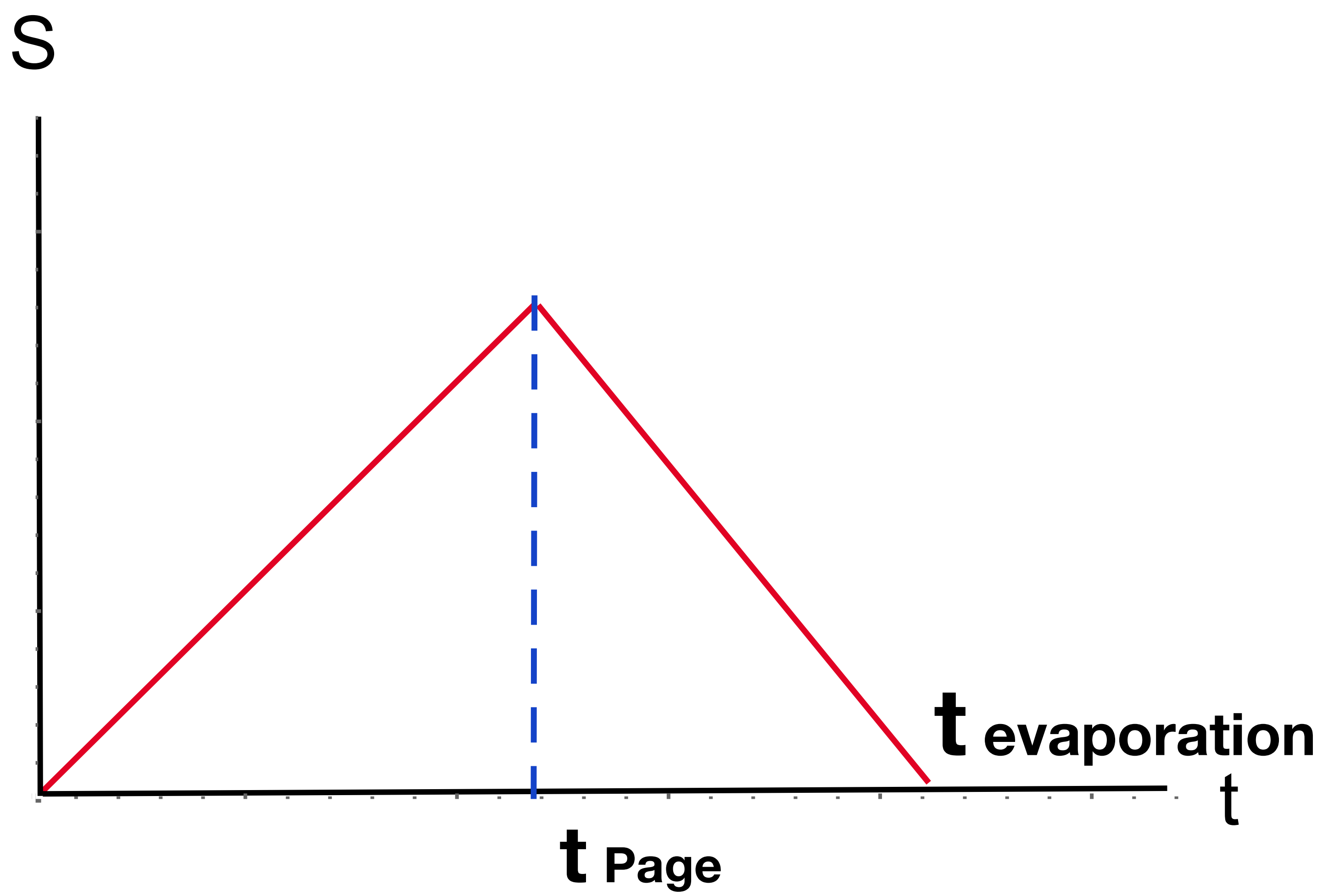
Information paradox

Entropy of Hawking radiation of black holes grows up to infinity during evaporation and it is a manifestation of the information paradox.

S.W. Hawking, Particle creation by black holes, CMP 43 (1975) 199.

Information paradox

This increase contrasts with Page's hypothetical behavior, in which entropy decreases after the so-called Page time and which ensures the unitarity of quantum mechanics



D.N.Page, Information in black hole radiation, PRL 71 (1993) 3743

Information paradox

An approach to treating the problem of black hole information was proposed

G. Penington, 1905.08255

A.Almheiri, N.Engelhardt, D.Marolf,H.Maxfield,1905.08762

"Island formula" for the entanglement entropy of Hawking radiation (based on quantum extremal surfaces)

$$S(R) = \min\left\{\operatorname*{ext}_{\mathcal{I}}\left[\frac{\text{Area}(\partial\mathcal{I})}{4G} + S_{\text{matter}}(R\cup\mathcal{I})\right]\right\}$$

$\mathcal{I}$ is the island, Area$[\partial\mathcal{I}]$ its boundary area

S_{matter} is the von Neumann entropy $S_{\text{vN}}(R\cup I)$ of union of the island and the region R.

An extremization on any possible island and then taking the minimum entropy is supposed.

Information paradox

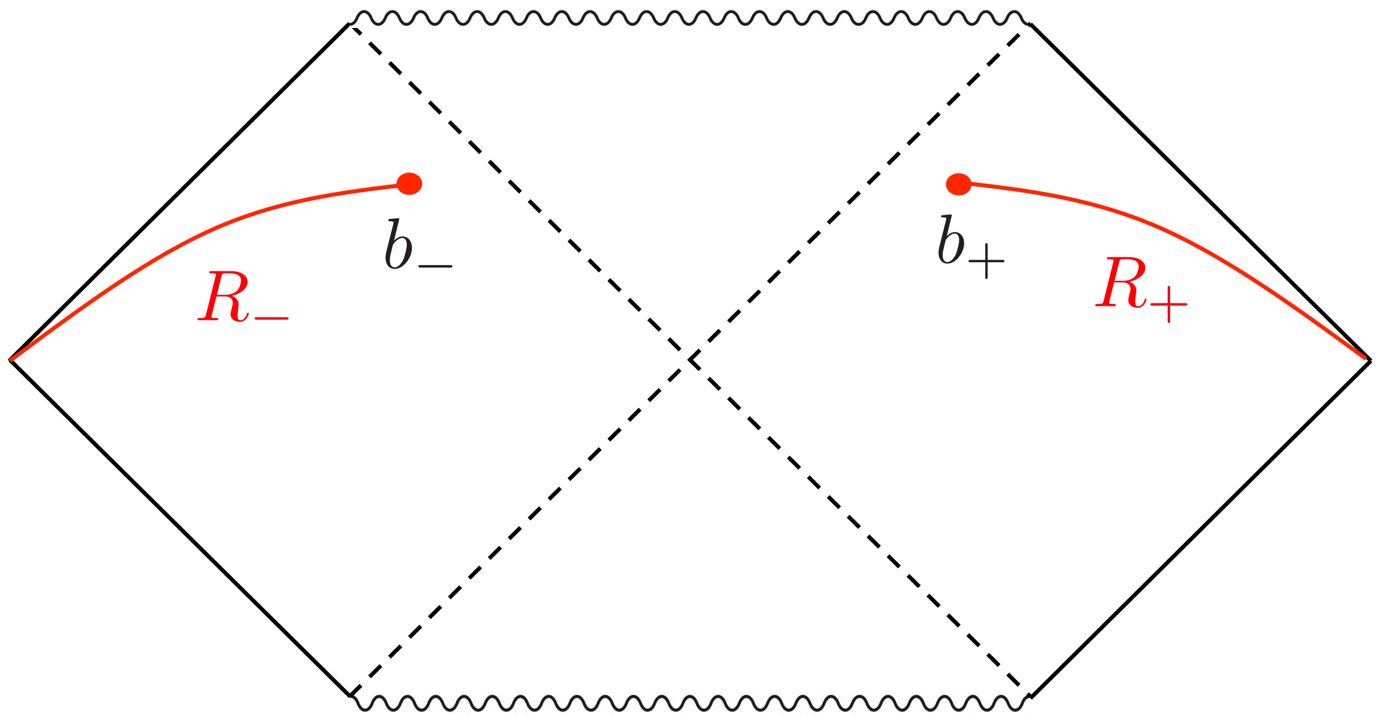
$$S(R) = \min\left\{\operatorname*{ext}_{\mathcal{I}}\left[\frac{\text{Area}(\partial\mathcal{I})}{4G} + S_{\text{matter}}(R \cup \mathcal{I})\right]\right\}$$

For 2 dim gravity the island rule has been derived by making use of replica trick

Information paradox

One of simple example explicitly demonstrated how an island can help to make bounded entanglement entropy of the Hawking radiation is two sided black hole

K.Hashimoto, N.Iizuka, Y.Matsuo, 2004.05863

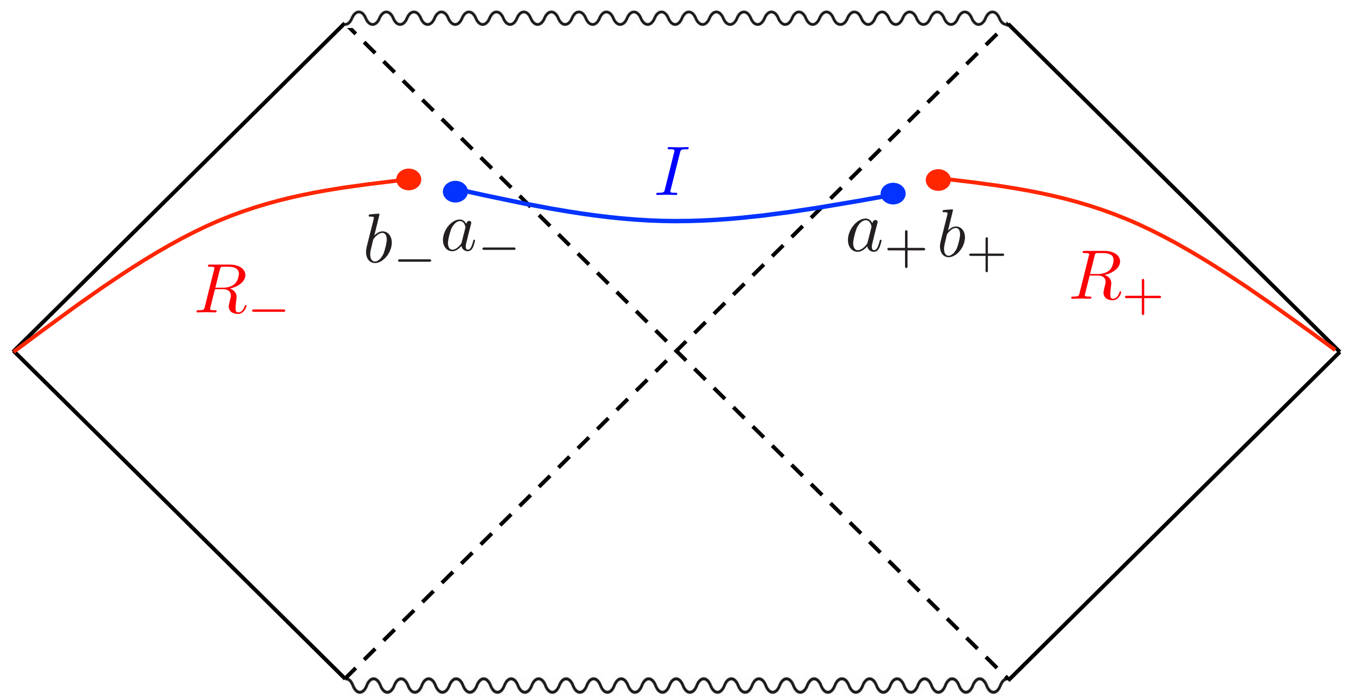


Penrose diagram of the static Schwarzschild spacetime

$$S_{n\mathcal{I}} = \frac{2\pi b^2}{G_N} + \frac{c}{6}\log\left[\frac{16r_h^2(b-r_h)}{b}\cosh^2\frac{t_b}{2r_h}\right]$$

Linear in time at large time

Information paradox



Penrose diagram of the static Schwarzschild spacetime with an island

Hawking radiation has two parts R_+ and R_-

The boundaries of I are located at a_+ and a_-

$$S_{\mathcal{I}} = \frac{2\pi r_h^2}{G} + \frac{c}{6}\frac{b - r_h}{r_h} + \frac{c}{6}\log\frac{16 r_h^3 (b - r_h)^2}{G^2 b}$$

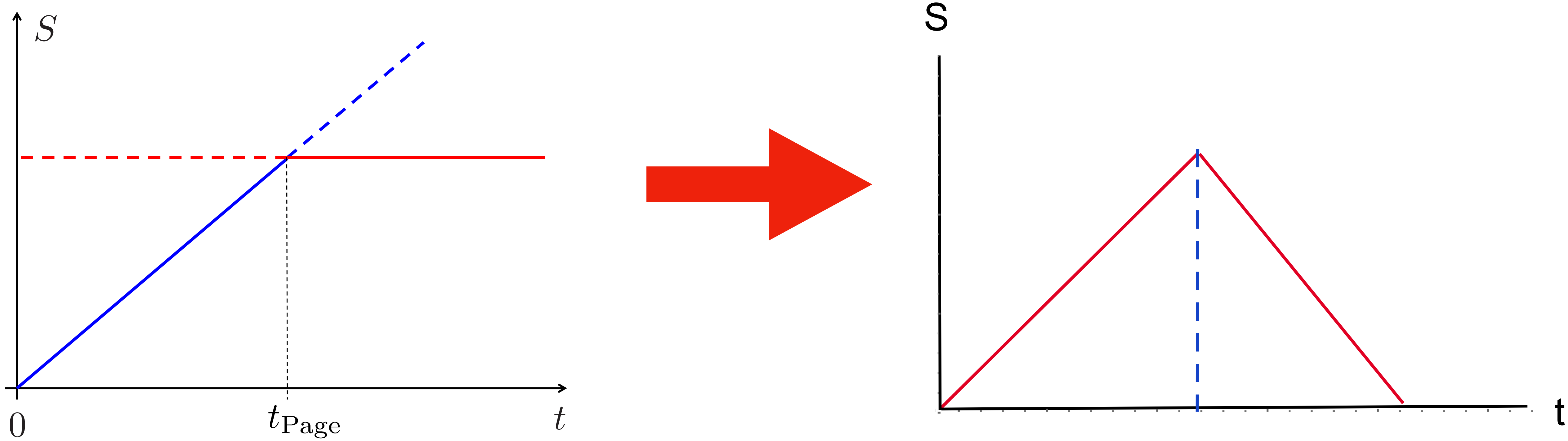
No time dependence

Information paradox

The Page curve for the

eternal Schwarzschild black hole

evaporating black hole

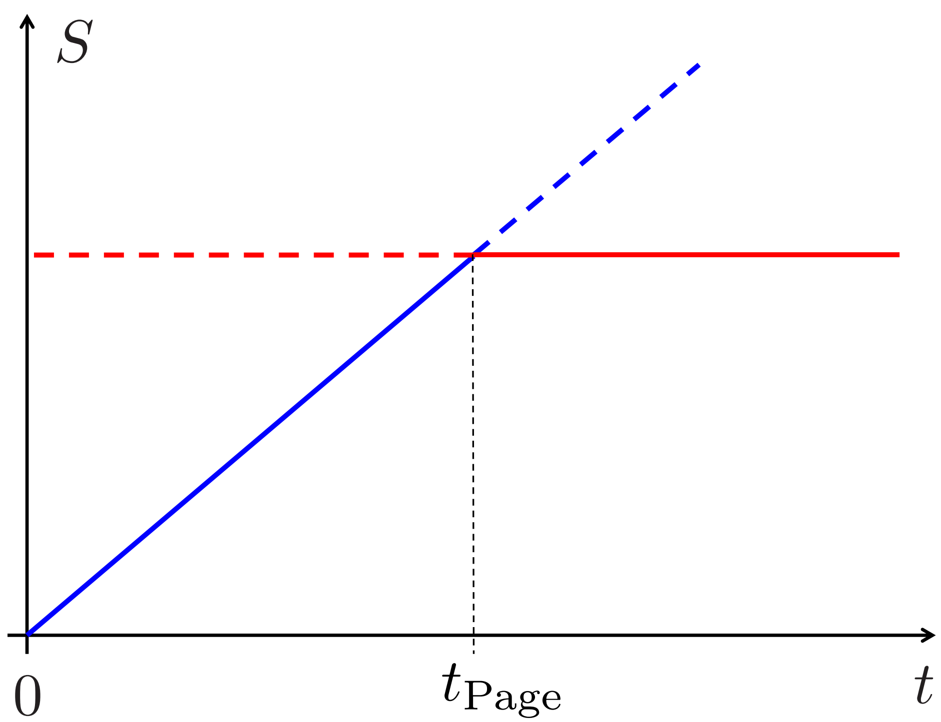


K.Hashimoto, N.Iizuka, Y.Matsuo, 2004.05863

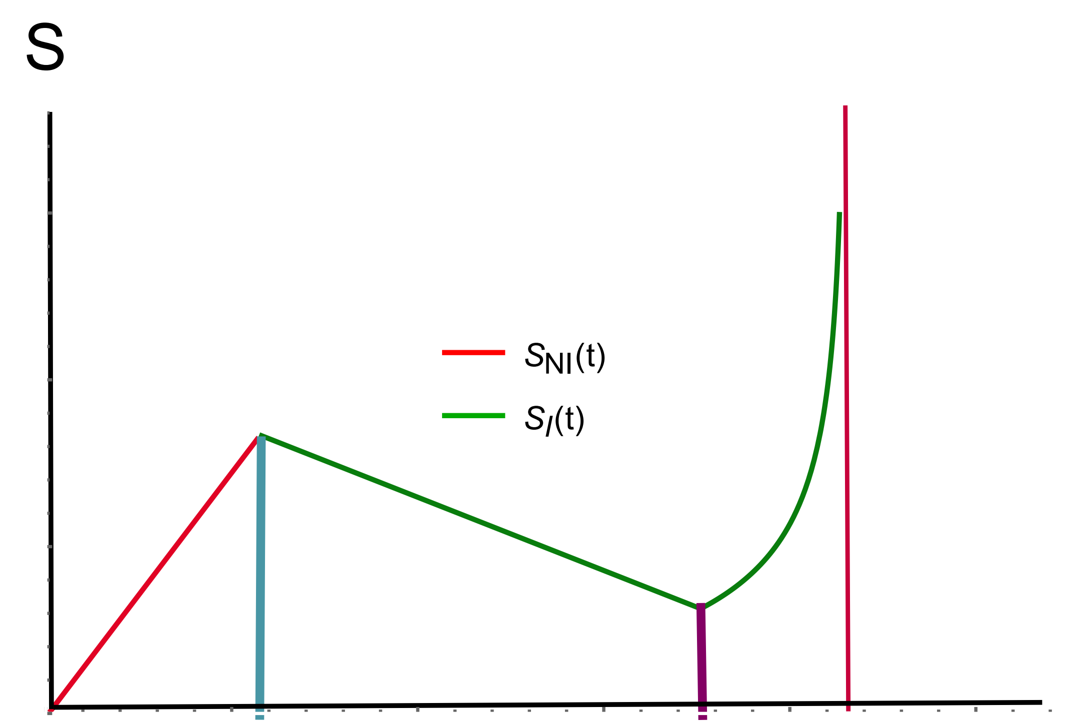
Information paradox

With I.Volovich

The Page curve for the eternal Schwarzschild black hole



Island entropy increas in the end of evaporation.

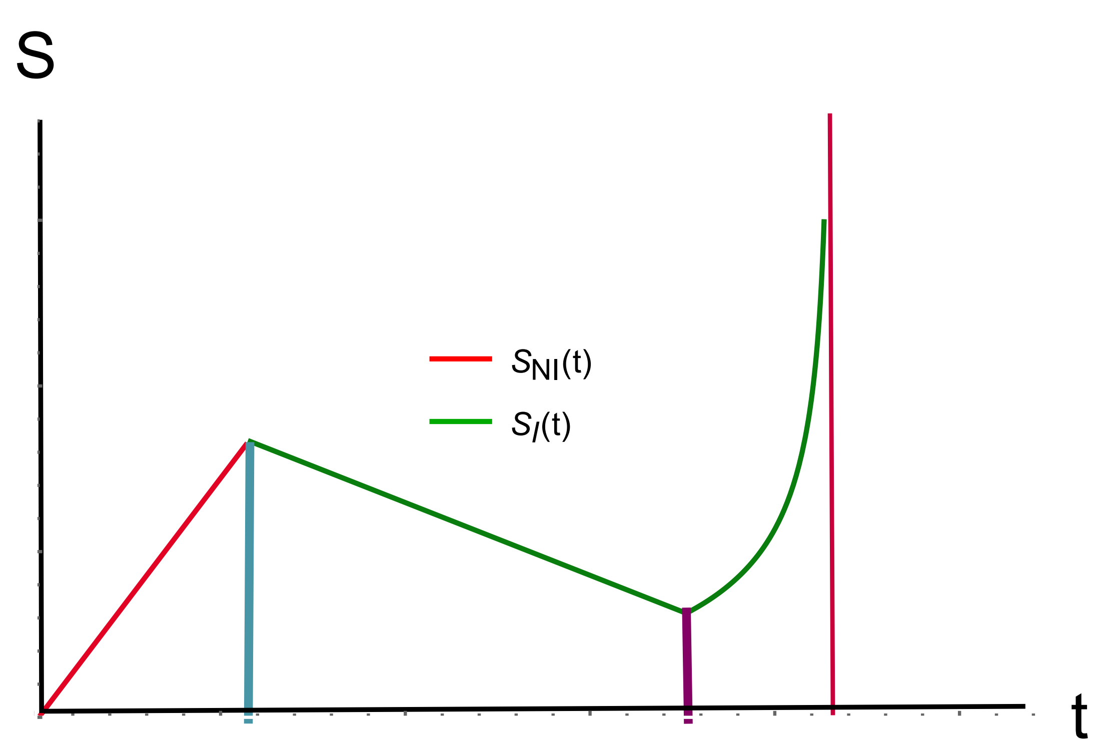


Increasing starts after some decreasing time.

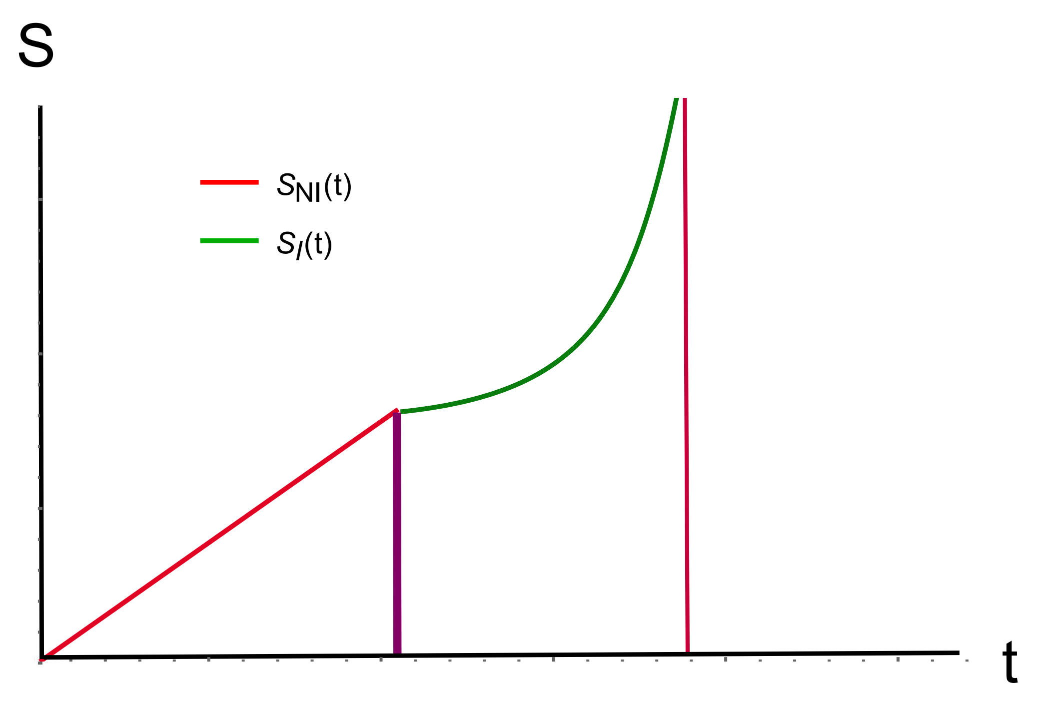
Information paradox

With I.Volovich

Island entropy increas in the end of evaporation.



Increasing starts after some decreasing time.



There is no decreasing period.

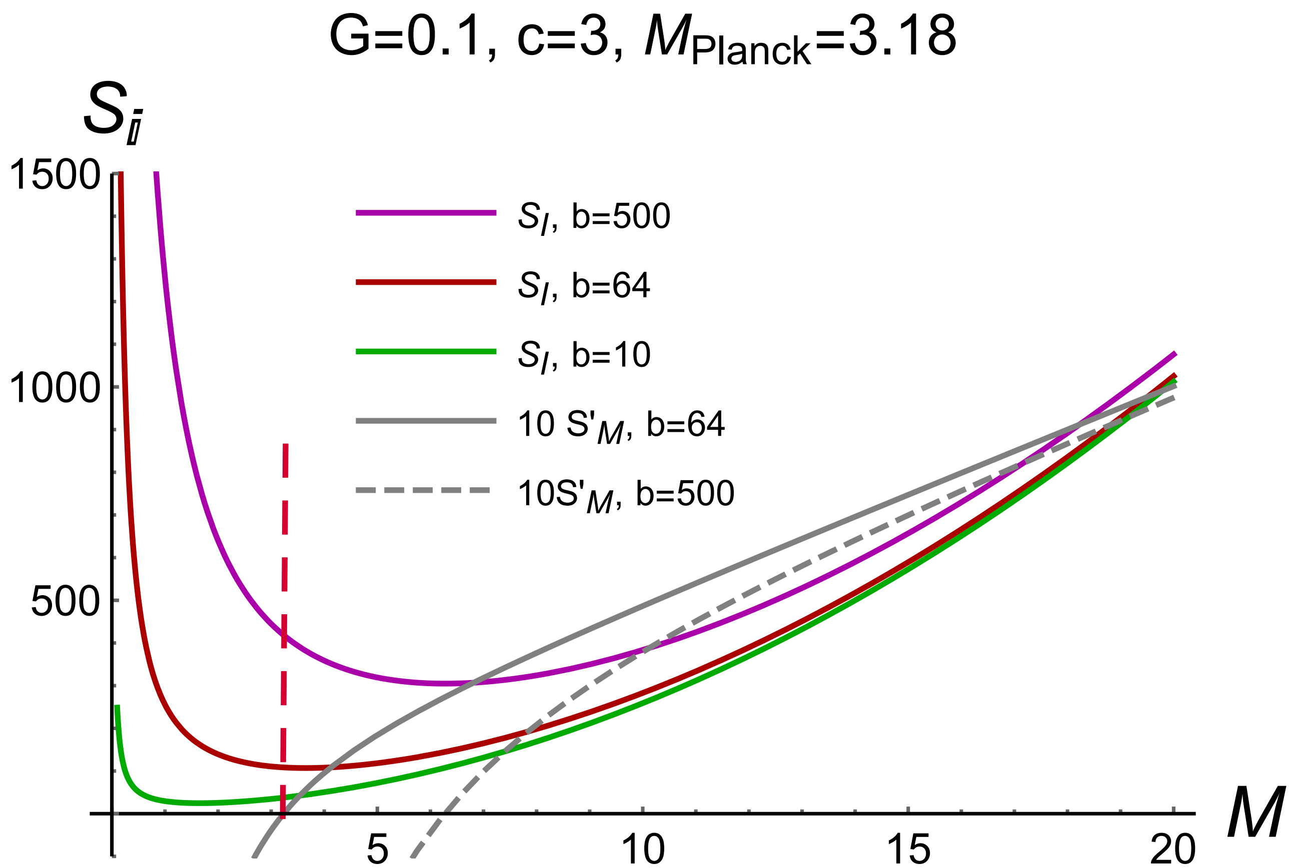
Information paradox

Conclusion.

We consider evaporation of the Schwarzschild black hole and note that, generally speaking, an island doesn't provide a bounded entanglement entropy in the end of the black hole evaporation.

Thank you for your attention!

Backup



$$M_{min} = \frac{1}{4}\left(\frac{bc}{3\pi G^2}\right)^{1/3}$$

$$b > r_h$$

$$M_{min} > M_{Planck} \simeq 1/\sqrt{G},$$

In four dimensions, due to radiation the mass M of the black hole is reduced as

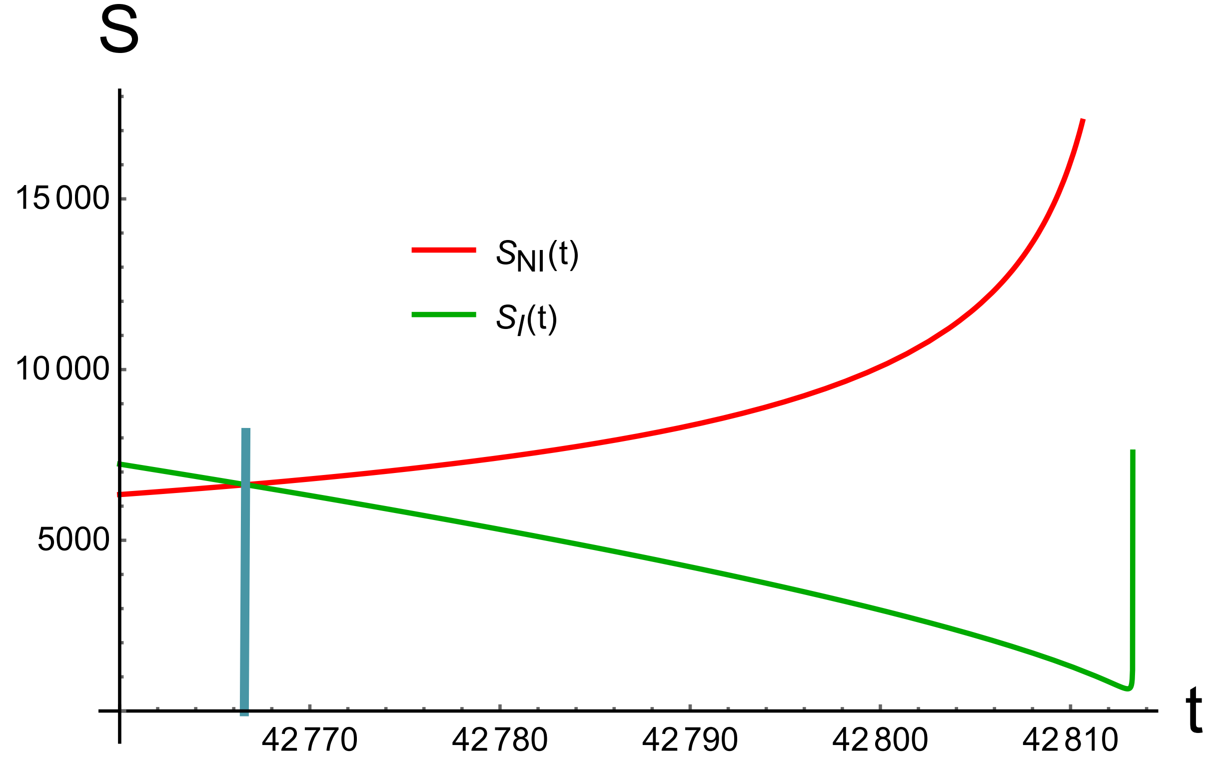
Page

$$M(t) = \frac{r_0}{2G}\left(1 - \frac{24\alpha\, c\, Gt}{r_0^3}\right)^{1/3} \qquad t_{evaporate} = \frac{r_0^3}{24c\,\alpha G}$$

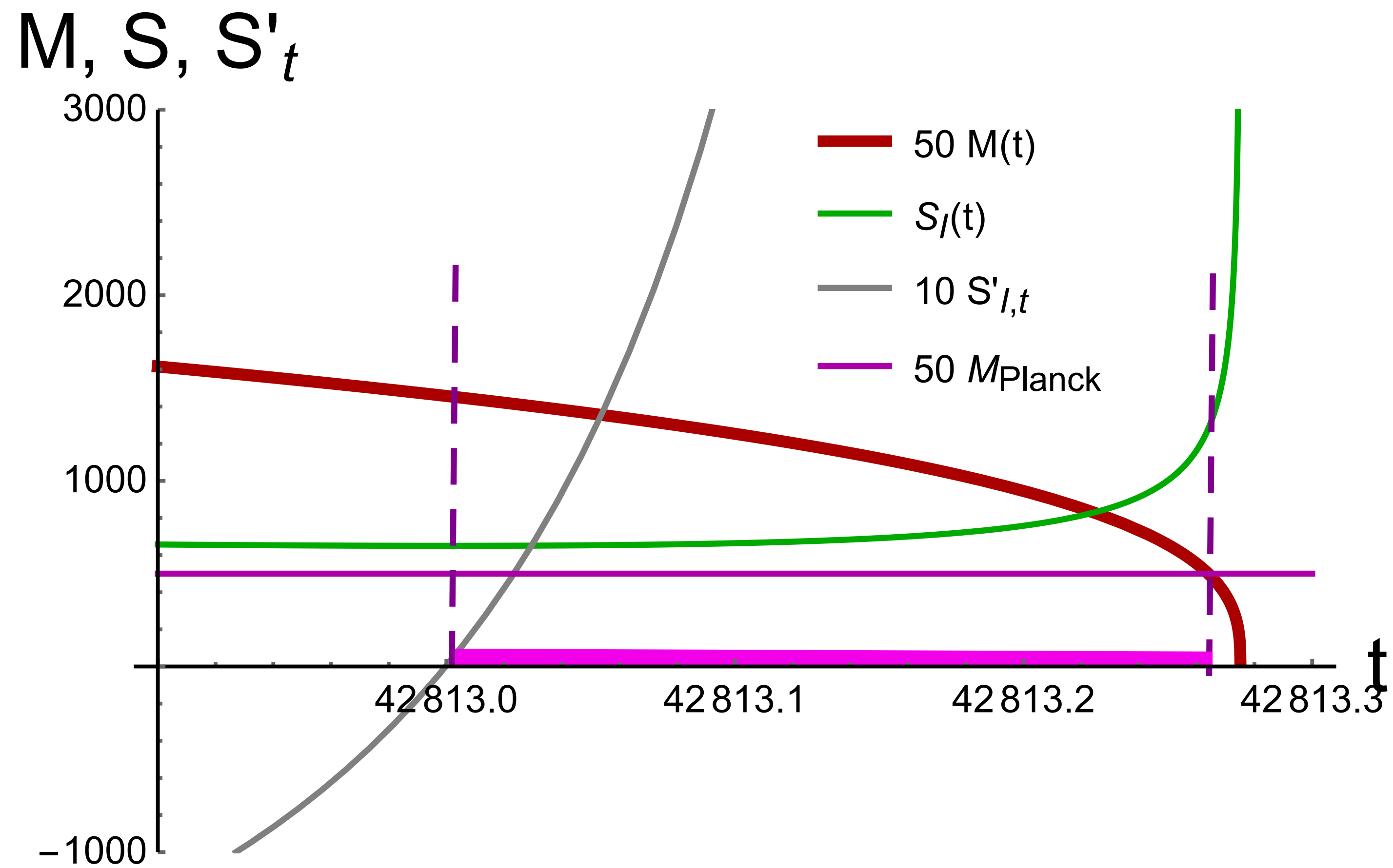
α is a constant dependent on the spin of the radiating particle

Backup

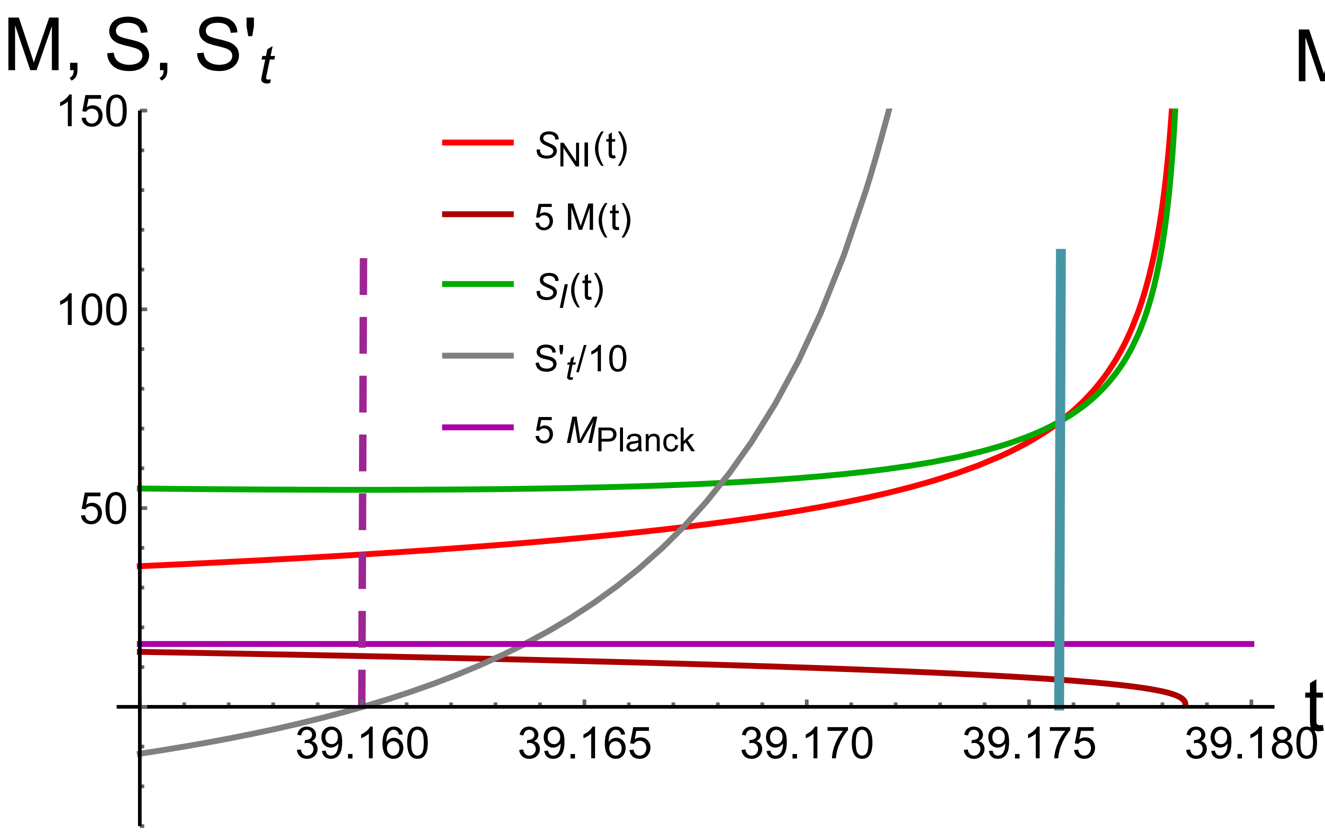
b=500, κ=−30.771, α=1, G=0.01, c=3



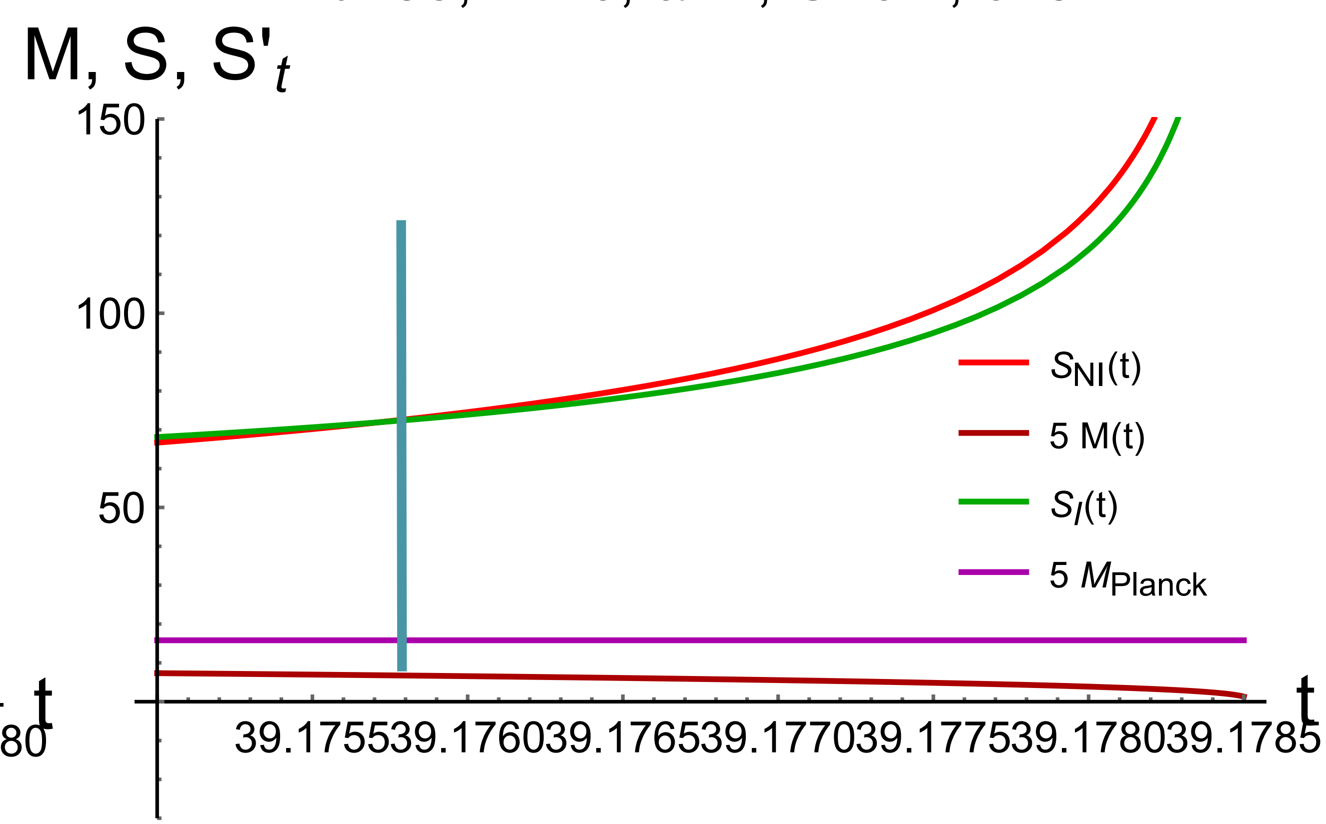
b=500, κ=−30.771, α=1, G=0.01, c=3



b=35, k=−6, α=1, G=0.1, c=3



b=35, k=−6, α=1, G=0.1, c=3



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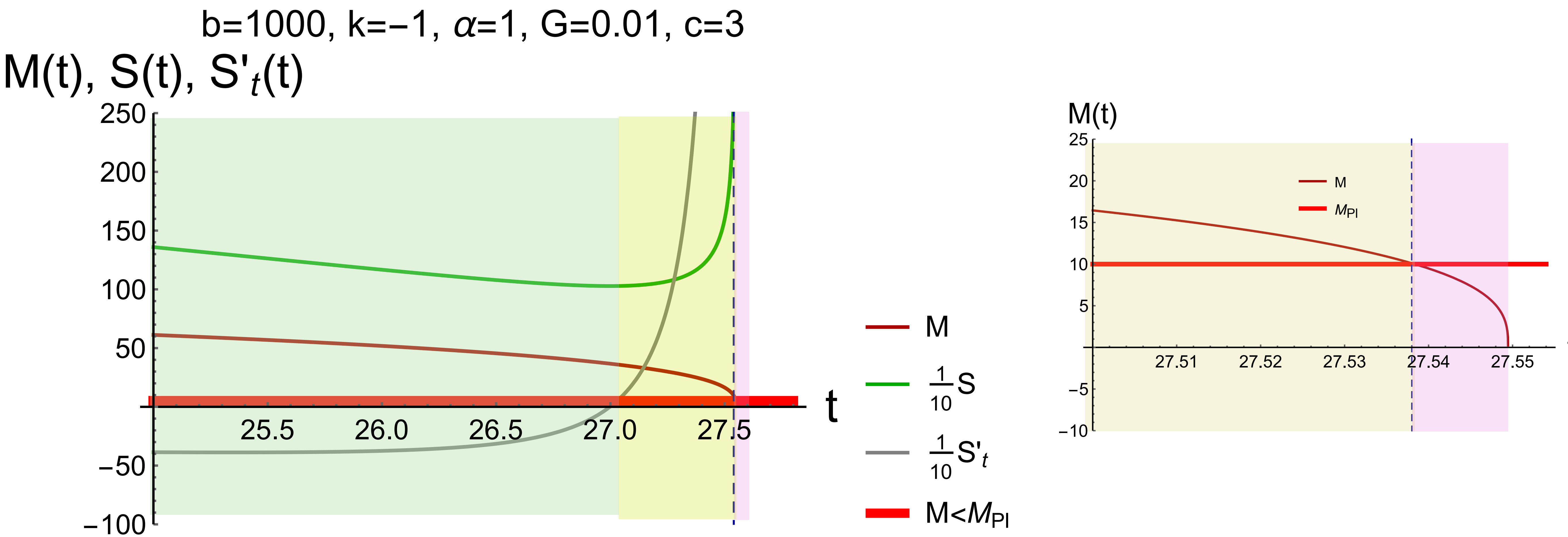

Green area shows the area of decreasing entropy (the derivative shown by gray line is negative), the yellow one shows increasing entropy (the derivative is positive) and the pink one corresponds to the Planck scale. In plot near the Planck scales. The darker red line show the decreasing of mass of evaporated BH, the red line shows the Plank mass, $M_{Plank} = 1/\sqrt{G}$ and for $G = 0.01$ we have $M = 10$.