Лекция 13 по курсу "Квантовые тензорные сети"

(30.11.2021)

$A_k = I \otimes I \otimes \sigma_x \cdot \sqrt{\varepsilon}$ ← переворот

$|0_L\rangle = \dots\dots$ многокубитное состояние

$|1_L\rangle = \dots\dots$ многокубитное состояние

$\boxed{\text{Если } A_k = I \otimes I \otimes \sigma_z \cdot \sqrt{\varepsilon} \text{ фазов. ошибка}}$

$0 \to 000$

$1 \to 111$

$\boxed{010} \xrightarrow{\text{испр.}} \boxed{000}$

110

избыточное кодирование

$$\mathbb{C}^2 \begin{cases} |0_L\rangle = |000\rangle \in \mathbb{C}^8 \\ |1_L\rangle = |111\rangle \in \mathbb{C}^8 \end{cases}$$

изометрия

$w|0_L\rangle = |000\rangle$

$w|1\rangle = |111\rangle$

$w^+ w|0\rangle = |0\rangle$

$w^+ w|1\rangle = |1\rangle$

$w^+ w = I$, но $\underset{P.}{\underset{\parallel}{w w^+}} \neq I$

$|\psi\rangle = a|0\rangle + b|1\rangle$

$w|\psi\rangle = a|000\rangle + b|111\rangle$

кв. ошибка

$\Phi(\rho) = \sum_k A_k \rho A_k^+$

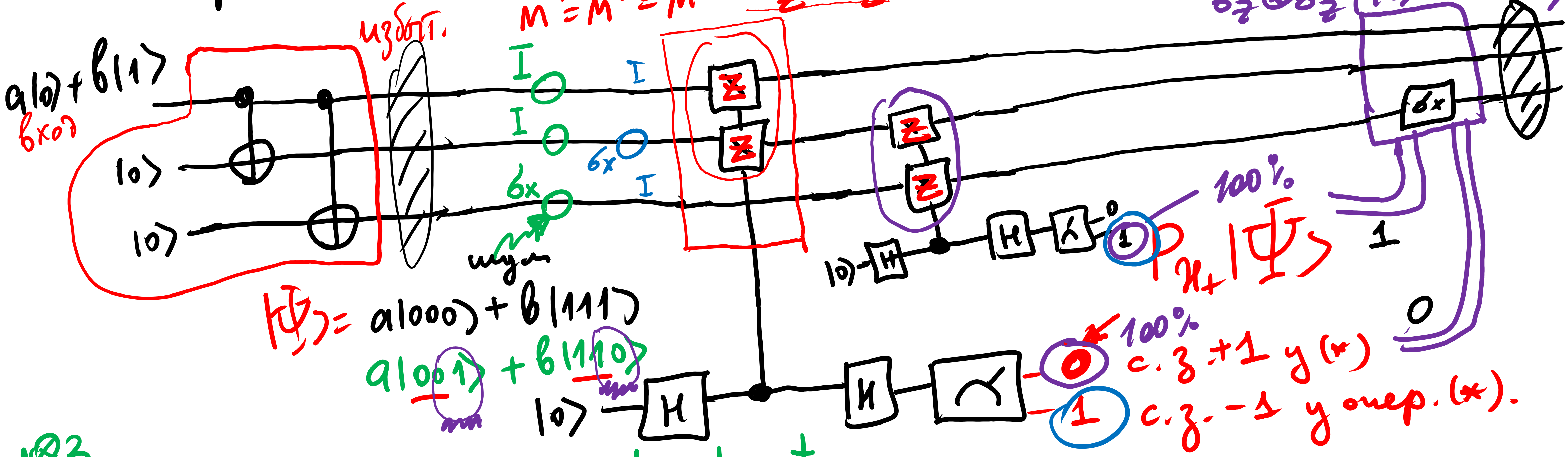
$\Phi \approx Id +$ малая ошибка ε

Исправл. перев. спина

$M^{-1}=M^{+}=M=\sigma_z\otimes\sigma_z\otimes I \quad (*)$

$\sigma_z\otimes\sigma_z|01\rangle=-|01\rangle$

$\sigma_z\otimes\sigma_z|10\rangle=-|10\rangle$



$|\Psi\rangle=a|000\rangle+b|111\rangle$

$a|001\rangle+b|110\rangle$

$$\Phi^{\otimes 3}(\rho)=\sum_{klm}A_k\otimes A_l\otimes A_m\,\rho\,A_k^{+}\otimes A_l^{+}\otimes A_m^{+}$$

$I \quad I \quad \sigma_x$

ε

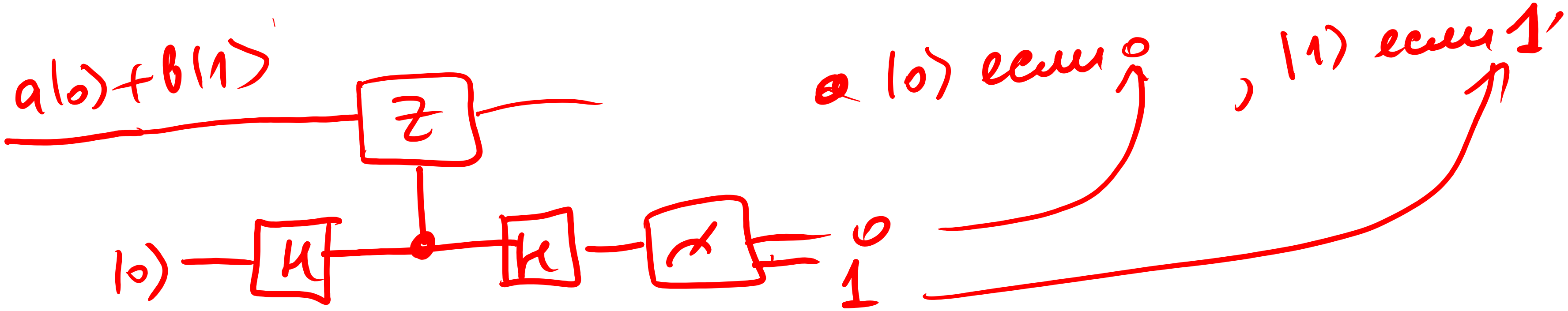
$\sigma_z\otimes\sigma_z|00\rangle=|00\rangle$

$\sigma_z\otimes\sigma_z|11\rangle=|11\rangle$

$M|\psi\rangle=\pm|\psi\rangle$

$\mathcal{H}_+=\mathrm{Span}\left(|\psi\rangle:\ M|\psi\rangle=|\psi\rangle\right)$

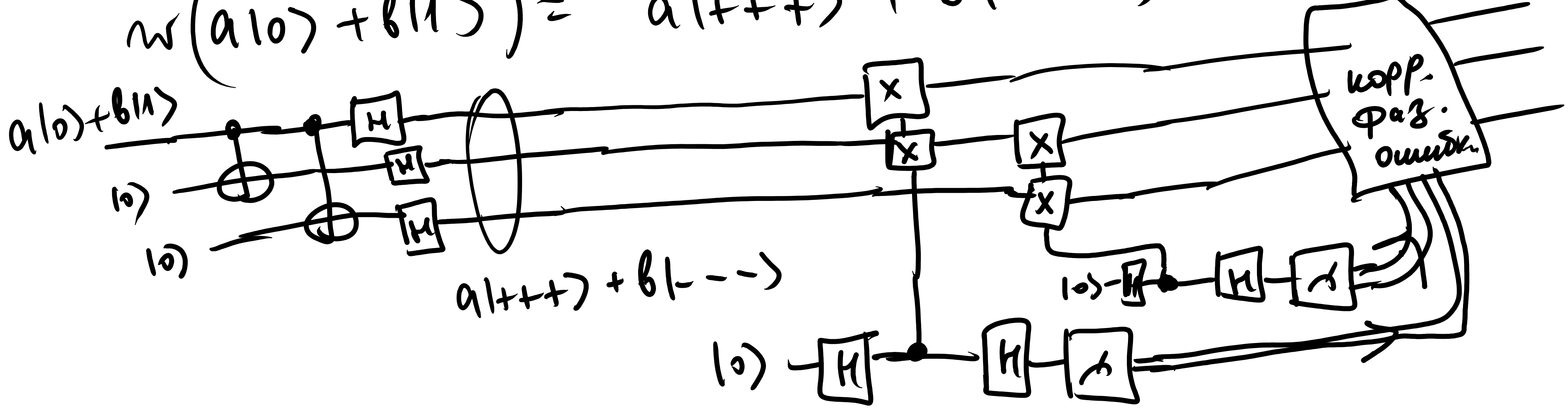
$\mathcal{H}_-=\mathrm{Span}\left(|\psi\rangle:\ M|\psi\rangle=-|\psi\rangle\right).$


a|0⟩+b|1⟩
Z
|0⟩
H
H
0
1
|0⟩ если 0 , |1⟩ если 1

$$|\pm\rangle = \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} \quad \text{с. в. } \sigma_x.$$

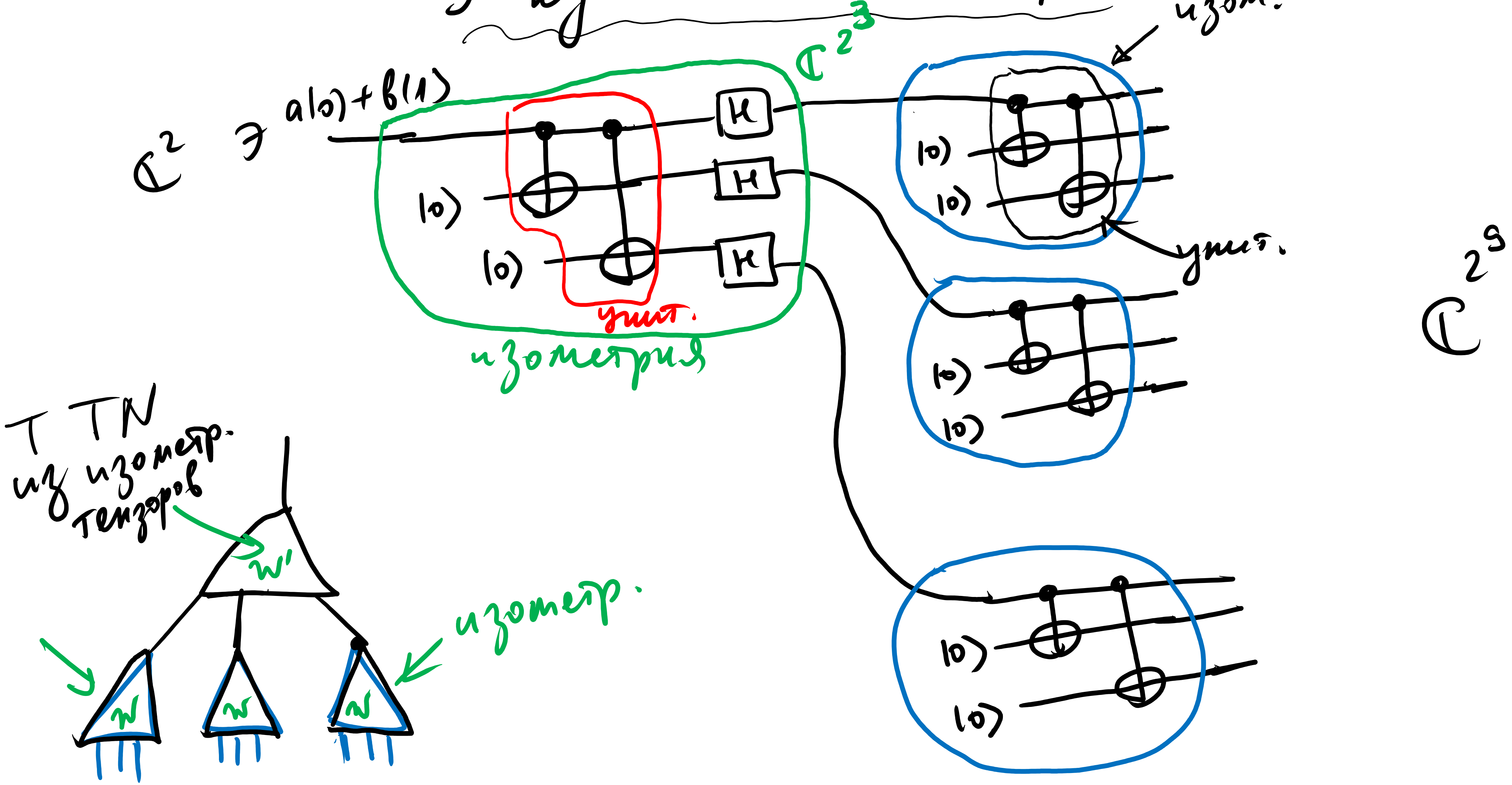
Испр. фаз. ошибки $\quad A = I \otimes I \otimes \ldots \otimes I \otimes \sigma_z \otimes I \otimes \ldots I$

$$w(a|0\rangle + b|1\rangle) = a|+++\rangle + b|---\rangle$$



$\underline{\sigma_x}$, $\underline{\sigma_z}$, $\quad \sigma_x \sigma_z = -i\underline{\sigma_y}$ Нильсен, Чанг.

9-кубитный код Шора



Multiscale entanglement-renormalization ansatz (MERA)

огр.

дет.

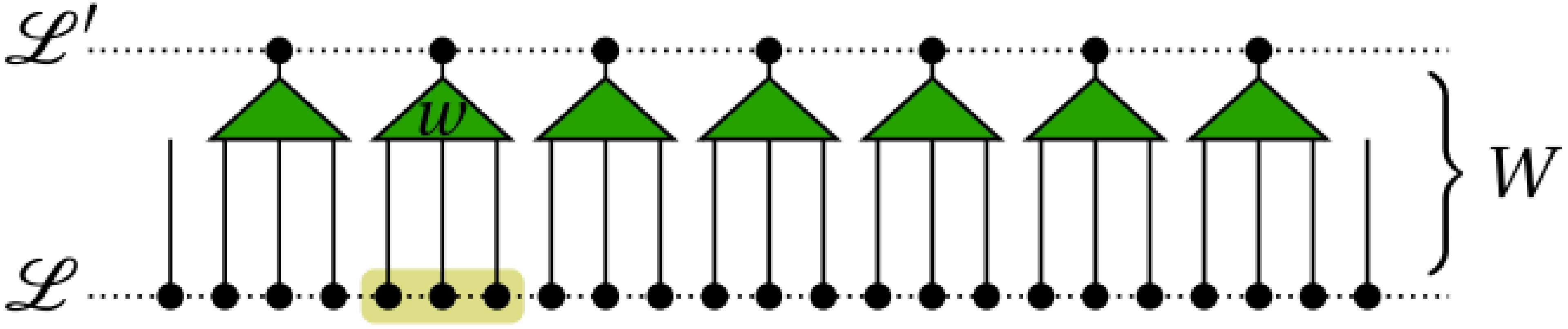
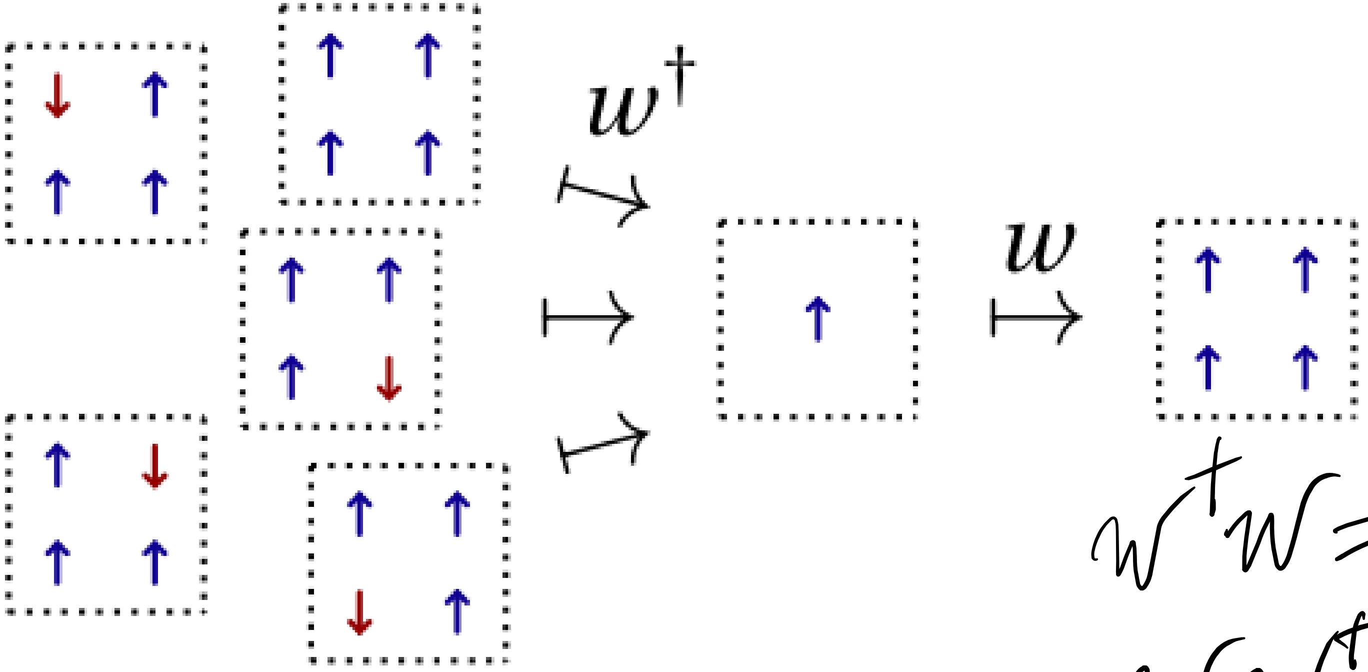


Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

Multiscale entanglement-renormalization ansatz (MERA)



$w^\dagger w = I$

$w w^\dagger = P \neq I$

Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

Multiscale entanglement-renormalization ansatz (MERA)

renorm.
flow

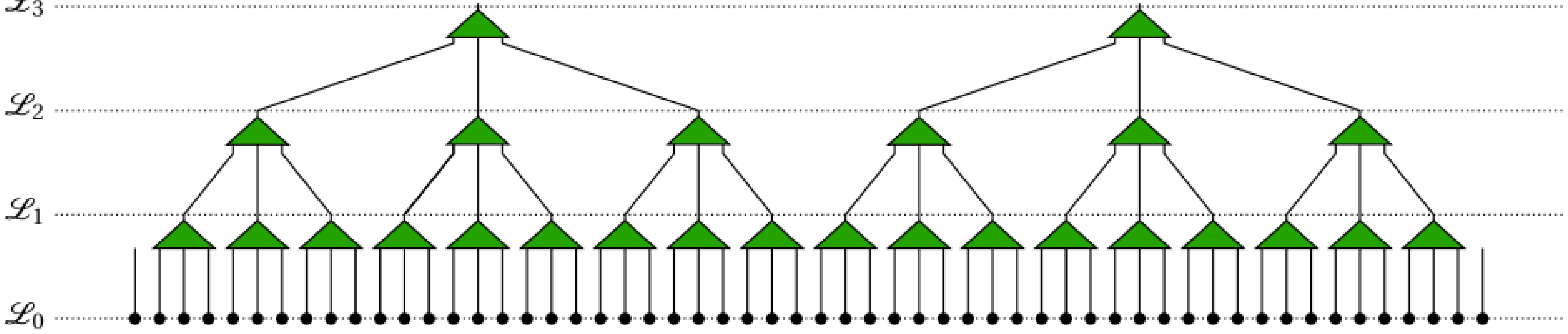


Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

Multiscale entanglement-renormalization ansatz (MERA)

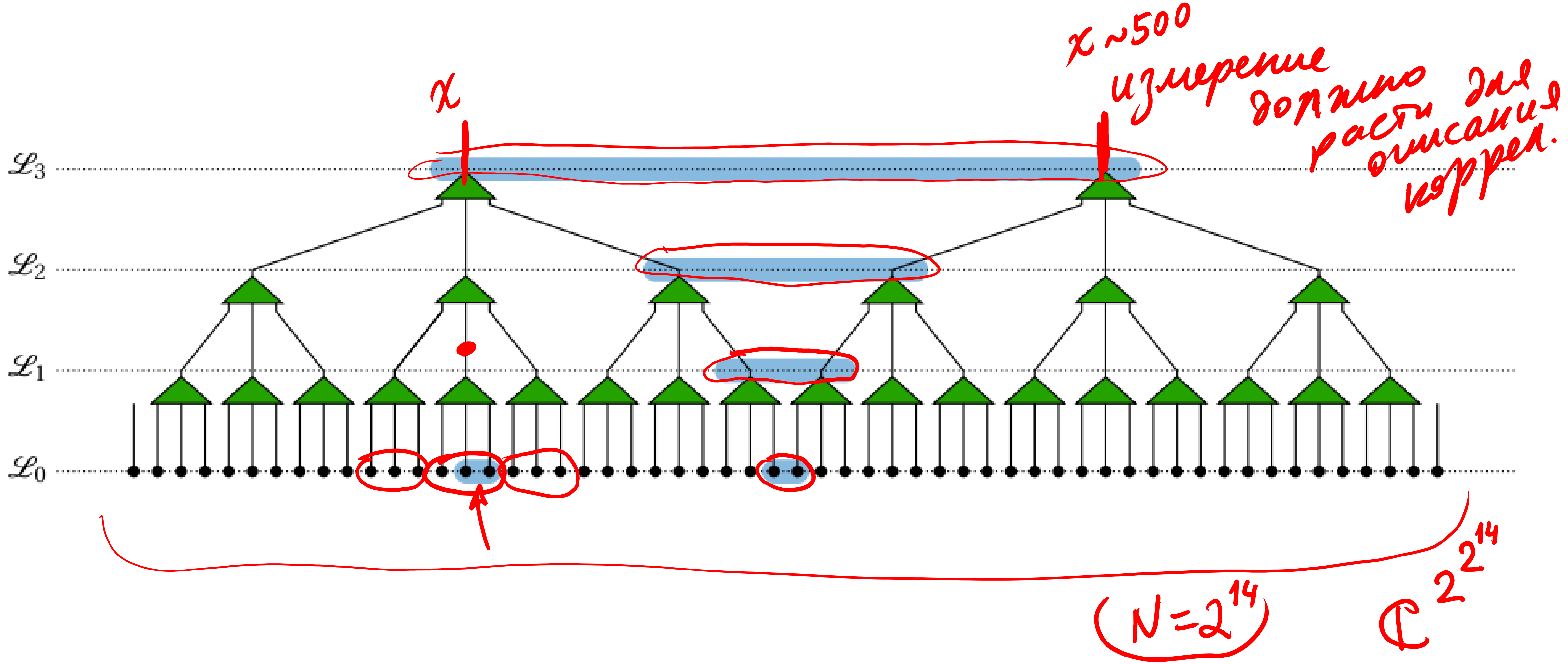


Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

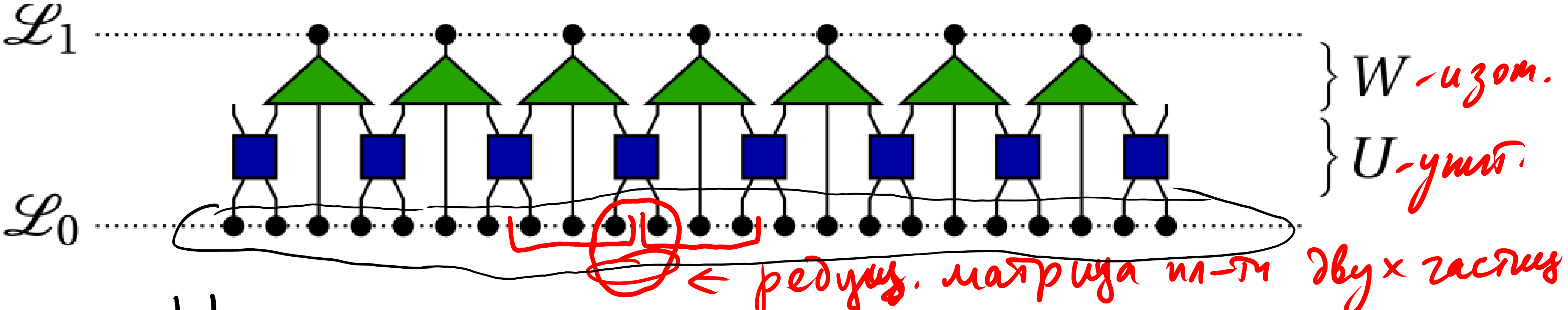
$u\rho u^+ = \sum_k \lambda_k |\varphi_k\rangle\langle\varphi_k| \otimes |\chi_k\rangle\langle\chi_k|$

расцепл. состояние

$|ij\rangle = |i\rangle \otimes |j\rangle$ — несцепл. состояние.

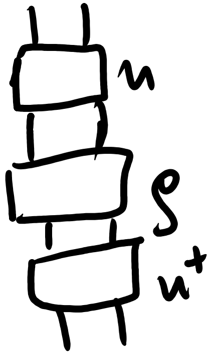
Multiscale entanglement-renormalization ansatz (MERA)

$$u = \sum_{k=1}^{4} |\varphi_k\rangle \otimes |\chi_k\rangle \langle \psi_k| = |00\rangle\langle\psi_1| + |01\rangle\langle\psi_2| + |10\rangle\langle\psi_3| + |11\rangle\langle\psi_4|$$

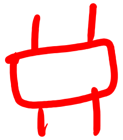


$|\psi_k\rangle \neq |\varphi_k\rangle \otimes |\chi_k\rangle$

$u\rho u^+ =$



$= \sum_k \lambda_k u|\psi_k\rangle\langle\psi_k|u^+$



$$\rho = \sum_{k=1}^{4} \lambda_k |\psi_k\rangle\langle\psi_k|$$

2 кубита

Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

Multiscale entanglement-renormalization ansatz (MERA)

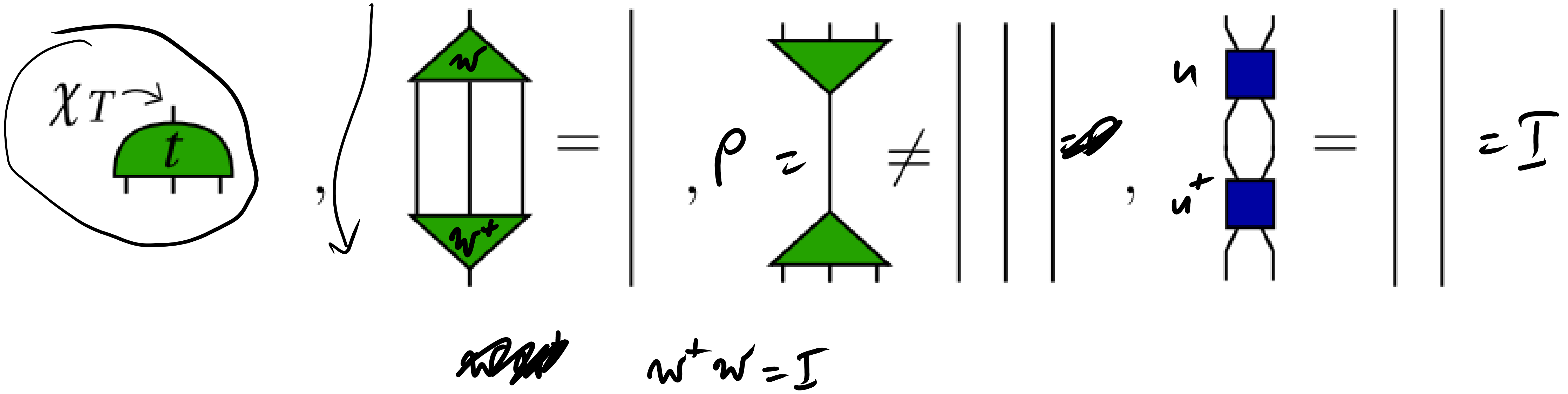
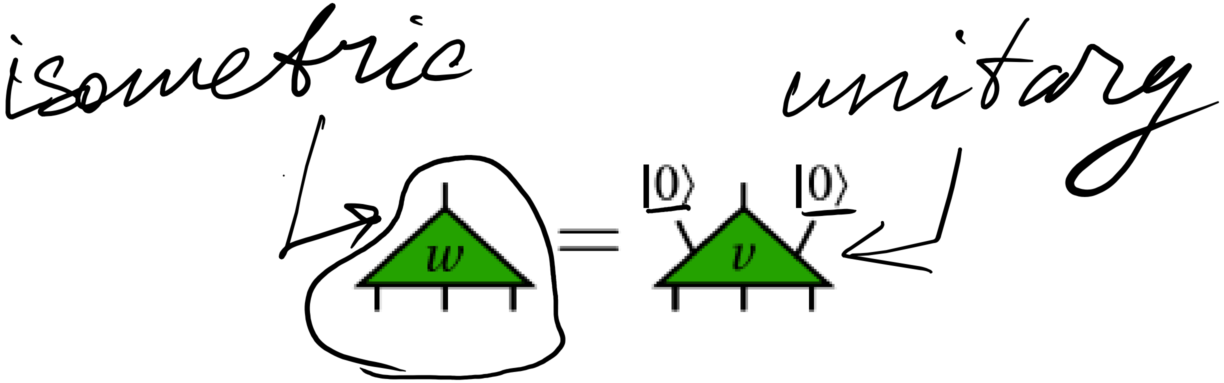


Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

Multiscale entanglement-renormalization ansatz (MERA)



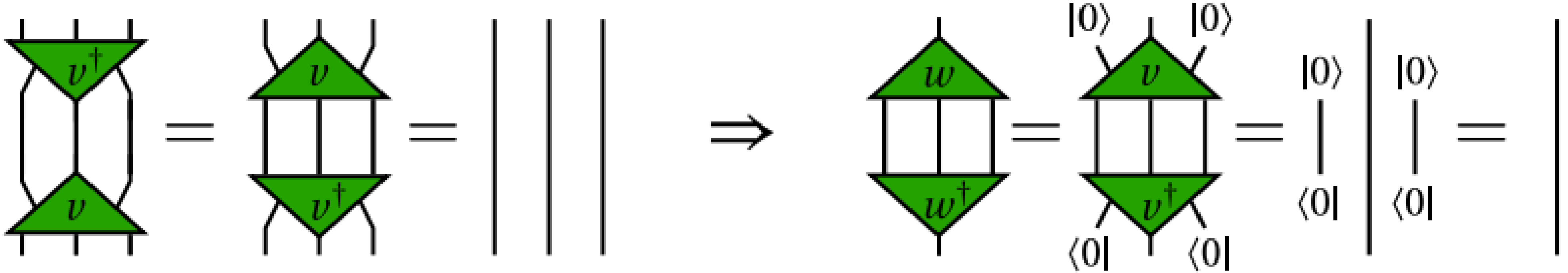


Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

На квант. компьютере

Multiscale entanglement-renormalization ansatz (MERA)

вектор $\in \mathbb{C}^{\chi^3}$, $\chi = 2$.

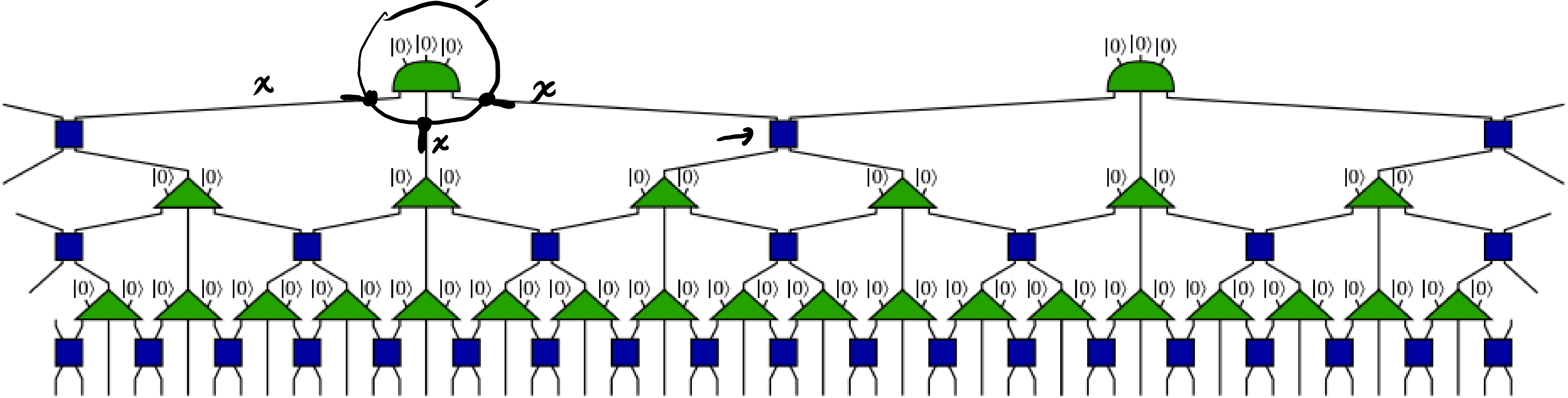
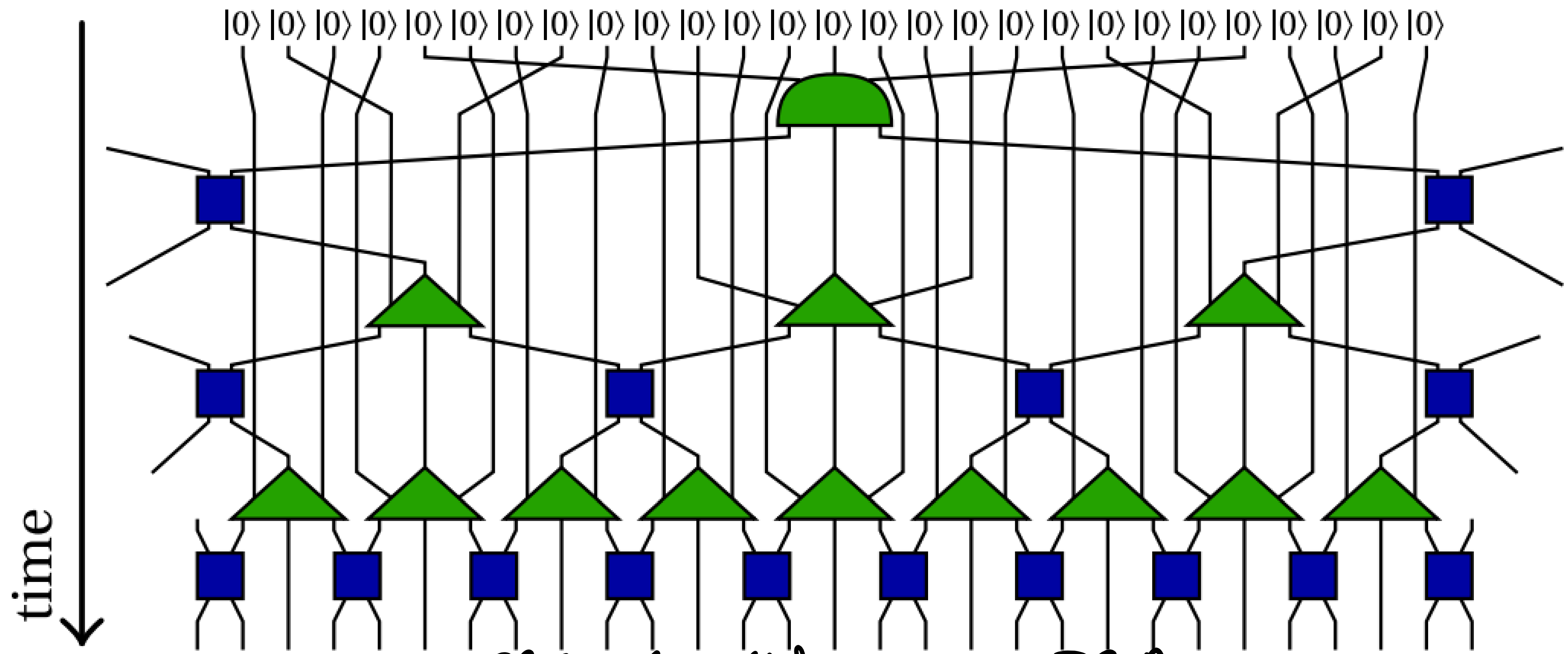


Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

Multiscale entanglement-renormalization ansatz (MERA)



реализация сост. MERA, которое можно использовать → в испр. ошибок
→ в поиске кодир. для суперадд. проп. спос.

Figure: M. Hauru, https://mhauru.org/MSc_thesis.pdf

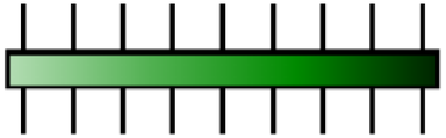
Optimization problem #4: Low-energy spectrum and eigenstates

Solution

$$H' = \sum_{k=1}^{8} E_k \underbrace{V|\psi_k\rangle}\underbrace{\langle\psi_k|V^+}$$

$$\underbrace{\mathrm{Tr}\left(V^{\dagger}HV\right)} \to \min_{V\in V_{n,m}}$$

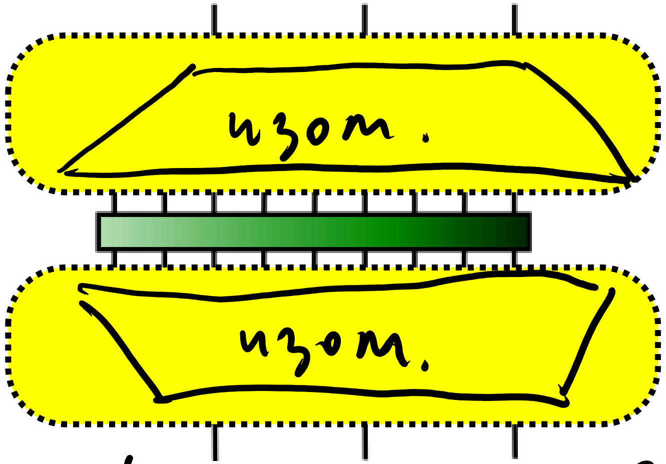
$$= \sum_{k=1}^{2^N} \langle\psi_k|V^+V|\psi_k\rangle \underline{E_k}$$

H  $= \sum_{k=1}^{2^N} E_k|\psi_k\rangle\langle\psi_k|$

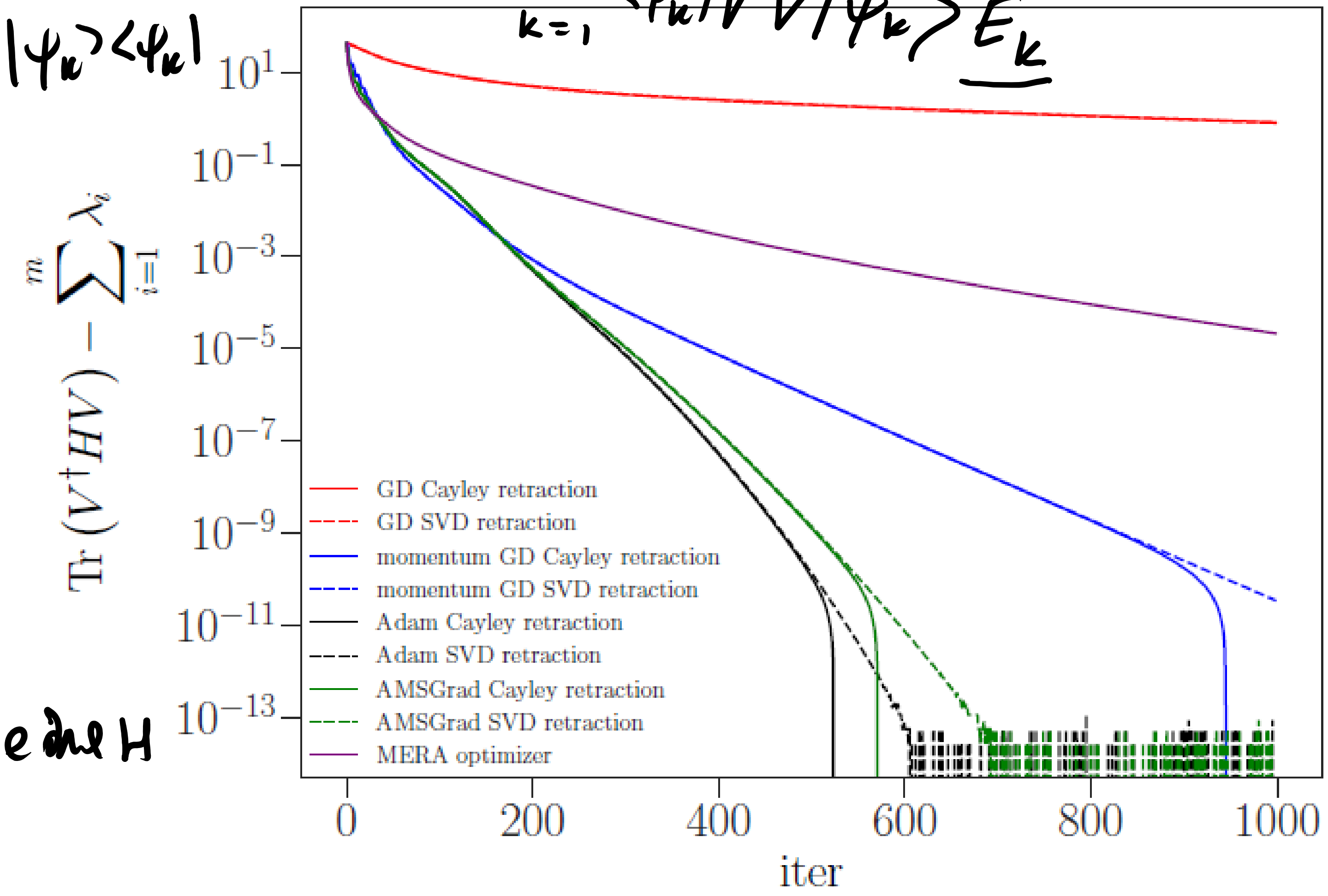
$E_1 \le E_2 \le \dots$

V

$V^{\dagger}$



$H' = VHV^+$ – огрубл. пр-ение H



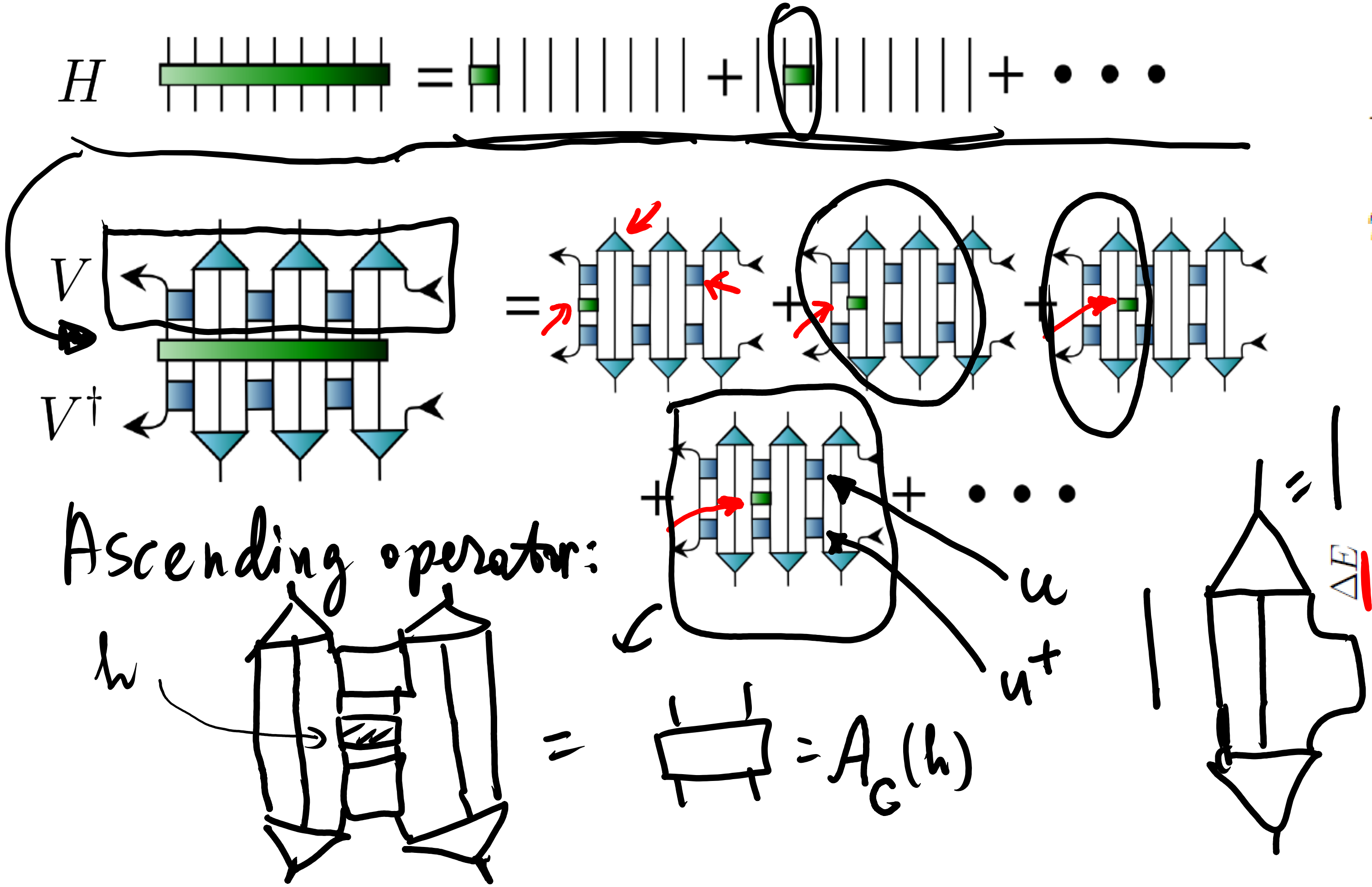
Optimization problem #5: MERA optimization for local Hamiltonians

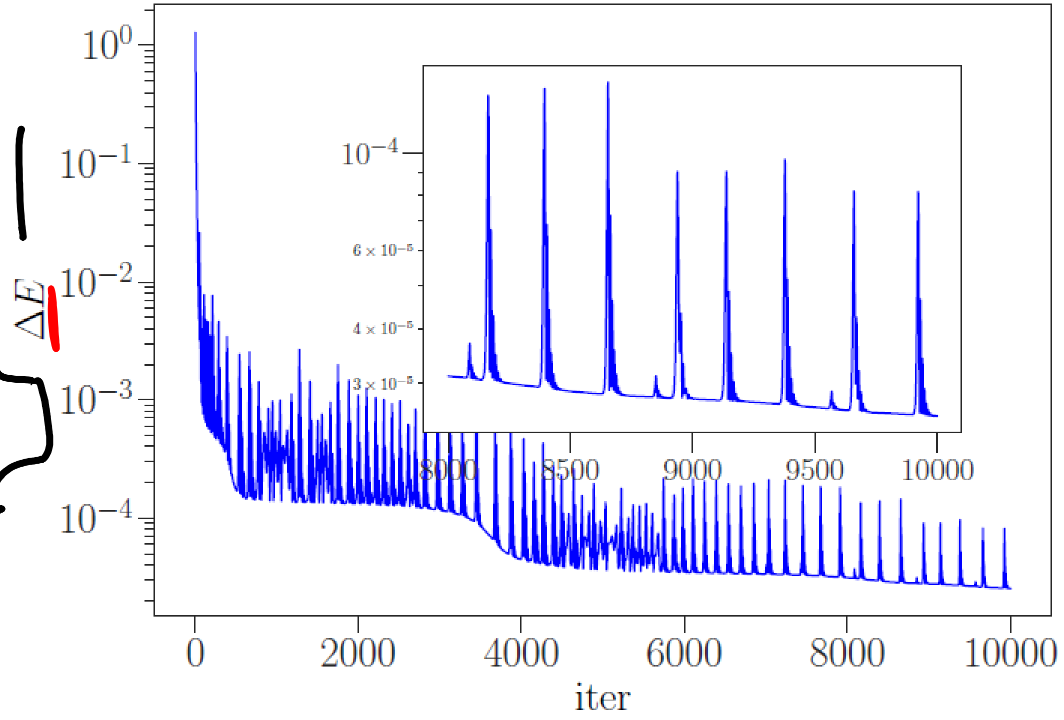
Solution

$$H = \sigma_N^z \sigma_1^z + \sum_{i=1}^{N-1} \sigma_i^z \sigma_{i+1}^z + h_x \sum_{i=1}^{N} \sigma_i^x$$

$N = 2 \times 3^5 = 486$ spins $\qquad h_x = 1$

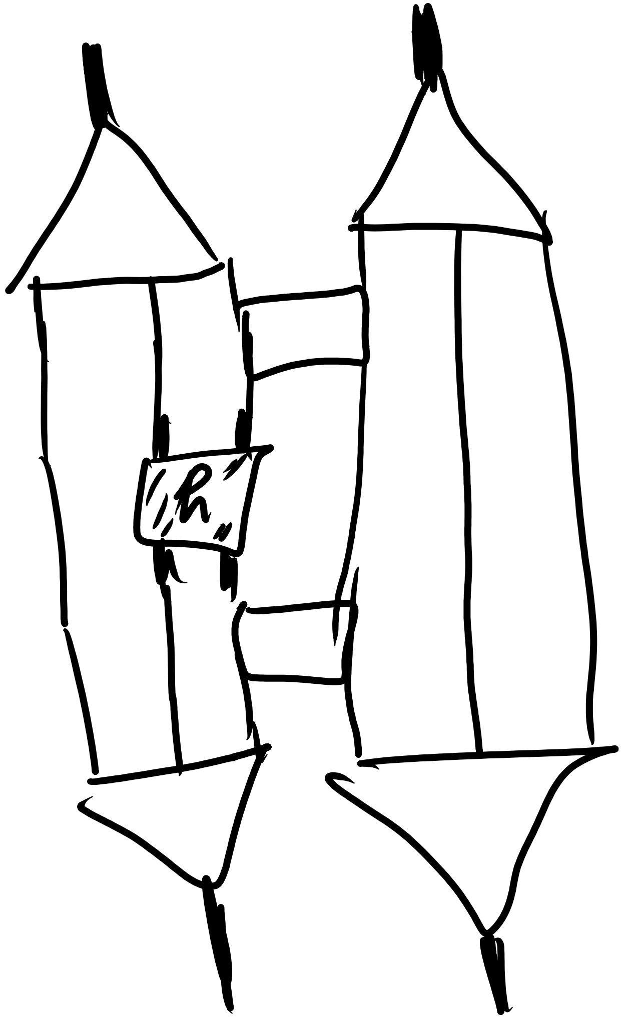
$n = 5$ renormalization layers





Пусть H – локальный гамильтониан вида

$$H = \sum_i h^{i,i+1}$$

$A_L(h) =$ 

Локальная структура H сохраняется при переходе к огрубл. описанию

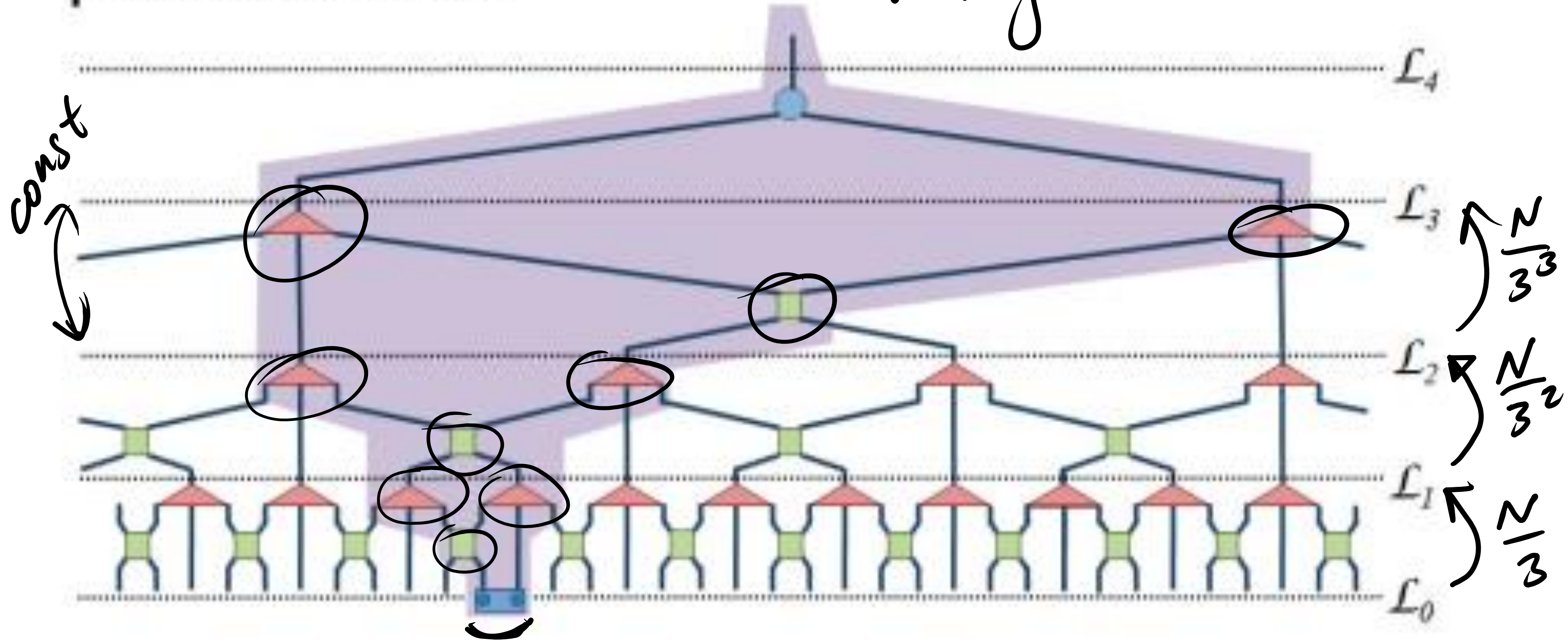
arXiv 0912.1651
arXiv 0707.1454

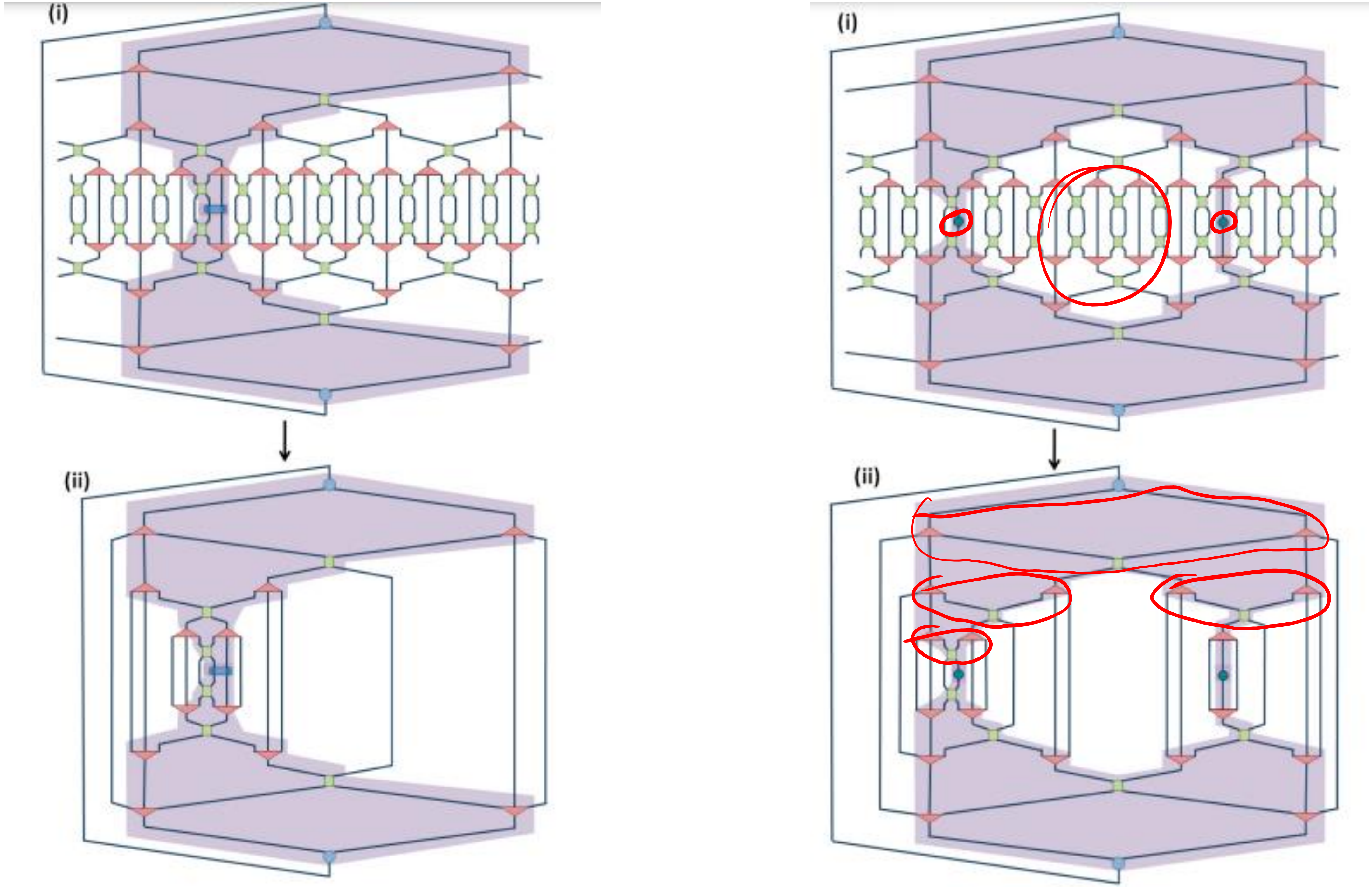
см. также мин. среднее энергии.

past causal cone:

конус влияния

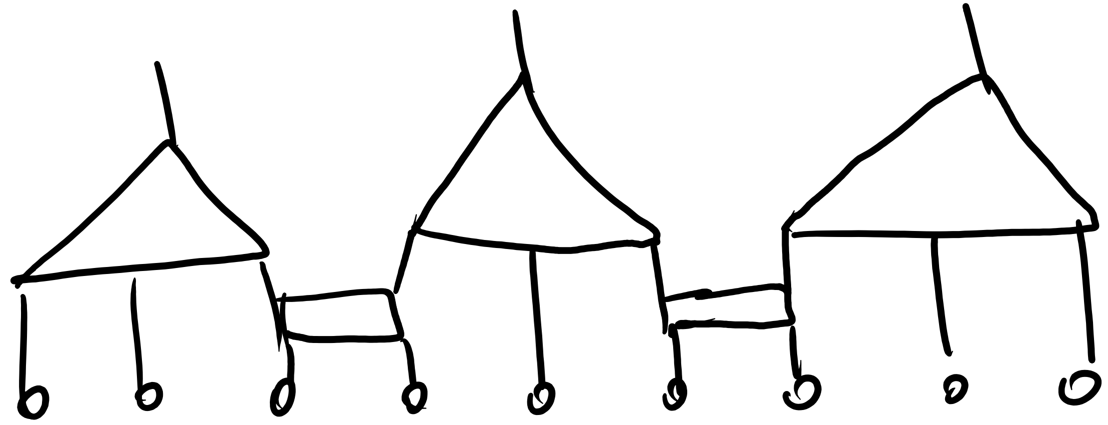
const. глуб. тенз. сети



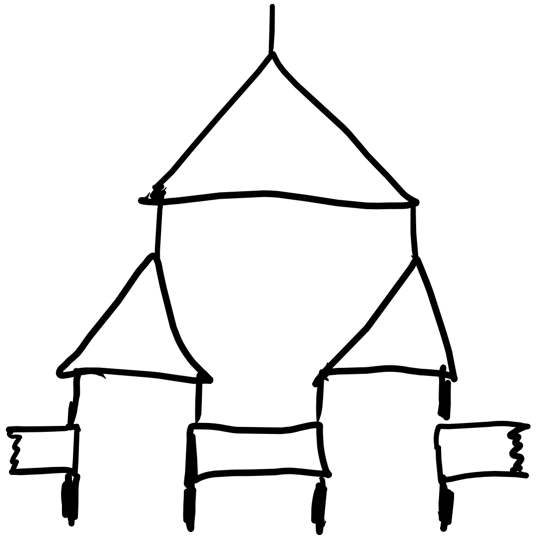


log N.
~
lays. × const

Упражнение*: Записать N кубитное сост. в виде тенз. сети MERA.

$|\psi\rangle =$ 

Упражнение*: Следующее состояние 4-х кубитов яв-ся основным для гам-на
$$H = -\sum_i \sigma_z^i \sigma_z^{i+1} + \sum_i \sigma_x^i$$ с период. гран. условиями.



$|\psi\rangle \in \mathbb{C}^{16}$

$$|\psi\rangle = \frac{2+\sqrt{2(2+\sqrt{2})}}{8}\left(|0000\rangle + |1111\rangle\right) - \frac{1}{4\sqrt{2}}\left(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle + |1110\rangle + |1101\rangle + |1011\rangle + |0111\rangle\right) + \frac{\sqrt{4-2\sqrt{2}}}{8}\left(|0011\rangle + |0110\rangle + |1100\rangle + |1001\rangle\right) + \frac{\sqrt{2(2+\sqrt{2})}-2}{8}\left(|0101\rangle + |1010\rangle\right).$$

Найдите представление для $|\psi\rangle$ в виде MERA.