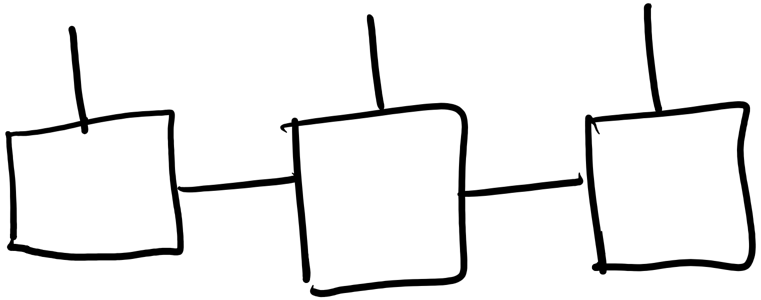
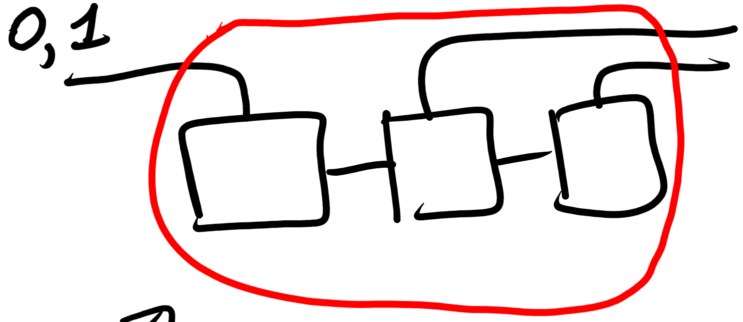
Лекция 4 по курсу "Квантовые тензорные сети" (28.09.2021)



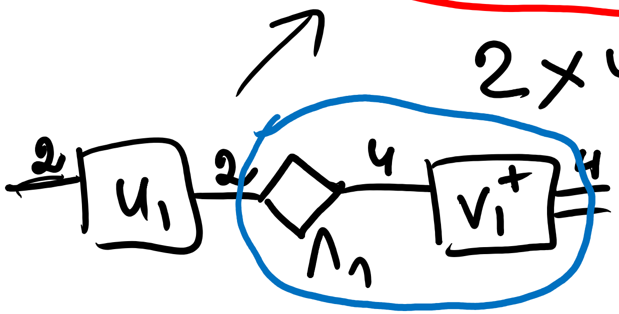
Пример: W – сост. 3-х кубитов

$$|W\rangle = \frac{1}{\sqrt{3}}\left(|100\rangle + |010\rangle + |001\rangle\right)$$

Хотим получить левую канон. форму



= мультииндекс 00, 01, 10, 11



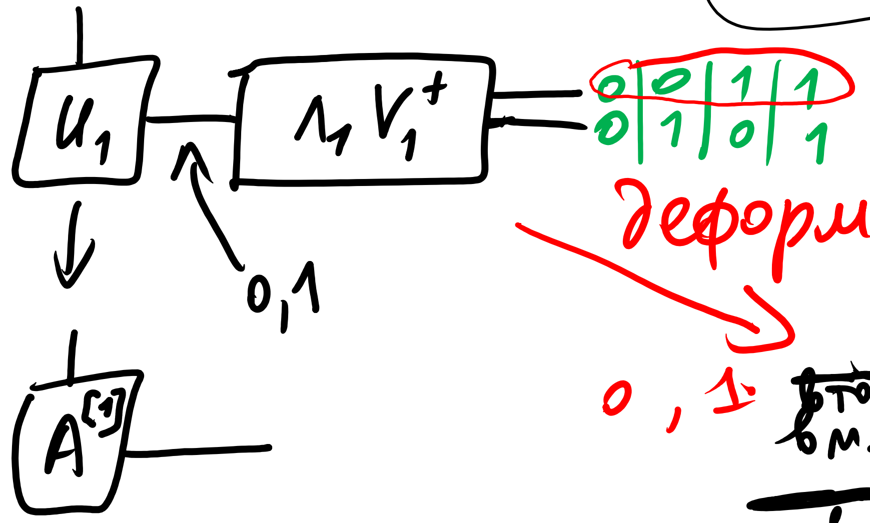
2×4

	00	01	10	11
0 →	0	$1/\sqrt{3}$	$1/\sqrt{3}$	0
1 →	$1/\sqrt{3}$	0	0	0

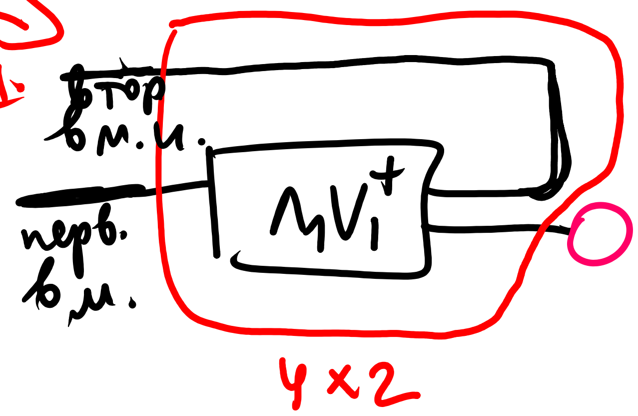
→ синг. разлож.

$$= \begin{pmatrix}1&0\\0&1\end{pmatrix} \cdot \begin{pmatrix}\sqrt{\frac{2}{3}}&0&0&0\\0&\sqrt{\frac{1}{3}}&0&0\end{pmatrix} \begin{pmatrix}0&1&0&0\\ \frac{1}{\sqrt{2}}&0&0&-\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}&0&0&\frac{1}{\sqrt{2}}\\ 0&0&1&0\end{pmatrix}$$

$$A^{[1]} = U_1 \qquad \Lambda_1 \qquad V_1^+$$



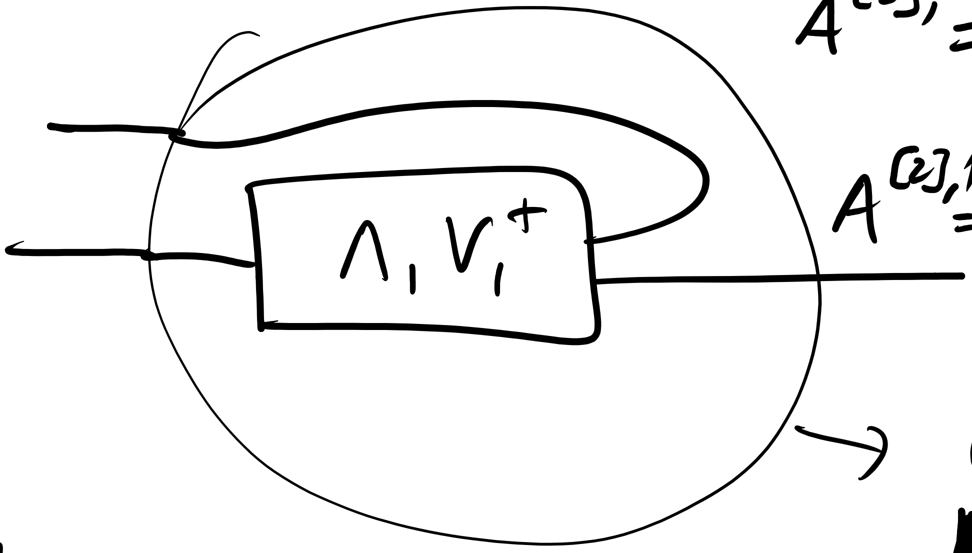
$$\Lambda_1 V_1^+ = \begin{array}{c} \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ \frac{1}{\sqrt{6}} & 0 & 0 & -\frac{1}{\sqrt{6}} \end{pmatrix} \end{array} \begin{array}{l} \leftarrow 0 \\ \leftarrow 1 \end{array}$$



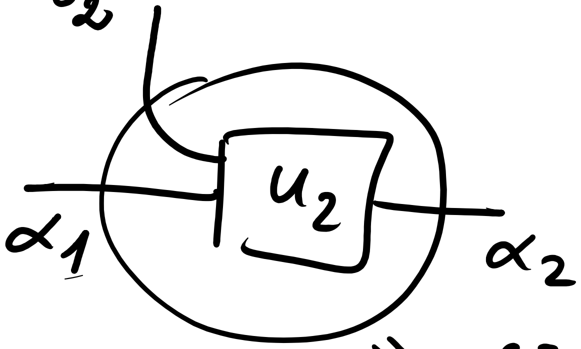
$$\begin{array}{c|cc} & 0 & 1 \\ \hline 00 & 0 & \sqrt{\frac{2}{3}} \\ 01 & 0 & 0 \\ 10 & \frac{1}{\sqrt{6}} & 0 \\ 11 & 0 & -\frac{1}{\sqrt{6}} \end{array}$$

$\rightarrow A^{[1],0} = (1 \;\; 0)$

$\rightarrow A^{[1],1} = (0 \;\; 1)$

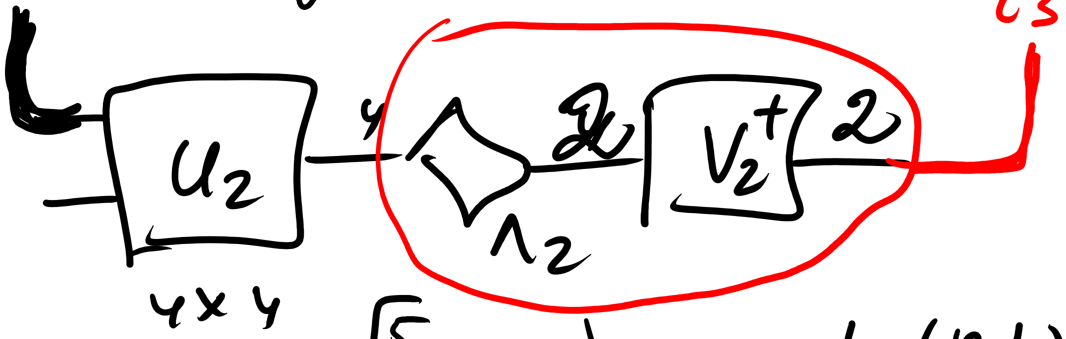


$$A^{[2],0} = \begin{pmatrix} \frac{2}{\sqrt5} & 0 & \frac{1}{\sqrt5} & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad A^{[2],1} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ -\frac{1}{\sqrt5} & 0 & \frac{2}{\sqrt5} & 0 \end{pmatrix}$$



$= A^{[2],i_2}$

→ синг. разл.

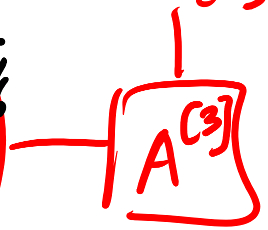


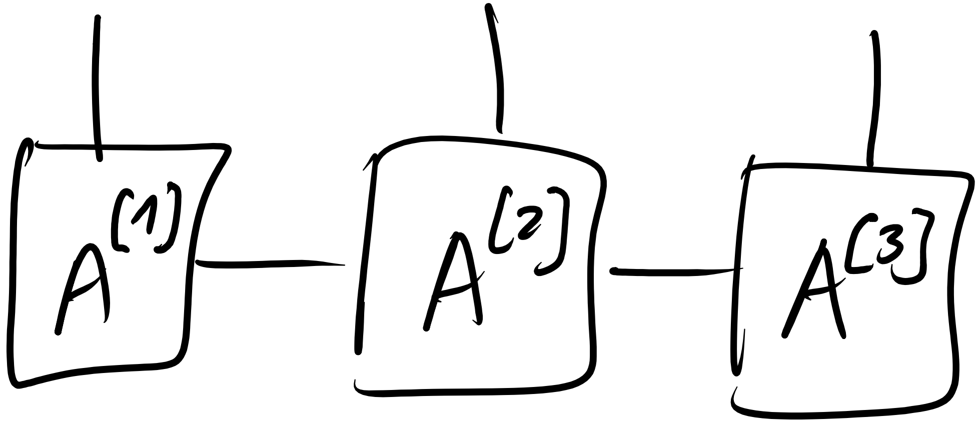
$$U_2 = \begin{array}{c} \alpha_1\, i_1 \\ 0\;0 \\ 0\;1 \\ 1\;0 \\ 1\;1 \end{array} \begin{pmatrix} \frac{2}{\sqrt5} & 0 & \frac{1}{\sqrt5} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt5} & 0 & \frac{2}{\sqrt5} & 0 \end{pmatrix}$$

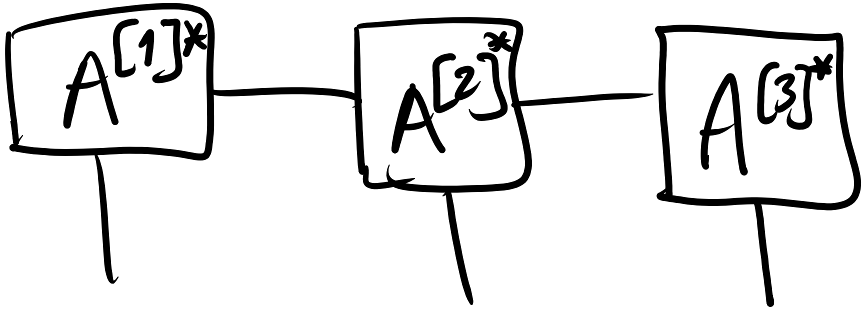
α_2

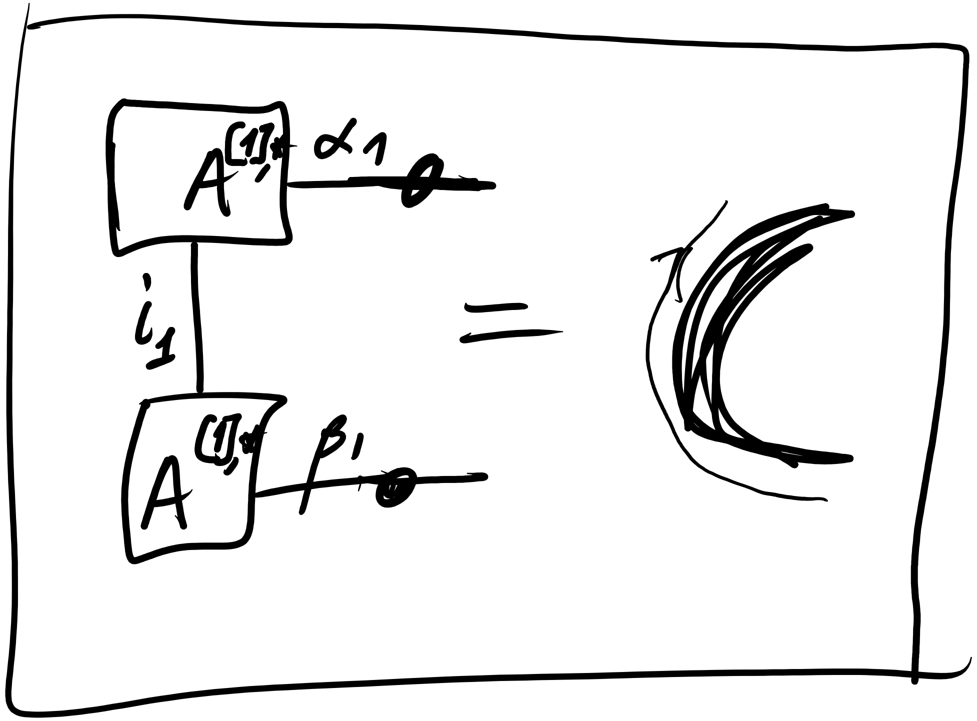
$$\Lambda_2 = \begin{pmatrix} \sqrt{\frac{5}{6}} & 0 \\ 0 & 1/\sqrt6 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad V_2^+ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A^{[3],0} = \begin{pmatrix} 0 \\ 1/\sqrt6 \\ 0 \\ 0 \end{pmatrix} \qquad A^{[3],1} = \begin{pmatrix} \sqrt{5/6} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



$|\overset{W}{\psi}\rangle =$ 

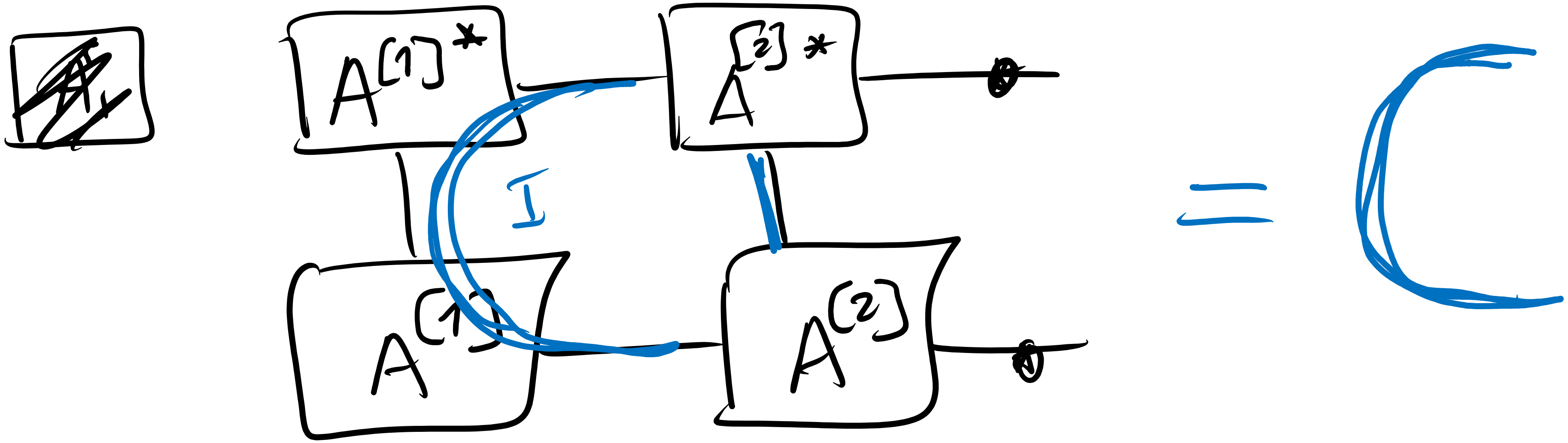
$\langle \overset{W}{\psi}| =$ 



$$\underbrace{\langle\alpha_1|\sum_{i_1}\left(A^{[1],i_1}\right)^{\dagger}} A^{[1],i_1}|\beta_1\rangle \Leftarrow \sum_{i_1}\underbrace{\left(A^{[1],i_1}\right)^{*}_{\alpha_1}}\left(A^{[1],i_1}\right)_{\beta_1} = \delta_{\alpha_1\beta_1}$$

$$\langle\alpha_1| I |\beta_1\rangle$$

$$\sum_{i_1} \left(A^{[1],i_1}\right)^{+} A^{[1],i_1} = I.$$

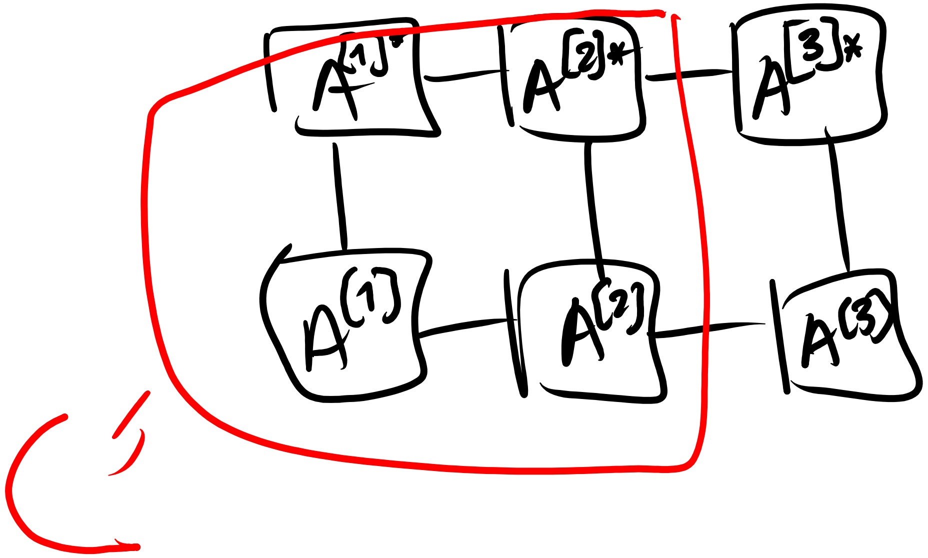


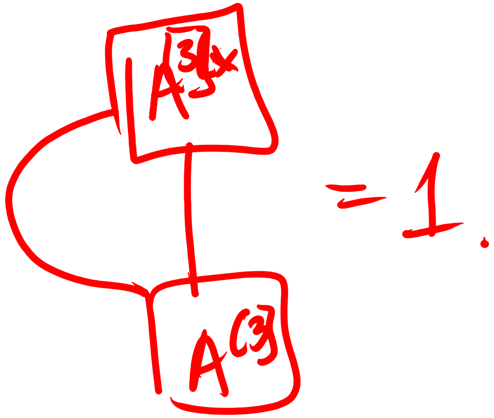
$$\boxed{\sum_{i_2} \left(A^{[2],i_2}\right)^{+} A^{[2],i_2} = I.}$$

левая канон.

$$A^{[3],0} = \begin{pmatrix} 0 \\ 1/\sqrt{6} \\ 0 \\ 0 \end{pmatrix} \; ; \quad A^{[3],1} = \begin{pmatrix} \sqrt{5/6} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

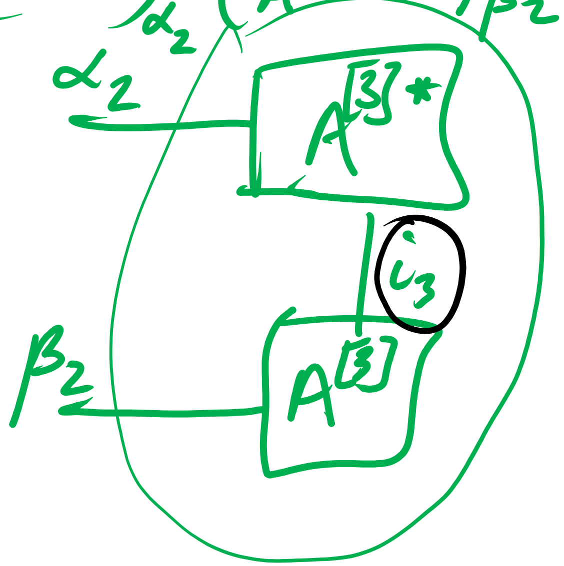
$$\sum_{i_3} \left(A^{[3],i_3}\right)^{+} A^{[3],i_3} = \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right) = (1)$$





$$\sum_{i_3} \left(A^{[3],i_3}\right)^{*}_{\alpha_2} \underbrace{\left(A^{[3],i_3}\right)_{\beta_2}}_{}$$

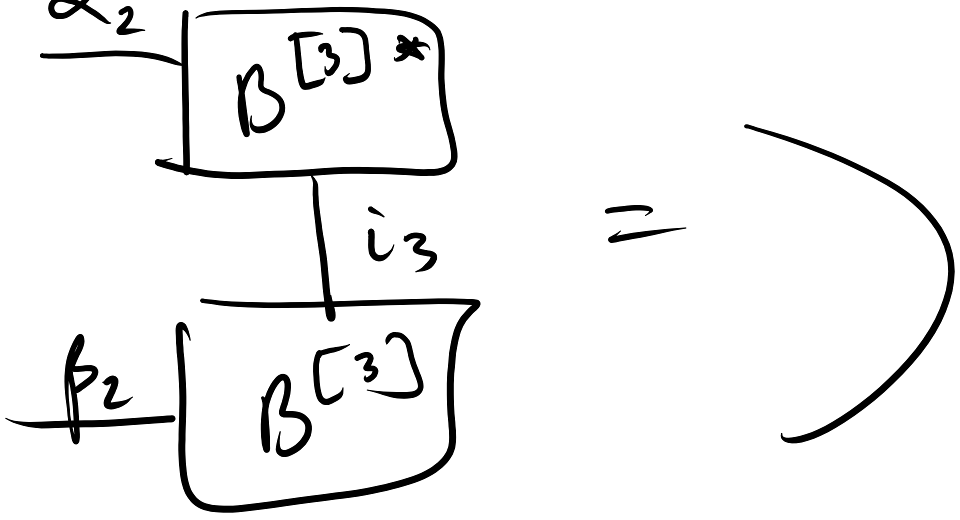
1×1



$$\begin{pmatrix} 0 \\ 1/\sqrt{6} \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 & 1/\sqrt{6} & 0 & 0 \end{pmatrix} + \begin{pmatrix} \sqrt{5/6} \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \sqrt{5/6} & 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 5/6 & & & \\ & 1/6 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \neq I.$$

Условие правой каноничности



$= \delta_{\alpha_2\beta_2}$

$$\sum_{i_3} \left(B^{[3],i_3}\right)^*_{\alpha_2} \left(\underline{B^{[3],i_3}}\right)_{\beta_2} = \delta_{\alpha_2\beta_2} = \delta_{\beta_2\alpha_2}$$

$$\langle \beta_2 | \sum_{i_3} B^{[3],i_3} \left(B^{[3],i_3}\right)^+ | \alpha_2 \rangle = \langle \beta_2 | I | \alpha_2 \rangle$$

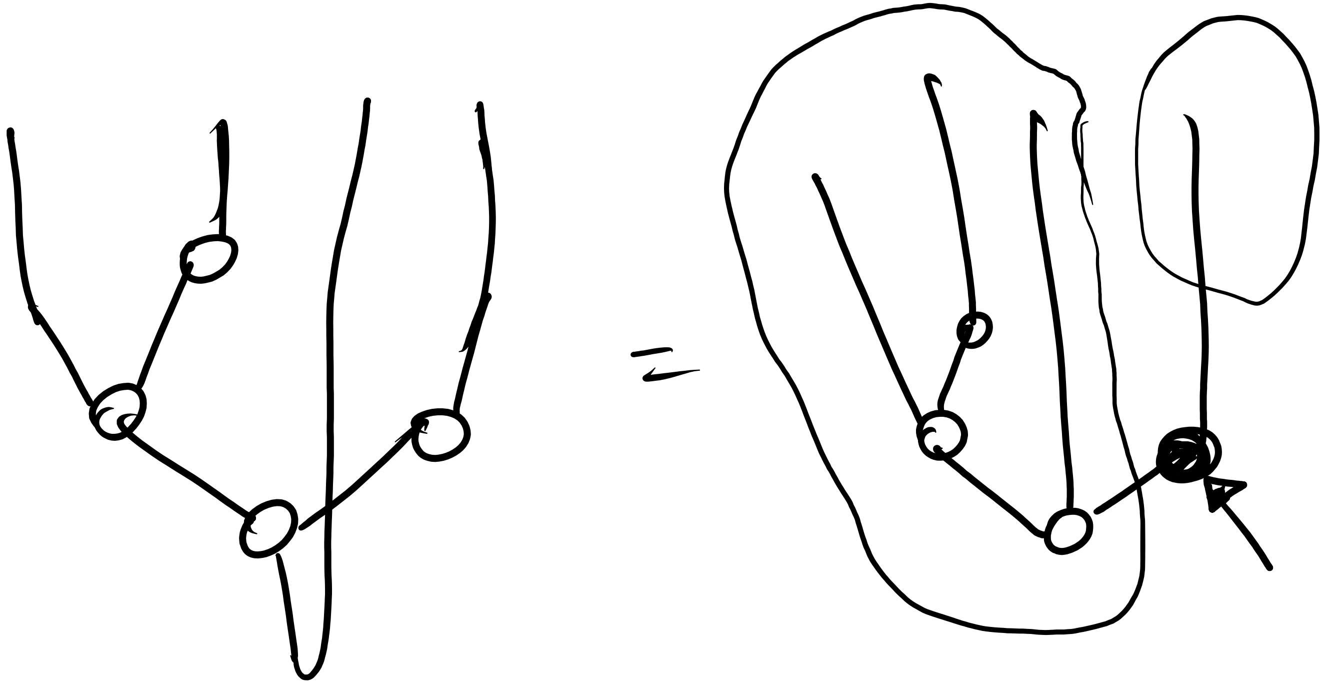
$$\sum_{i_3} \left(B^{[3],i_3}\right)\left(B^{[3],i_3}\right)^+ = I.$$

Аналогично

$$\boxed{\sum_{i_k} B^{[k],i_k}\left(B^{[k],i_k}\right)^+ = I.}$$

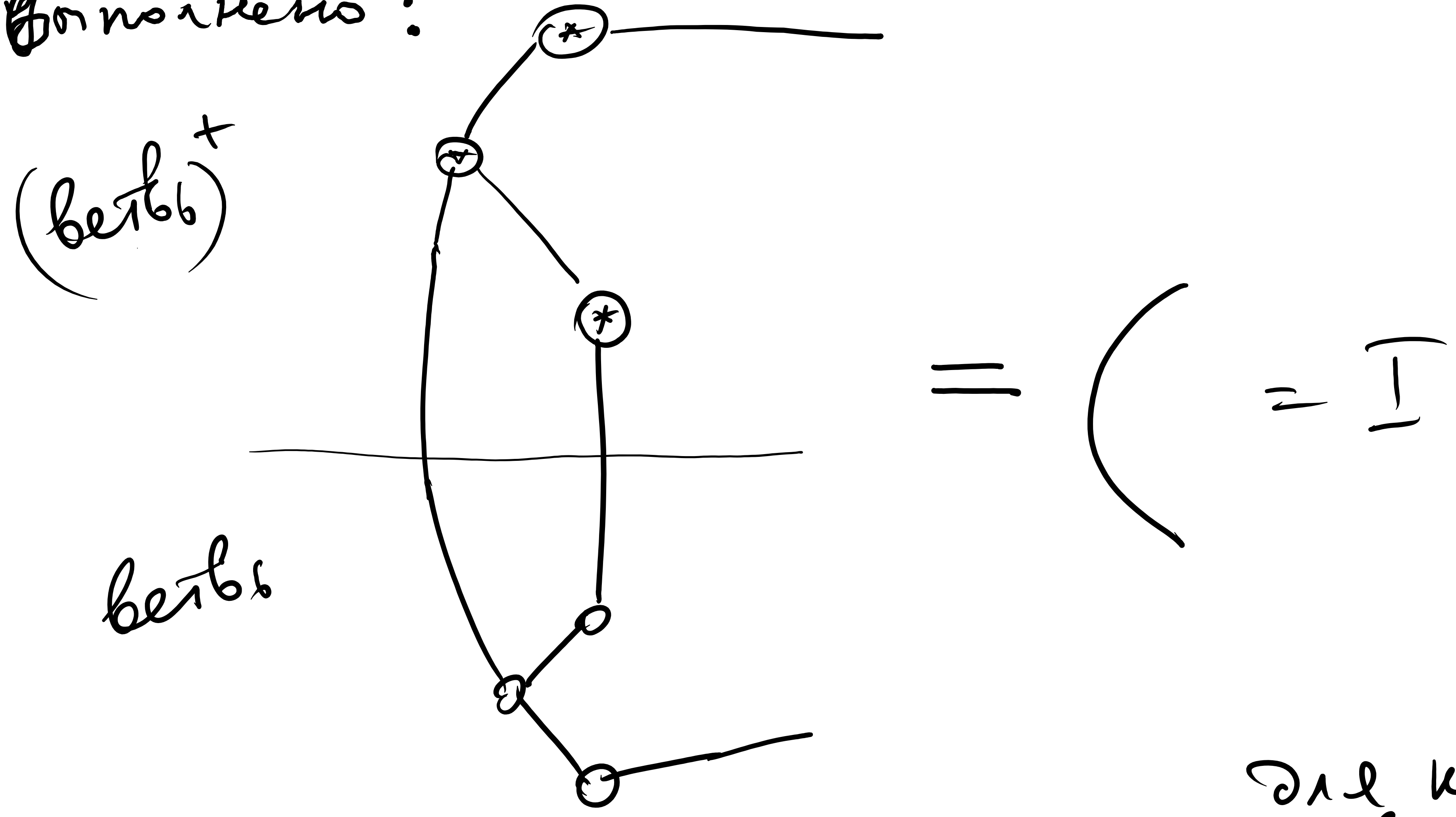
прав. канон.

Каноническая форма тензорного дерева



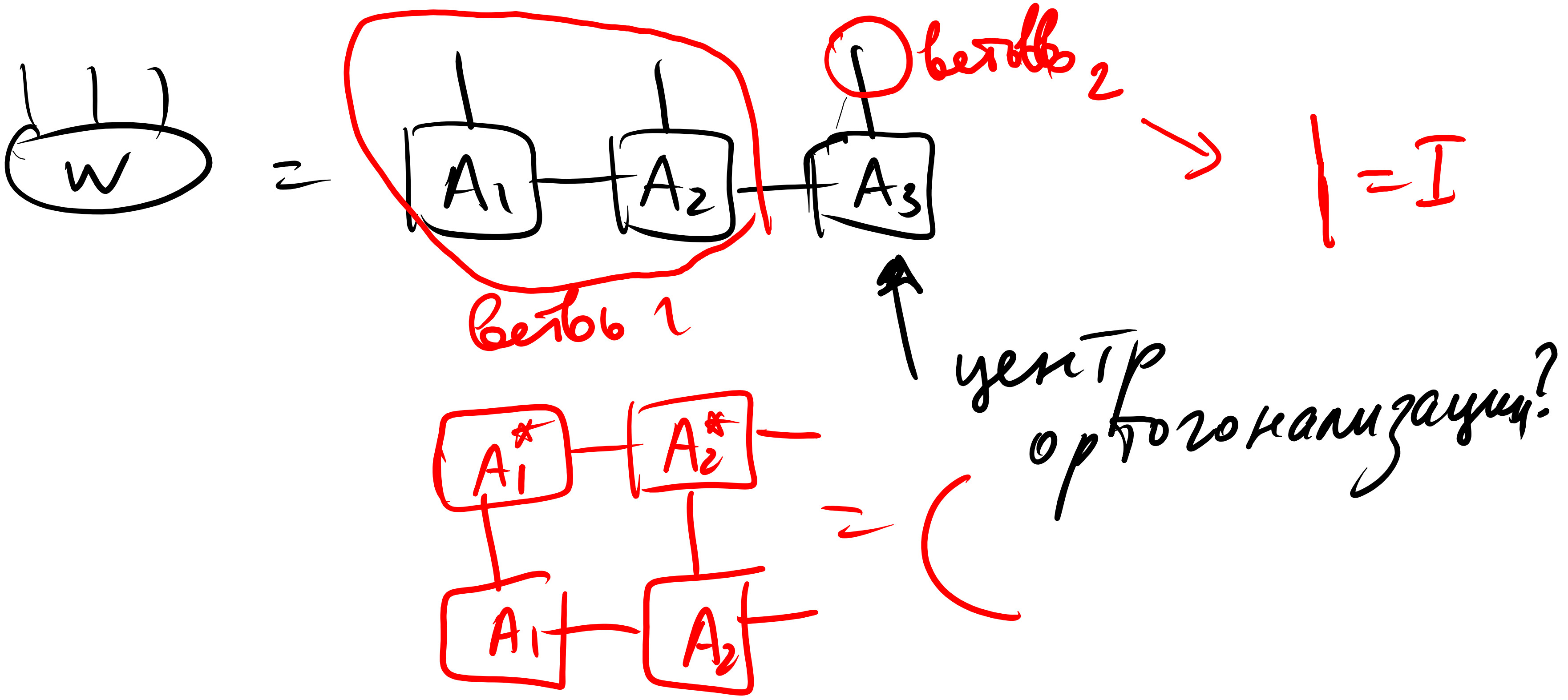
Определение
Тензор в
стр. тензорного
дерева называется
центром ортогонализации, если

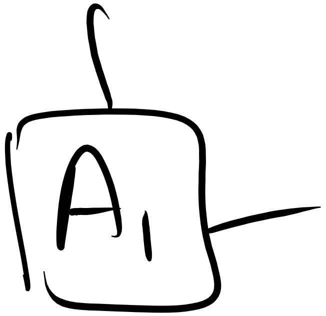
Выполнено:



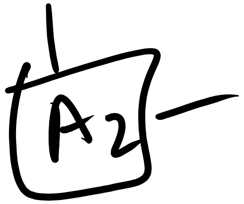
для каждой ветви дерева, исходящей из данного тензора.

Вопрос:

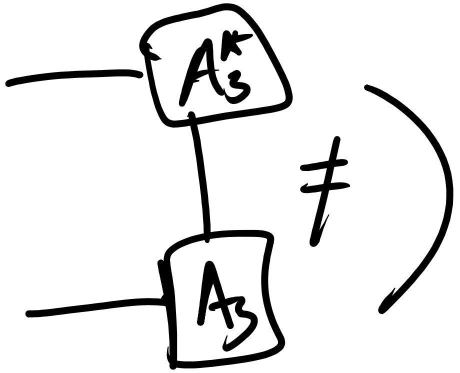




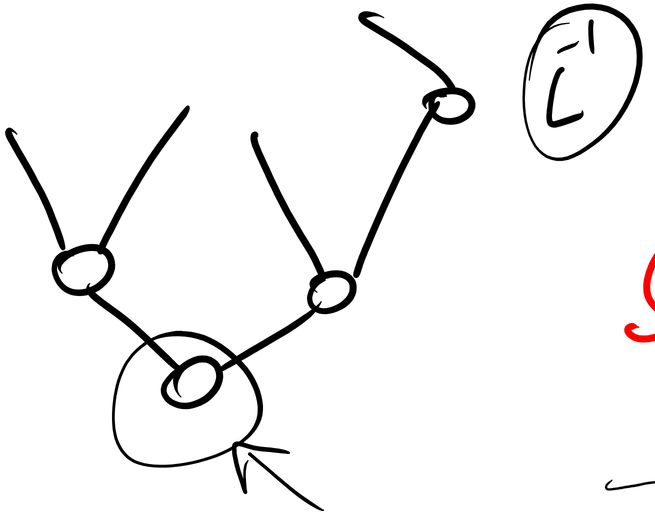
не центр орт.

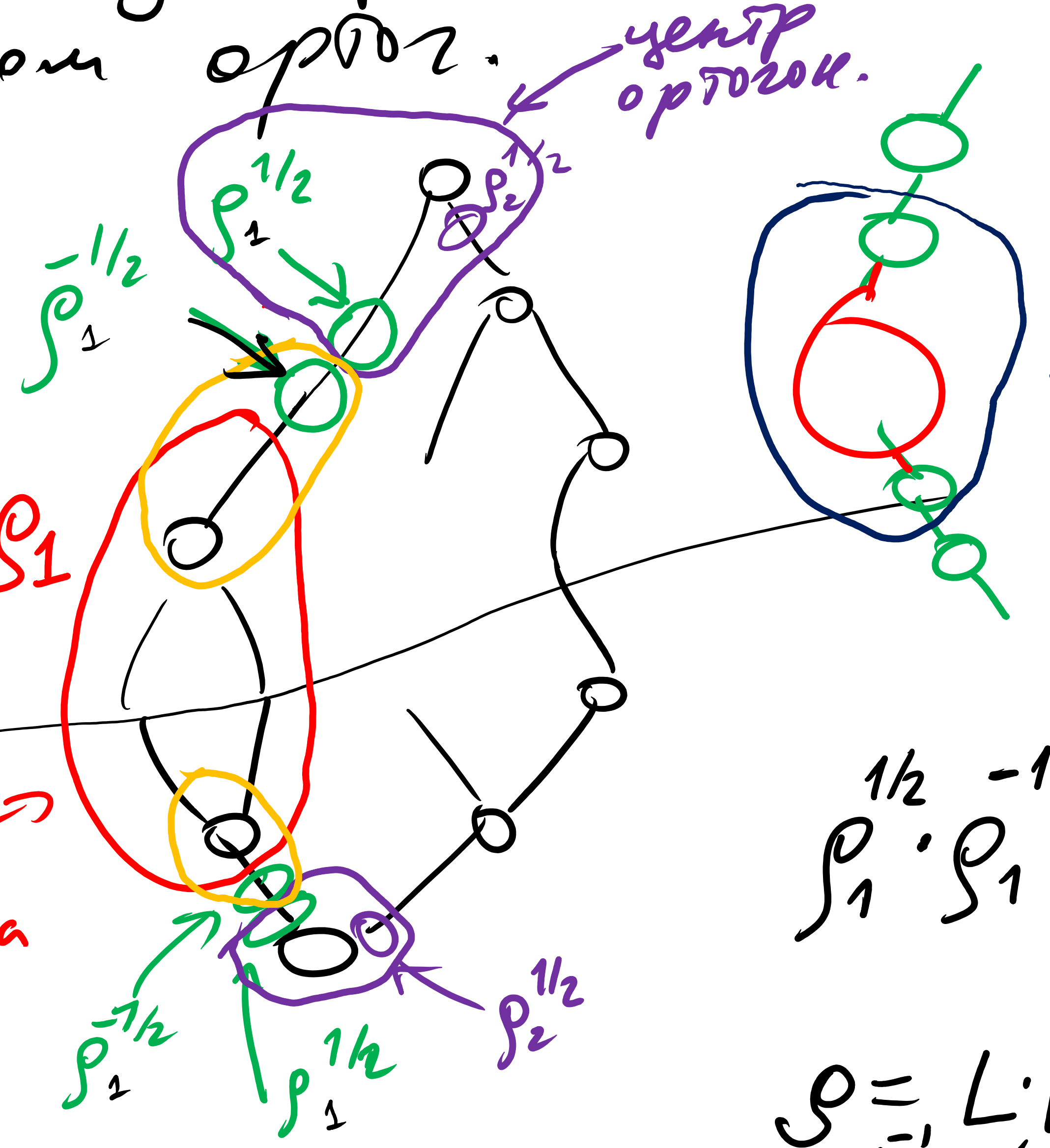


не ц.о.

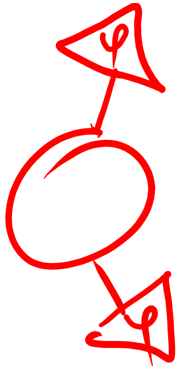


Утв. Любой узел ~~д~~ тенз. дерева можно сделать центром ортог.





$\rho_1^{-1/2}\rho_1\rho_1^{-1/2}$ $=$ $= (= I_{supp\,\rho}$

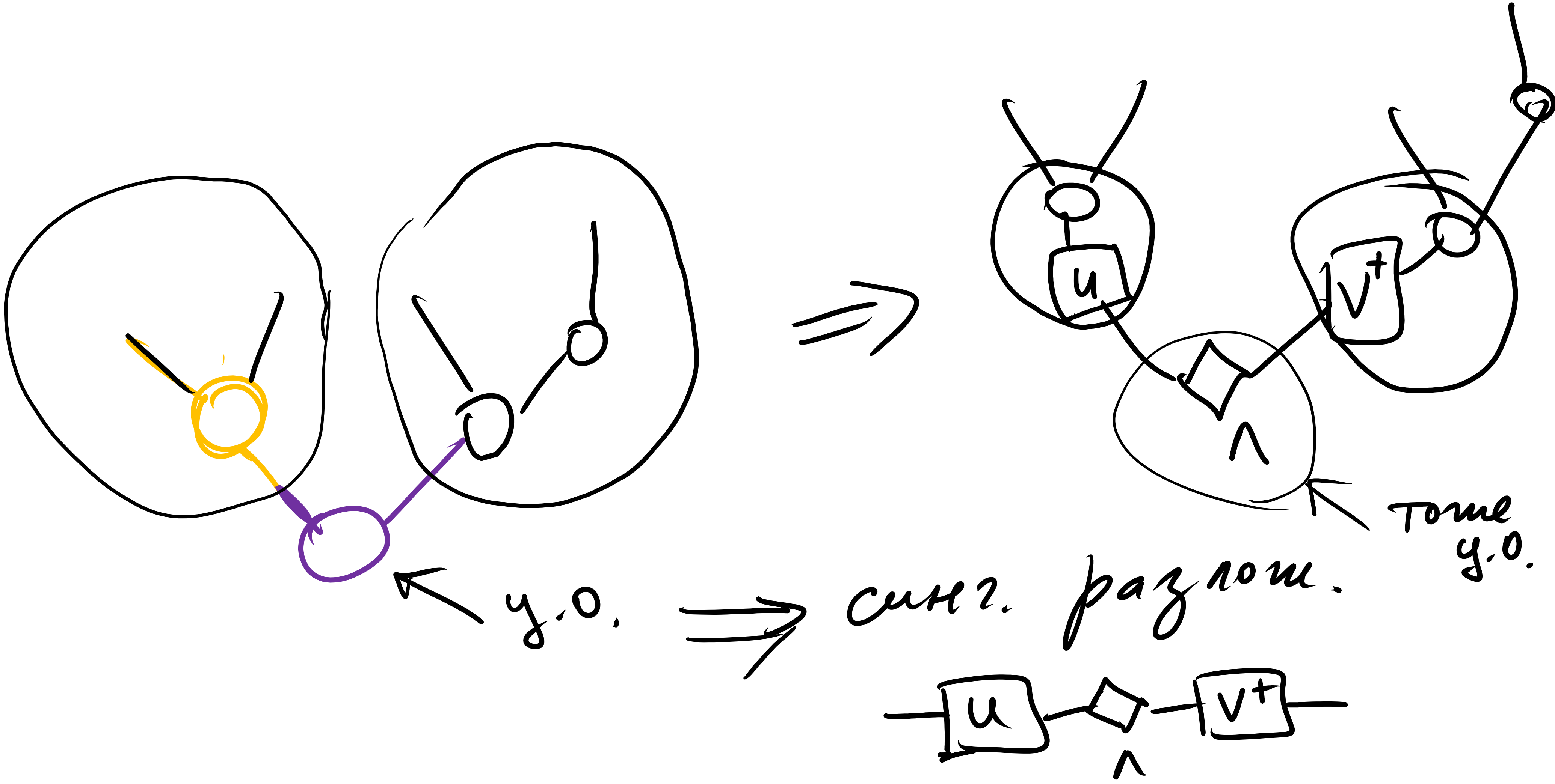


≥ 0.

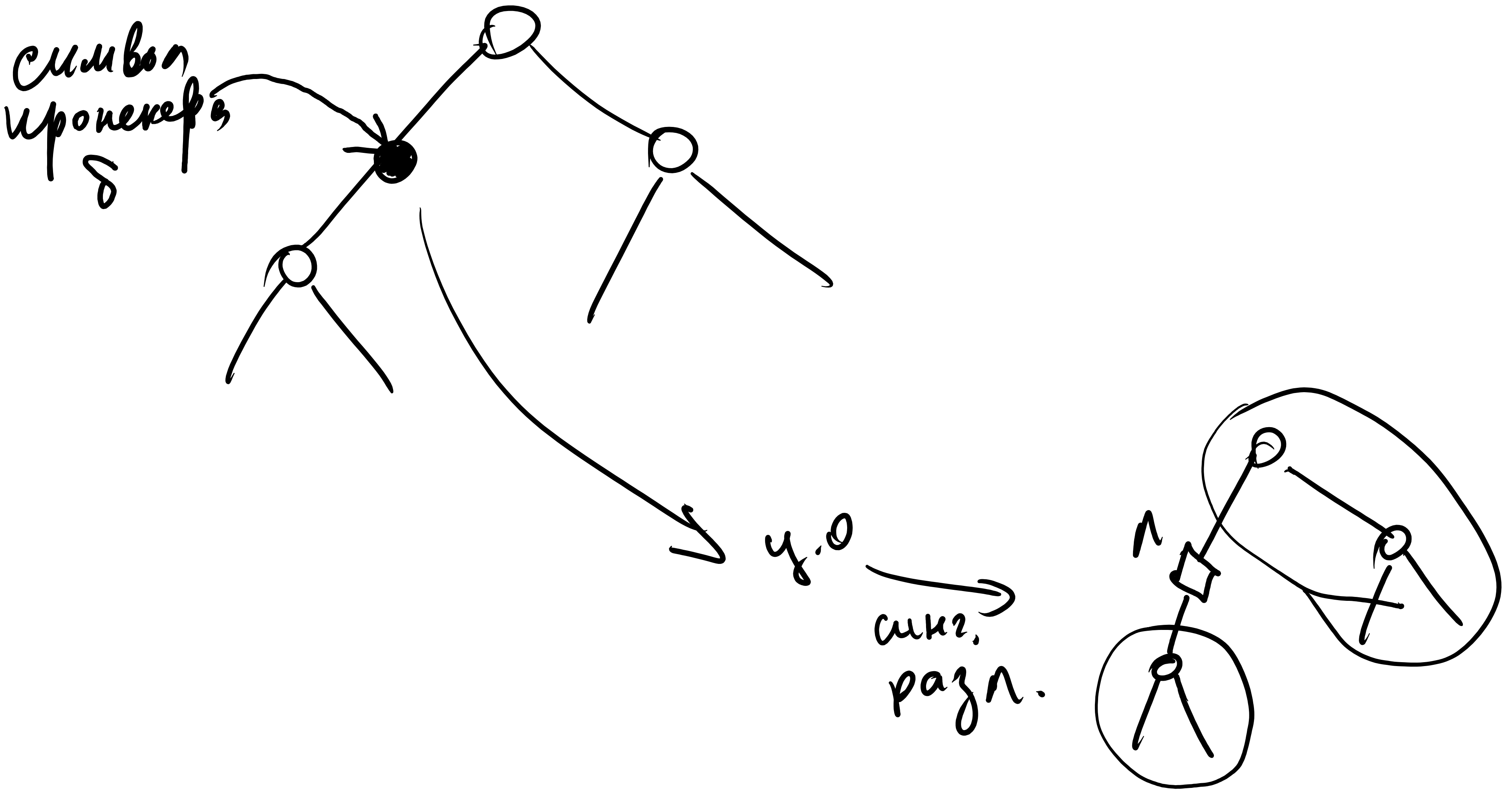
$$\rho_1^{1/2}\cdot\rho_1^{-1/2} = I_{supp\,\rho}$$

$$\rho = L\cdot L^{+}$$

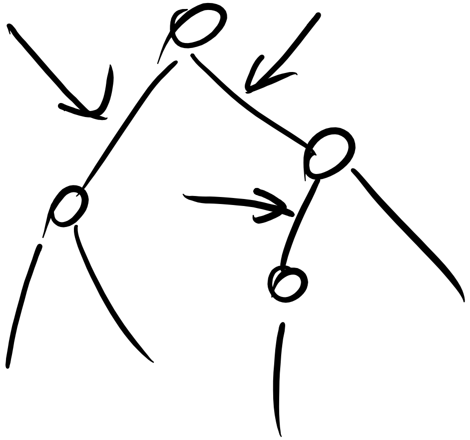
$$L^{-1}\rho(L^{+})^{-1} = I.$$

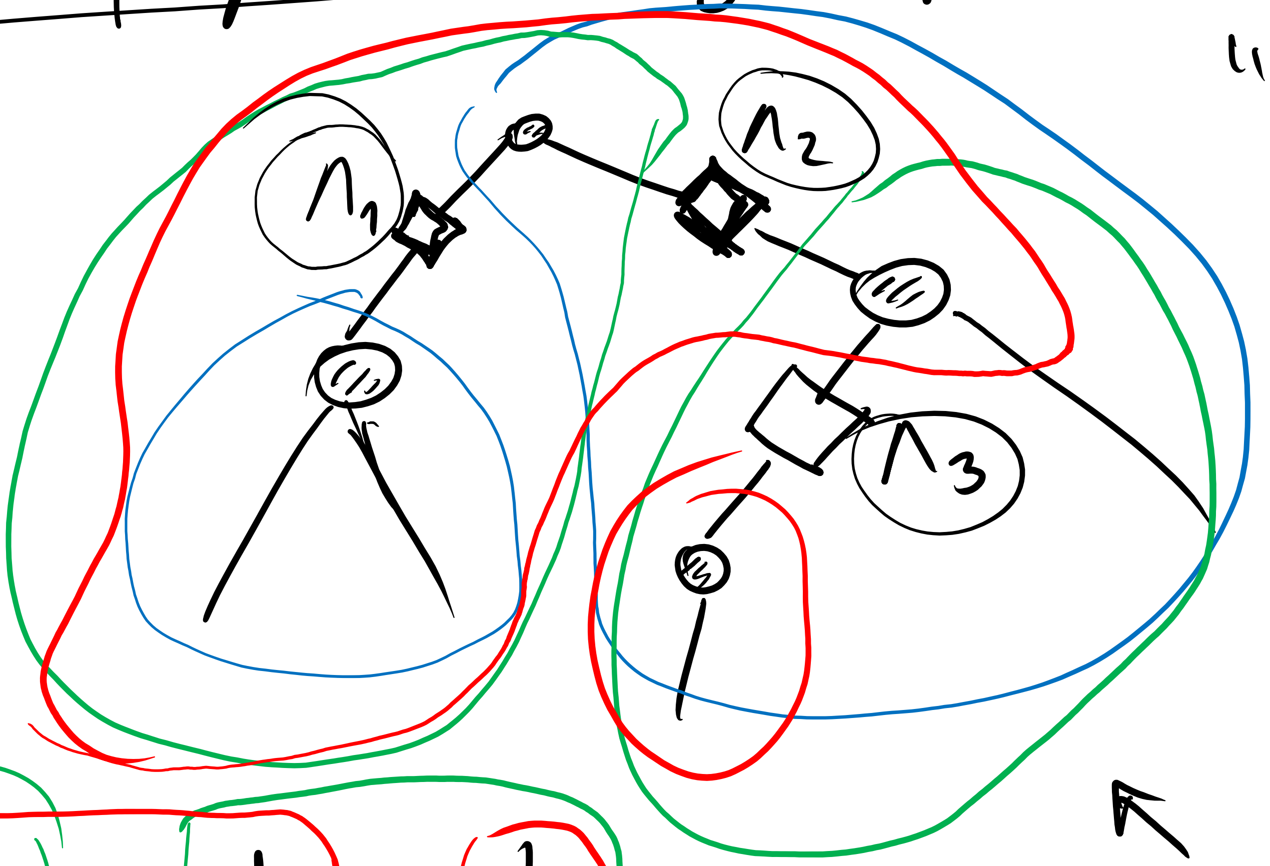

у.о.
⟹ синг. разлож.
U
V⁺
Λ
тоже у.о.

Канон. форма тензорного дерева.



Канон. форма Тенз. дерева



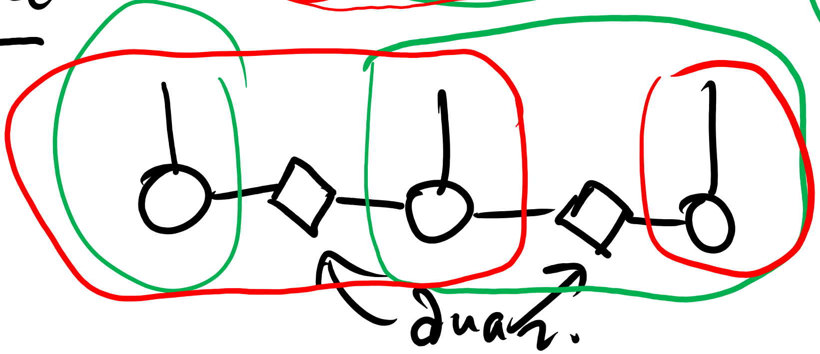


"gauge"
кан. калибровка
Тенз. дерева.

обобщ. разл. Шмидта.

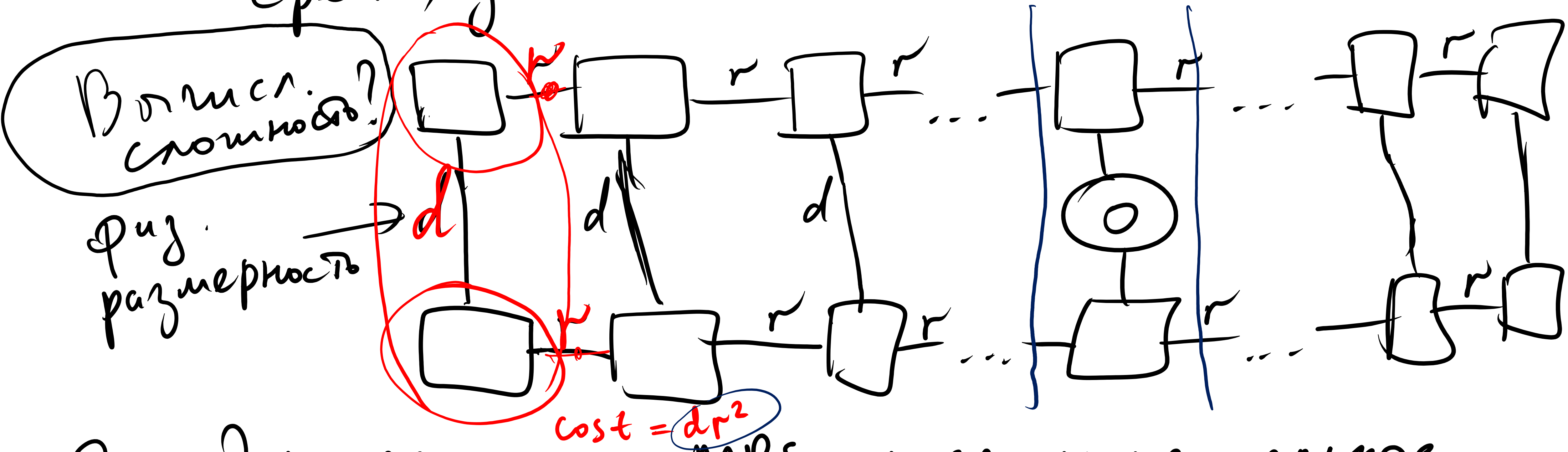
Упражнение:

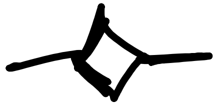
3 кубита



найти канон. форму.

Сворачивание MPS для расчёта средних знач. локальных операторов.

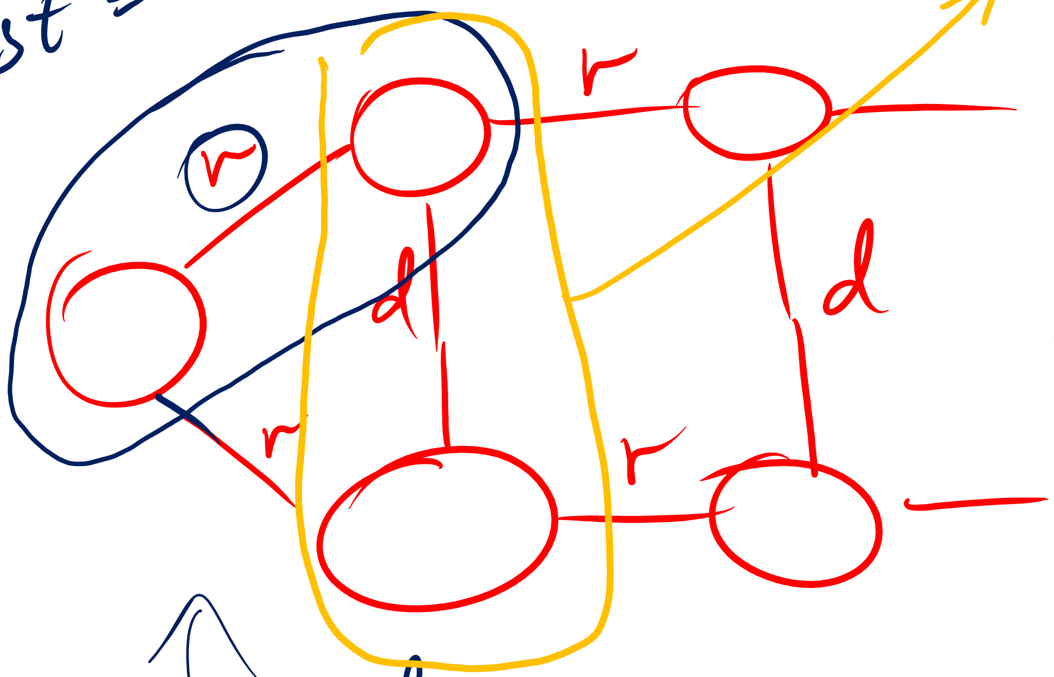


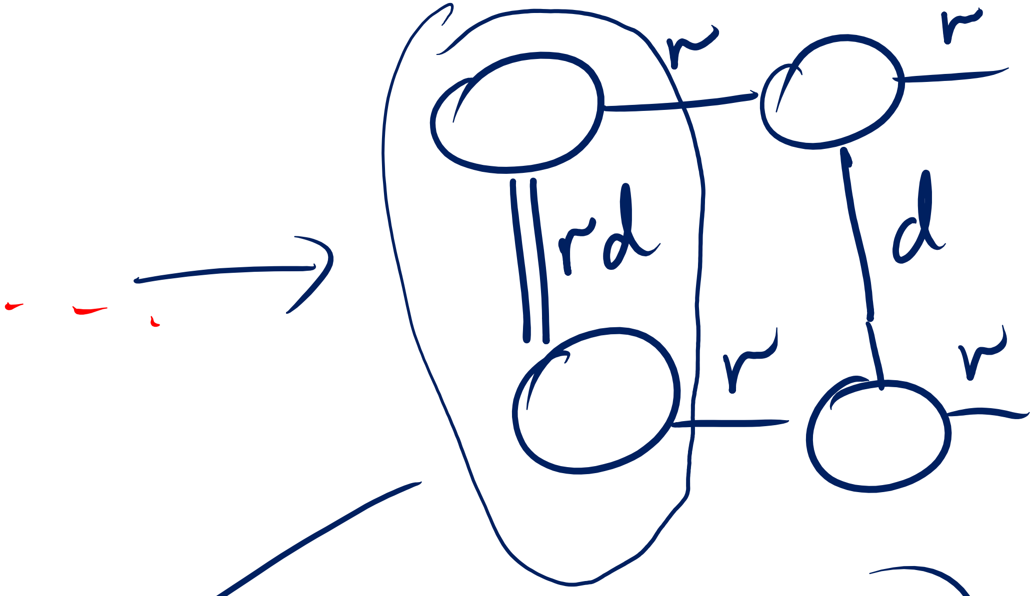
<u>Определение:</u> рангом MPS наз-ся максимальное число ненул. синг. зн. среди  канонической формы.

r — ранг MPS

$$\text{cost} = r \cdot d \cdot r \cdot r = dr^3$$

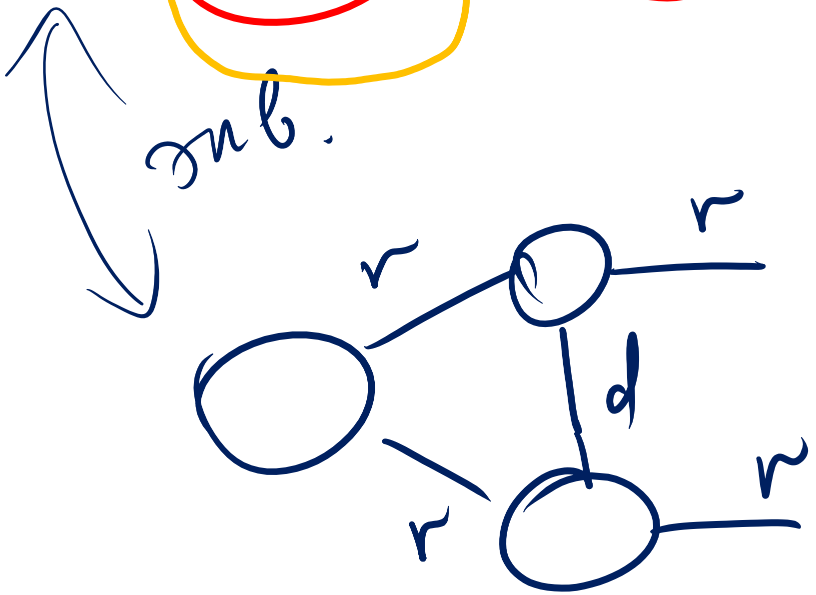
$$\text{cost} = dr^4$$





$$\text{cost} = r \cdot r \cdot rd = dr^3$$

экв.



$$\boxed{\text{total cost} \sim 2\underline{N}dr^3}$$

N - число частиц

Orus
Ann. Phys.
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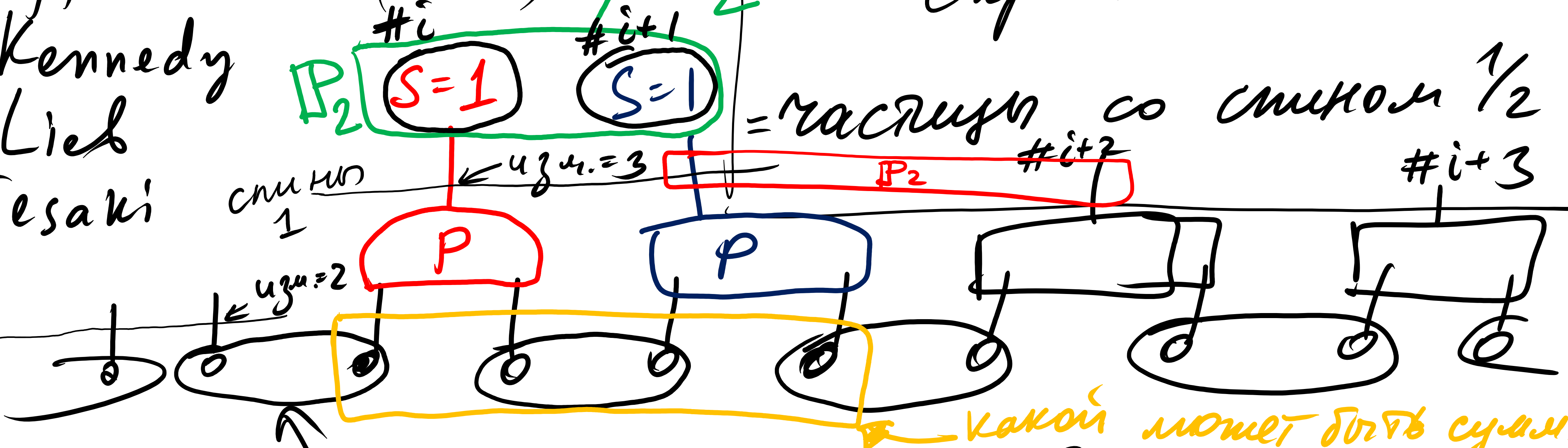
AKLT — состояние. — пример MPS состояния.

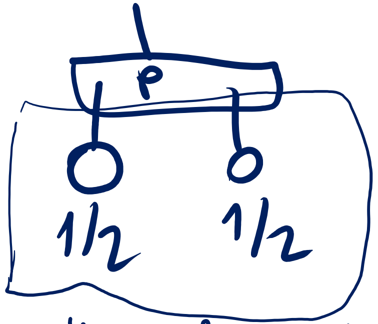
Affleck
Kennedy
Lieb
Tesaki
(1987)

скрытый слой сети = частицы со спином $1/2$.



какой может быть сумм. спин? (не может быть 2)

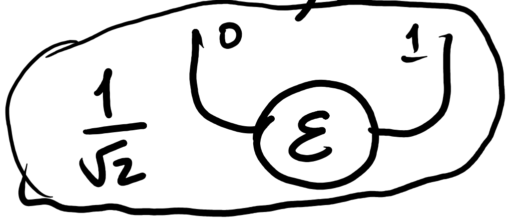
синг. сост. = сумм. спин 0

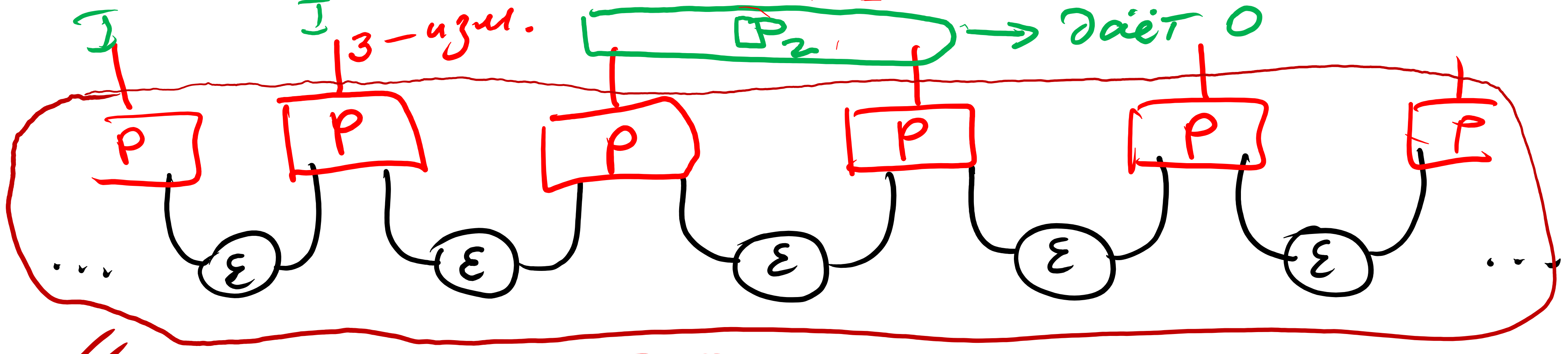


$\mathbb{C}^4 = \mathbb{C}^3 \oplus \mathbb{C}^1$

$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$$

$|\uparrow\rangle = 0$
$|\downarrow\rangle = 1$





многогр. состояние кутритов

$|\psi\rangle$

$P:\ \mathbb{C}^4 \to \mathbb{C}^3$

$|1\rangle, |0\rangle, |-1\rangle$

$|00\rangle, |01\rangle, |10\rangle, |11\rangle$

$|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$

$I\otimes I\otimes \ldots \otimes \mathbb{P}_2 \otimes I \otimes \ldots$

$|\psi\rangle = 0$

$$P = |1\rangle\langle\uparrow\uparrow| + |0\rangle\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}} + |-1\rangle\langle\downarrow\downarrow|.$$

$S_1 = \frac{1}{2}$ и $S_2 = \frac{1}{2}$

$S = 0$ или 1

$S = 0 \rightarrow M = 0$

$S = 1 \rightarrow M = 1, 0, -1$

$|S=1, M=1\rangle = |\uparrow\uparrow\rangle$

$|S=1, M=0\rangle = \frac{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle}{\sqrt{2}}$

$|S=1, M=-1\rangle = |\downarrow\downarrow\rangle$

$\mathbb{C}^4 \to \mathbb{C}^4 \quad P: \sum_{M=1,0,-1} |S=1, M\rangle\langle S=1, M|$ — проектор ранга 3 в $\mathbb{C}^4$.

$|S=1,M\rangle \rightarrow |1\rangle, |0\rangle, |-1\rangle.$

$S_1 = 1 \quad , \quad S_2 = 1$

$$S = 0, 1, 2.$$

$$\mathbb{P}_2 : (\mathbb{C}^3)^{\otimes 2} \longrightarrow (\mathbb{C}^3)^{\otimes 2}$$

$$\mathbb{C}^9 \rightarrow \mathbb{C}^9.$$

$$\mathbb{P}_2 = \sum_{M=2,1,0,-1,-2} |s=2, M\rangle \langle s=2, M|$$

parra 5.

$\mathbb{P}_2 \geq 0.$

$$H = \sum_i \mathbb{P}_2^{(i,i+1)} \geq 0$$ – гамильтониан AKLT

i – номера кутритов

$$H|\psi\rangle = 0$$

$\Rightarrow$ постр. нами MPS-сост. яв-ся основным сост. для данного гам-на.