Mathematical problems in compensation of fiber optic nonlinearity

Pavel Plotnikov, Ph.D.
Huawei Moscow R&D Center, Optic Algorithm Lab
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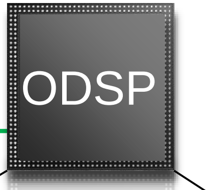
- Introduction
- Channel capacity and Nonlinear Shannon limit
- Models of fiber nonlinearity
- Task formulation
- References

Fiber Optic technologies - What we are focusing on?

Almost all electrical and optical components generate linear and nonlinear distortions, which reduce throughput and distance

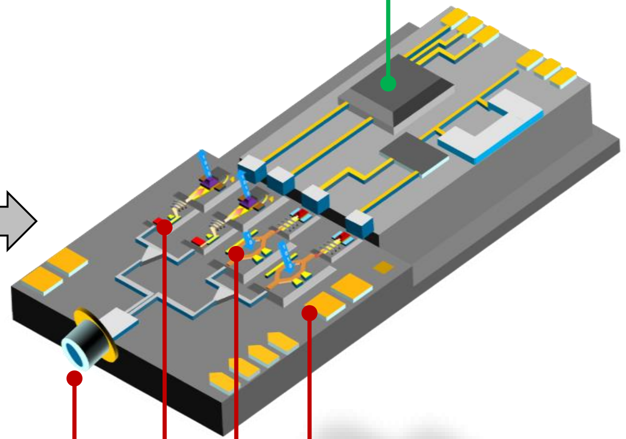
ODSP algorithms compensate distortions, suppress noise and correct errors

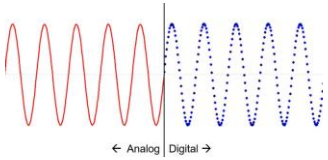


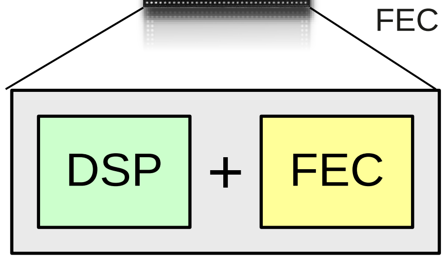
ODSP ASIC - digital "heart" of optical transceiver and contains 2 main parts:
DSP (signal processing) and FEC (forward error correction)

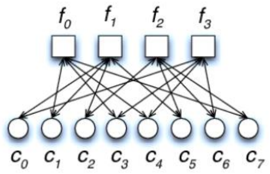
Electrical layer of optical link

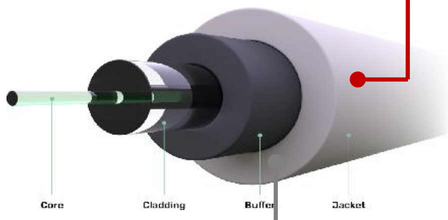
CFP Optical Module



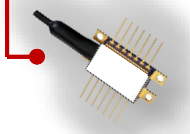




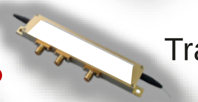




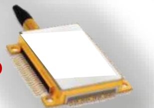
Fiber



Optical source



Transmitter



Receiver

<u>DSP features:</u>
- pre- and post correction of device linear and nonlinear distortions;
- compensation of fiber nonlinearity and chromatic dispersion;
- synchronization and tracking (TR/CR/LOFO/SOP)

<u>Main challenges:</u>
- high performance models for fiber and device nonlinear distortions;
- low complexity and power consumption

Innovative DSP algorithms - responsibility of Optic Algorithm Lab

<u>FEC features:</u>
- encoding/decoding algorithms (correction of bit errors);
- constellation shaping algorithms (achieving capacity limit)

<u>Main challenges:</u>
- high performance code constructions with low error floor;
- low latency, small complexity and power consumption

Innovative FEC algorithms - responsibility of Coding Competence Center

2 labs deeply cooperate together and have common projects

Fiber optic evolution roadmap

short term | middle term | long term

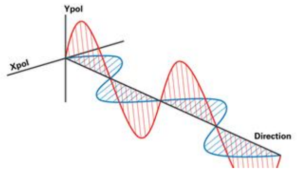
Main targets:
- increase link capacity;
- reduce cost and power consumption per bit

>10x

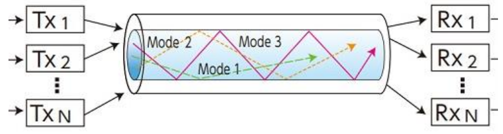
$$C = M \cdot B \cdot \log_2(1 + S/N)$$

additional degrees of freedom:

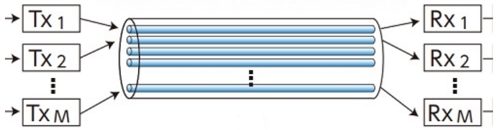
- polarization multiplexing



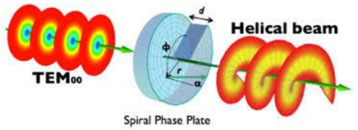
- mode division multiplexing



- multicore fiber

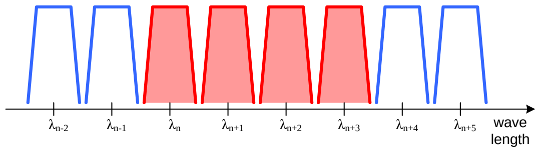


- orbital angular momentum

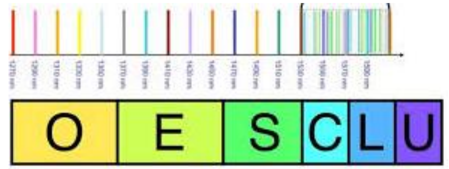


extending of bandwidth:

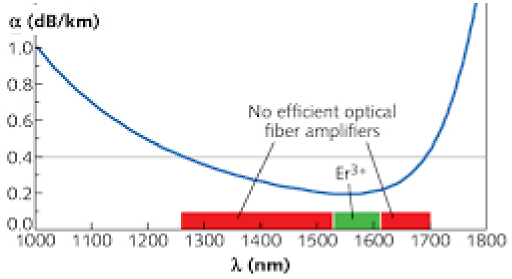
- multiple wavelengths



- C+L(+O) band

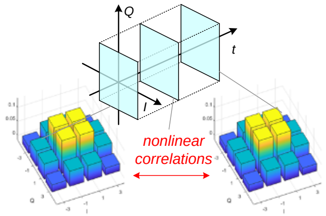


- wideband optical amplifiers (Er+Bi/SOA)

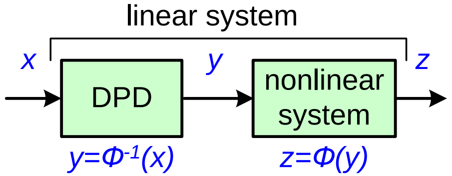


increasing of signal power:

- nonlinear modulation and constellation shaping

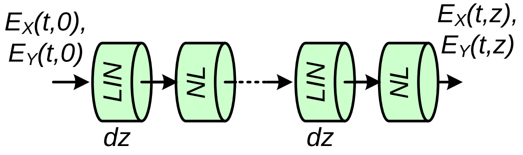


- digital predistortion

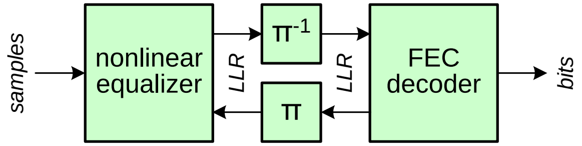


reducing of noise power:

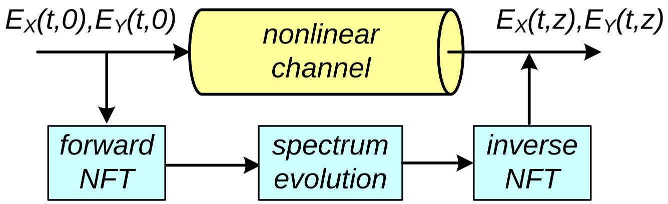
- nonlinear equalization

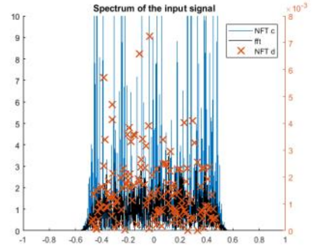


- iterative nonlinear equalizer and decoder



- Nonlinear Fourier Transform (NFT)





Content

Modulation and constellation shaping

In optic usually information is transmitted by QAM (quadrature amplitude modulation) modulated signal,

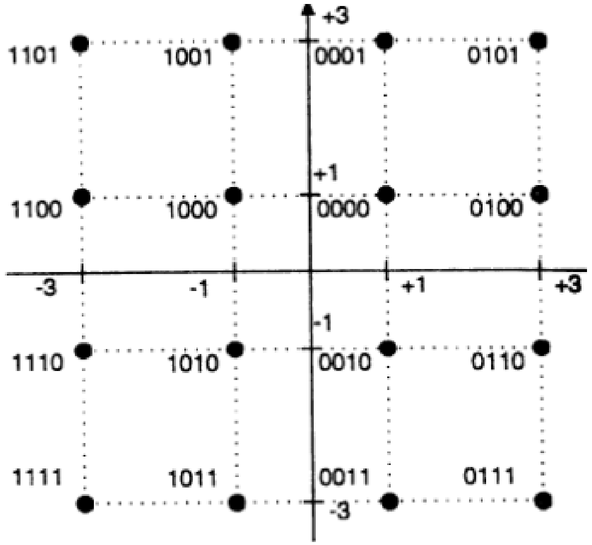
Bits are mapped to symbols with Gray scheme (neighbor symbols has one different bit), number of bits in one symbols (*M* - modulation order)

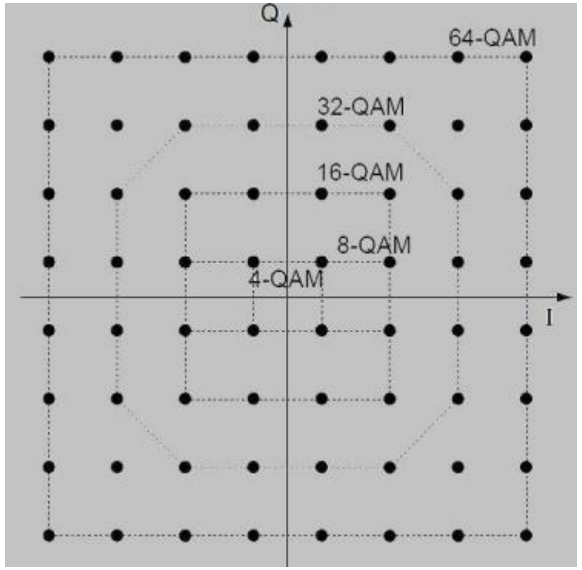
$$b = \log_2 M$$

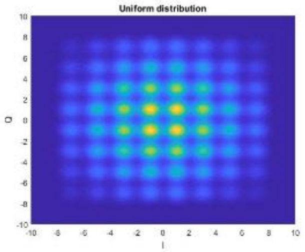
Each QAM symbol is represented by point on complex plane and occupies one time slot (duration about 10..20 ps), symbols transmitted one by one

As higher modulation order, as more transmission rate, but influence of noise is also stronger (because of smaller Euclidean distance between neighbor symbols)

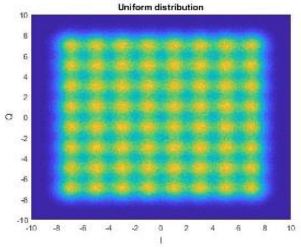
For achieving capacity limit, signal distribution in the channel should be close to Gaussian, hence geometric and probabilistic constellation shaping are used



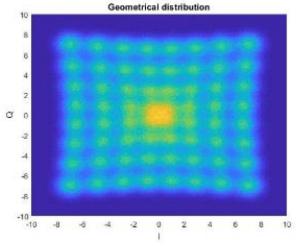




probability shaped



uniform



geometry shaped

Entropy, channel capacity and mutual information

For achieving capacity limit, signal distribution in the channel should be close to Gaussian, hence probabilistic constellation shaping is used

To do it, probabilities of different symbols set different values, hence information source has information entropy (number of bits per symbol)

$$H = -\sum_{i=0}^{M-1} P(x_i) \log_2 P(x_i)$$

Probabilities of symbols are defined with Maxwell-Boltzmann distribution

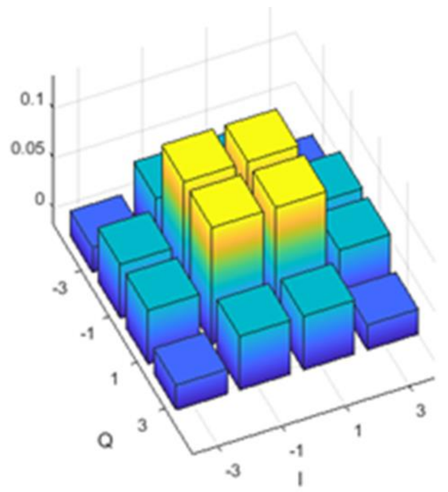
$$P(x_i) = e^{-\lambda |x_i|^2} / \sum_{i=0}^{M-1} e^{-\lambda |x_i|^2}$$

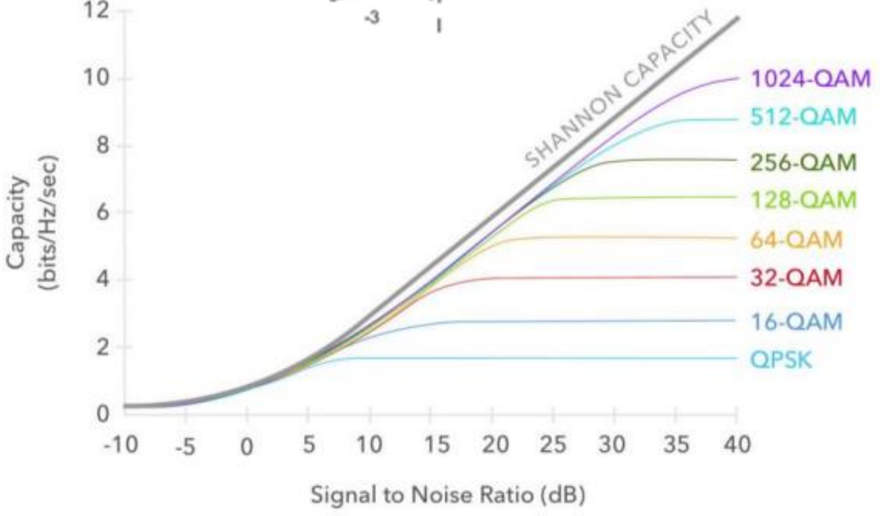
Maximum capacity of AWGN (additive white Gaussian noise) channel is calculated with Shannon-Hartley theorem

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

Mutual information - upper bound of information rate for current transmission conditions, can be estimated with Monte-Carlo method

$$I(X;Y) \geq \frac{1}{N} \sum_{n=0}^{N-1} \log_2 \frac{q_{Y|X}(y(n)|x(n))}{q_Y(y(n))} \qquad q_Y(y(n)) = \sum_{i=0}^{M-1} P_X(x_i) q_{Y|X}(y(n)|x_i) \qquad q_{Y|X}(y(n)|s_j) = \frac{1}{\sqrt{2\pi \det(\Sigma_j)}} \exp\left(-\frac{1}{2} \begin{pmatrix} y_I - \mu_{I,j} \\ y_Q - \mu_{Q,j} \end{pmatrix}^T \Sigma_j^{-1} \begin{pmatrix} y_I - \mu_{I,j} \\ y_Q - \mu_{Q,j} \end{pmatrix}\right)$$





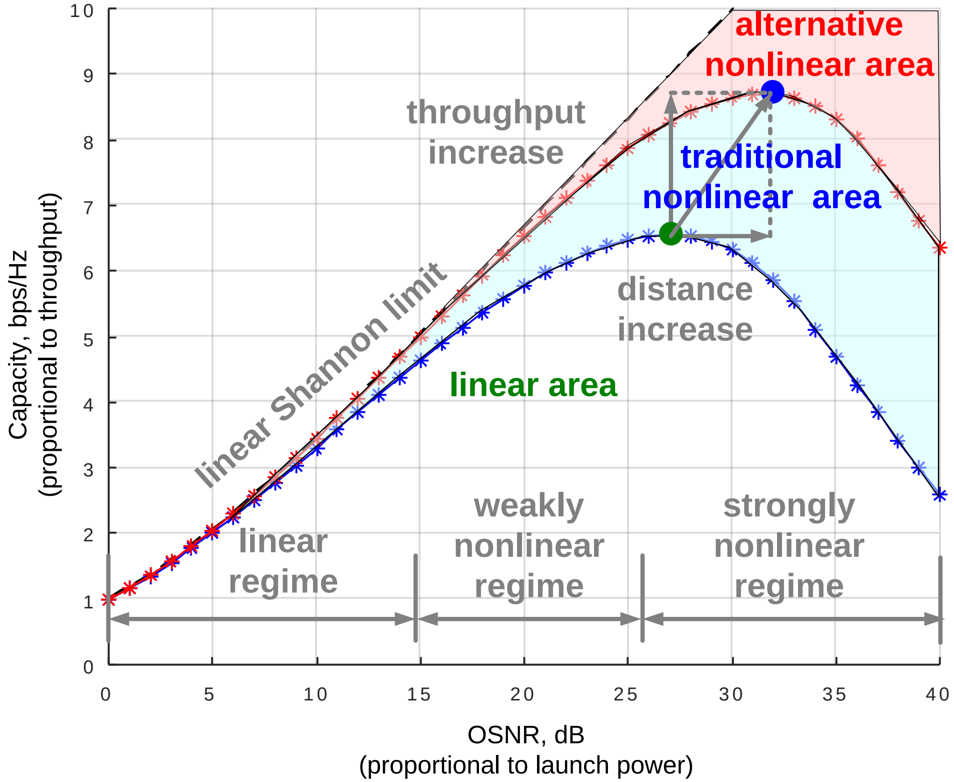
Nonlinear Shannon limit

Nonlinear Schrodinger equations (Manakov equations):

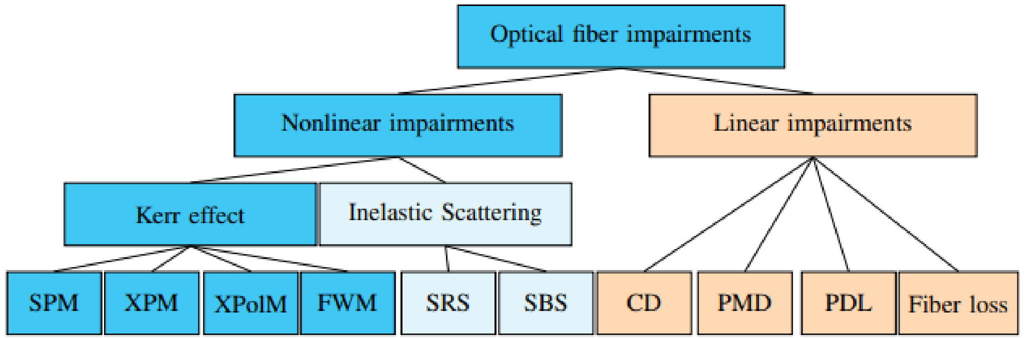
$$\frac{\partial \mathbf{E}(t,z)}{\partial z} = -i\frac{\beta}{2}\frac{\partial^2 \mathbf{E}(t,z)}{\partial t^2} + i\gamma \mathbf{E}(t,z)\left\|\mathbf{E}(t,z)\right\|^2 - \alpha \mathbf{E}(t,z)$$

chromatic dispersion (β), Kerr nonlinearity (γ), fiber loss (α)

$$\mathbf{E}(t,z) = \left[E_X(t,z)\, E_Y(t,z)\right]^T$$



Linear and nonlinear distortions in optical fiber:



SPM: Self-phase mod., XPM: Cross-phase mod., XPolM: Cross-polarization mod., FWM: Four wave mixing, SBS: Stimulated Brillouin scat., SRS: Stimulated Raman scat., CD: Chromatic disp., PMD: Polarization mode disp., PDL: Polarization depend. loss

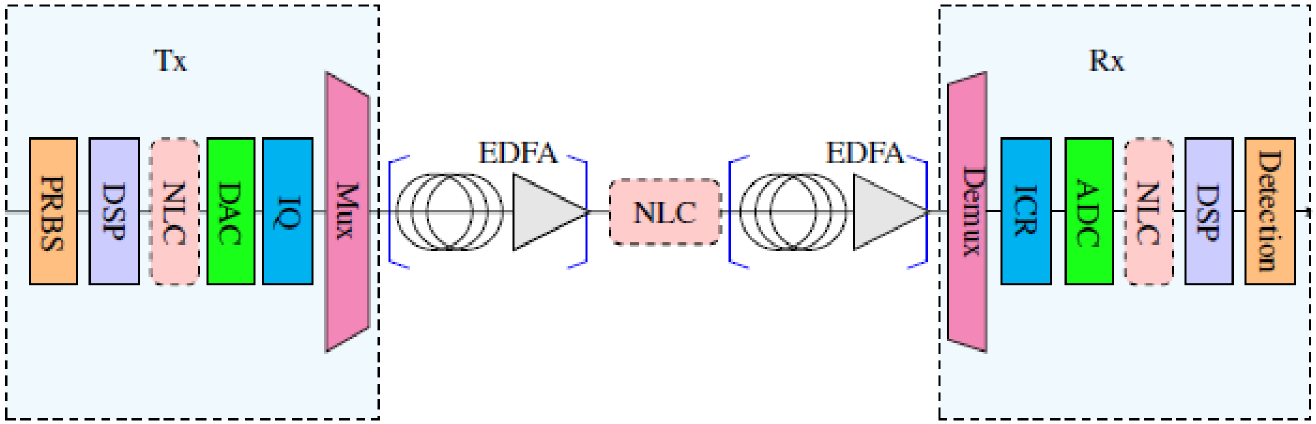
	Algorithm	Position	Domain
tradit. linear	Gaussian constellation shaping	TX/RX	bit/symbol
	Linear equalization	RX	sample
tradit. nonlin.	Nonlinear constellation shaping	TX/RX	bit/symbol
	Nonlinear equalization	RX	sample
altern. nonlin.	Nonlinear Fourier transform	TX/RX	bit/symbol/ sample

Fiber optic nonlinearity is the main limitation for increasing link distance and spectrum efficiency

Content

Techniques for fiber nonlinearity compensation

<u>Transmission diagram: possible NLC locations</u> (PRBS: Pseudo-random binary sequences, DSP: Digital signal processing, NLC: Nonlinearity compensation, DAC: Digital-to-analog converter, IQ: In-phase and quadrature modulator, Mux: Multiplexer, Demux: De-multiplexer, ICR: Intergrated coherent receiver, ADC: Analog-to-digital converter)



<u>TX side:</u>
+ digital domain processing;
+ input signal without noise;
+ no information about neighbor channels;
- requires feedback for adaptation;
- increases PAPR (peak to average power ratio)

<u>In channel:</u>
+ contains information about all channels;
+ optical domain processing;
- hard to implement and make adaptive

<u>RX side:</u>
+ digital domain processing;
+ limited information about neighbor channels;
+ easy to adapt;
- input signal with noise

Amari Abdelkerim and others, A Survey on Fiber Nonlinearity Compensation for 400 Gb/s and Beyond Optical Communication Systems, 2017

Digital back propagation (DBP)

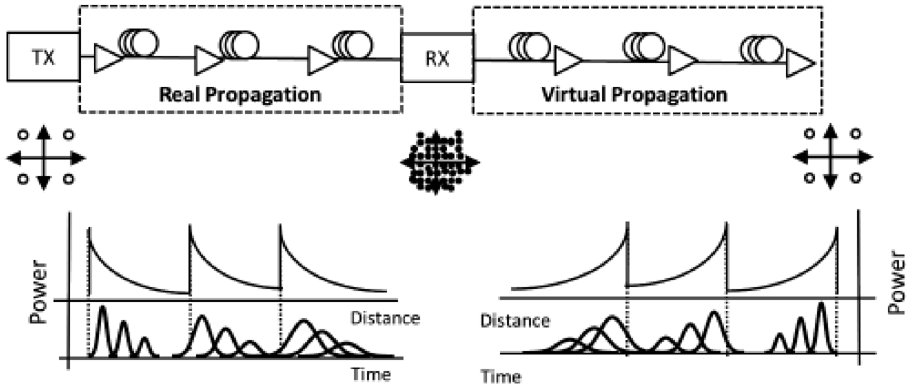
idea: α, β, and γ parameters are set to be the opposite values to those in the transmission fiber, and the optical amplifiers are replaced with digital attenuators.

$$\frac{\partial U(z,t)}{\partial z} = [\hat{L} + \hat{N}]U(z,t)$$

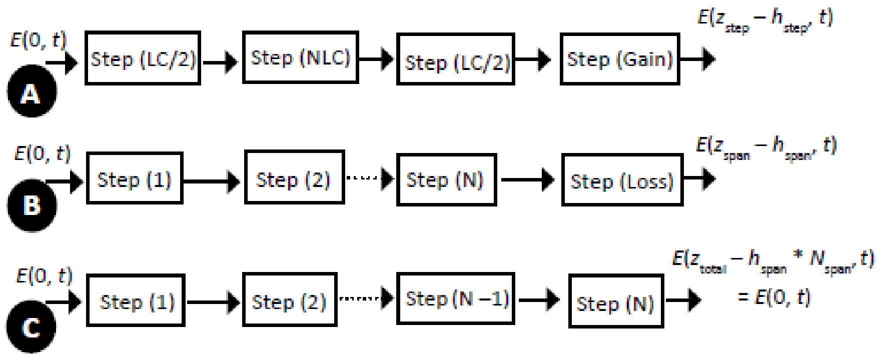
$$\hat{L} = -\frac{\alpha}{2} + j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial t^3}$$

$$\hat{N} = -j\gamma(1-f_r)|U(z,t)|^2 - j\gamma f_r \int h_r(\tau)|U(z,t-\tau)|^2 \delta\tau$$

Typical EDFA-based lumped amplified transmission system employing digital back propagation. Qualitative representation of pulse broadening as a function of transmission distance and DBP-based compensation.



Split step Fourier method-based solution of NLSE for digital back propagation. (a) Per DBP step, (b) per span, and (c) per link. E, z, h, and t represent signal electric field, step/span distance, step/span size,and time instant,



Andrew Ellis, Mariia Sorokina, Optical Communication Systems: Limits and Possibilities, Stanford Publishing Pte. Ltd. 2020

Digital back propagation (DBP)

Manakov equation:

$$\frac{\partial V_{x/y}}{\partial z} + j\frac{\beta_2}{2}\frac{\partial^2 V_{x/y}}{\partial t^2} + \frac{\alpha}{2}V_{x/y} = j\gamma'(|V_x|^2 + |V_y|^2)V_{x/y}$$

$V = [V_x, V_y]$ $\quad\quad$ $\gamma' = \frac{8}{9}\gamma$ i

DBP linear section:

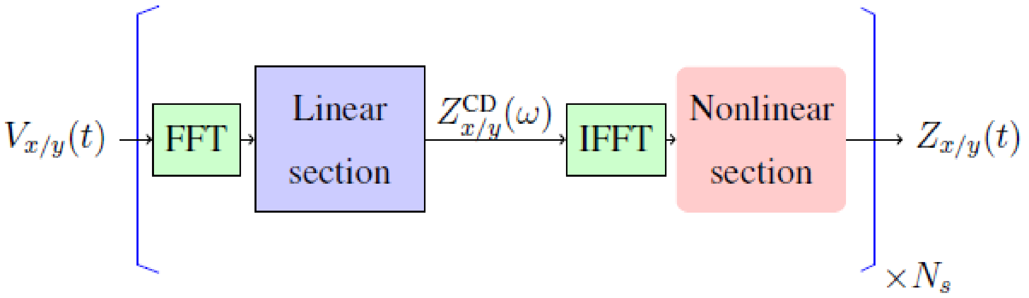
$$Z^{\mathrm{CD}}_{x/y}(\omega, z) = V_{x/y}(\omega, z)e^{-jh\left(\frac{\alpha}{2}+\frac{\beta_2}{2}\omega^2\right)}$$

DBP nonlinear section:

$$Z_{x/y}(t, z) = Z^{\mathrm{CD}}_{x/y}(t, z)e^{-j\varphi\gamma' h(|Z^{\mathrm{CD}}_x|^2+|Z^{\mathrm{CD}}_y|^2)}$$

$0 < \varphi < 1$ is a real-valued optimization parameter.

DBP implementation principle:



DBP key features:
- multiple FFT/IFFT are used for converting signal between time to frequency domains;
- multistep approximation of NLSE;
- usually used on RX side;
- hard to make adaptive due to sequential structure;
- computationally complicated, requires multiple steps per span and multiple samples per symbol

Amari Abdelkerim and others, A Survey on Fiber Nonlinearity Compensation for 400 Gb/s and Beyond Optical Communication Systems, 2017

Perturbation based model (PBM)

For a quasi-linear propagation regime in which the dispersive effects are the dominant source of signal distortion, the nonlinearities can be modeled as a small perturbation to the linear solution

$$\frac{\partial}{\partial z}\mathbf{u}(t,z) + j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\mathbf{u}(t,z) + \frac{\alpha - g(z)}{2}\mathbf{u}(t,z) = j\frac{8}{9}\gamma|\mathbf{u}(t,z)|^2\mathbf{u}(t,z).$$

$g(z)$ is added to model the signal amplification through the link

Set $\mathbf{u}(t,z) = \mathbf{v}(t,z)f(z),$ with $\frac{df(z)}{dz} = \frac{g(z)-\alpha}{2}f(z),$

$f(z) = e^{-\alpha z/2}$ for one span of fiber with no distributed amplification

Then

$$\frac{\partial}{\partial z}\mathbf{v}(t,z) + j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\mathbf{v}(t,z) = j\frac{8}{9}\gamma f^2(z)|\mathbf{v}(t,z)|^2\mathbf{v}(t,z).$$

Considering the following expansion

$$\mathbf{v}(t,z) = \mathbf{v}^{(0)}(t,z) + \gamma\mathbf{v}^{(1)}(t,z) + \ldots + \gamma^n\mathbf{v}^{(n)}(t,z)$$
$$= \begin{pmatrix} v_x^{(0)}(t,z) \\ v_y^{(0)}(t,z) \end{pmatrix} + \gamma \begin{pmatrix} v_x^{(1)}(t,z) \\ v_y^{(1)}(t,z) \end{pmatrix} + \ldots + \gamma^n \begin{pmatrix} v_x^{(n)}(t,z) \\ v_y^{(n)}(t,z) \end{pmatrix}$$

where $\mathbf{v}^{(n)}(t,z)$ are n-th order perturbation solutions.

Ali Bakhshali, Nonlinearity Compensation for Next Generation Coherent Optical Fiber Communication Systems, 2017

Perturbation based model (PBM)

We arrive at

$$\frac{\partial}{\partial z}\mathbf{v}^{(0)}(t,z) + j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\mathbf{v}^{(0)}(t,z) = 0,$$
$$\frac{\partial}{\partial z}\mathbf{v}^{(1)}(t,z) + j\frac{\beta_2}{2}\frac{\partial^2}{\partial t^2}\mathbf{v}^{(1)}(t,z) = j\frac{8}{9}\gamma f^2(z)|\mathbf{v}^{(0)}(t,z)|^2\mathbf{v}^{(0)}(t,z).$$

First equation can be solved in the frequency domain to obtain the following solution

$$\mathbf{v}^{(0)}(t,z) = \mathcal{F}^{-1}\left\{\mathbf{V}_{\text{in}}(\omega)e^{j\beta_2\omega^2 z/2}\right\},$$ where $$\mathbf{V}_{\text{in}}(\omega) = \mathcal{F}\{\mathbf{v}(t,0)\} = f^2(0)\mathcal{F}\{\mathbf{u}(t,0)\}$$

Similarly, the solution of the second order inhomogeneous differential equation corresponding to the first order perturbation solution is

$$\mathbf{v}^{(1)}(t,z) = \mathcal{F}^{-1}\left\{e^{+j\beta_2\omega^2 z/2}\int_0^z \mathbf{S}(\omega,z')e^{-j\beta_2\omega^2 z'/2}dz'\right\},$$

where

$$\mathbf{S}(\omega,z) = j\frac{8}{9}\gamma f^2(z)\mathcal{F}\left\{|\mathbf{v}^{(0)}(t,z)|^2\mathbf{v}^{(0)}(t,z)\right\}$$
$$= j\frac{8}{9}\gamma f^2(z) \times \frac{1}{4\pi^2}[\mathbf{V}^{(0)H}(-\omega,z) * \mathbf{V}^{(0)}(\omega,z)] * \mathbf{V}^{(0)}(\omega,z)$$

Ali Bakhshali, Nonlinearity Compensation for Next Generation Coherent Optical Fiber Communication Systems, 2017

Perturbation based model (PBM)

Consider input signal as

$$\mathbf{u}(t,0) = \sum_{k=-\infty}^{+\infty} \mathbf{a}_k b_0(t-kT),$$

where $\mathbf{a}_k = [a_x[k] \quad a_y[k]]^T$ - vector of modulated symbols;

$b_0(t)$ - unit energy transmit pulse shape

The matched filter output of the transmitted signal after propagation over distance z and sampled at t = kT can be expressed as

$$\hat{\mathbf{a}}_k = \mathbf{a}_k + \mathbf{\Delta}_k$$

where $\mathbf{\Delta}_k = [\delta_x[k] \quad \delta_y[k]]^T$ is the first-order perturbation of the sampled output with,

$$\delta_{x/y}[k] = \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \left(a_{x/y}[k+n]a^*_{x/y}[k+n+m]a_{x/y}[k+m] + a_{y/x}[k+n]a^*_{y/x}[k+n+m]a_{x/y}[k+m]\right) C_{m,n},$$

$C_{m,n}$ are the perturbation coefficients with m and n indexing the symbols relative to the current symbol

$$C_{m,n} = j\gamma \int_0^z dz f^2(z) \times \int_{t=-\infty}^{+\infty} dt b_0^*(t,z) b_0(t-nT,z) b_0^*(t-(n+m)T,z) b_0(t-mT,z)$$

with $b_0(t,z)$ denoting the dispersed shaping pulse at distance z,

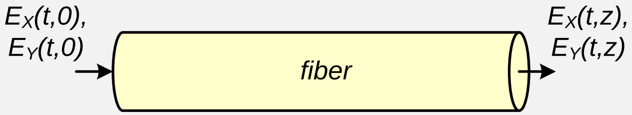
$$b_0(t,z) = \mathcal{F}^{-1}\left\{\mathcal{F}\{b_0(t)\} e^{j\beta_2\omega^2 z/2}\right\}.$$

Ali Bakhshali, Nonlinearity Compensation for Next Generation Coherent Optical Fiber Communication Systems, 2017

Comparison of PBM and DBP models

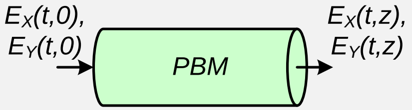
Nonlinear Schrodinger equations (Manakov equations for 2 polarizations):

$$\frac{\partial E_{X|Y}(t,z)}{\partial z} = -i\frac{\beta}{2}\frac{\partial^2 E_{X|Y}(t,z)}{\partial t^2} + i\gamma E_{X|Y}(t,z)\left(\left|E_X(t,z)\right|^2 + \left|E_Y(t,z)\right|^2\right) - \alpha E_{X|Y}(t,z)$$



single step approximation $\approx$ *multistep approximation*

Perturbation based model (PBM):

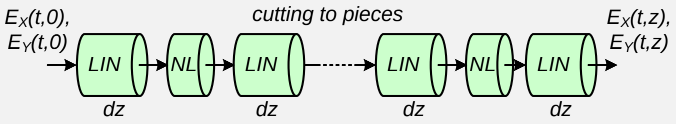


$$E_{X|Y}(t,z) = = E_{X|Y}(t,0) + \Delta E_{X|Y}(t,z)$$

$$\Delta E_{X|Y}(t,z) = \sum_{m,n\neq 0} c_{m,n} E_{X|Y}(t+n\tau,0)\begin{pmatrix} E_{X|Y}(t+m\tau,0)E^*_{X|Y}(t+m\tau+n\tau,0) + \\ + E_{Y|X}(t+m\tau,0)E^*_{Y|X}(t+m\tau+n\tau,0)\end{pmatrix} +$$

$$+\sum_{m\neq 0} c_{0,m} E_{X|Y}(t+m\tau,0)E_{Y|X}(t,0)E^*_{Y|X}(t+m\tau+n\tau,0)$$

Digital back propagation model (DBP):



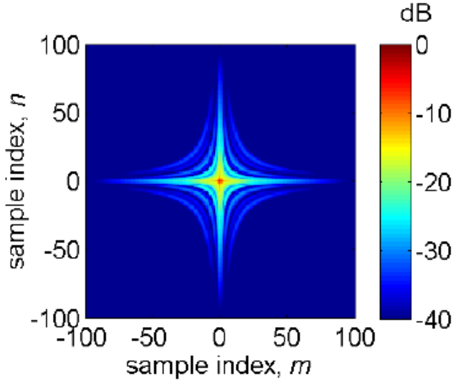
1) linear step (chromatic dispersion only):

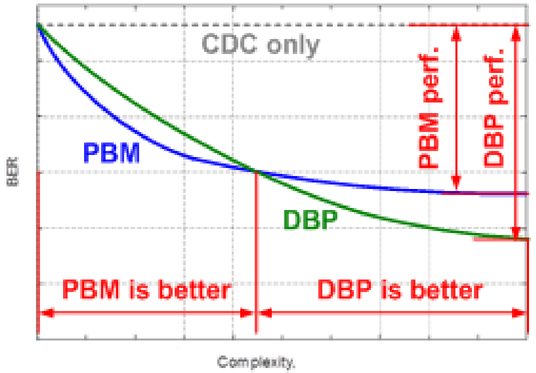
$$E_{X|Y}(t,z+dz) = E_{X|Y}(t,z) \otimes h_{CD}(dz)$$

2) nonlinear step (Kerr nonlinearity only):

$$E'_{X|Y}(t,z) = E_{X|Y}(t,z)e^{i\gamma\,dz\left(\left|E_X(t,z)\right|^2 + \left|E_Y(t,z)\right|^2\right)} \approx$$

$$\approx E_{X|Y}(t,z) + i\gamma\,dz\,E_{X|Y}(t,z)\left(\left|E_X(t,z)\right|^2 + \left|E_Y(t,z)\right|^2\right)$$

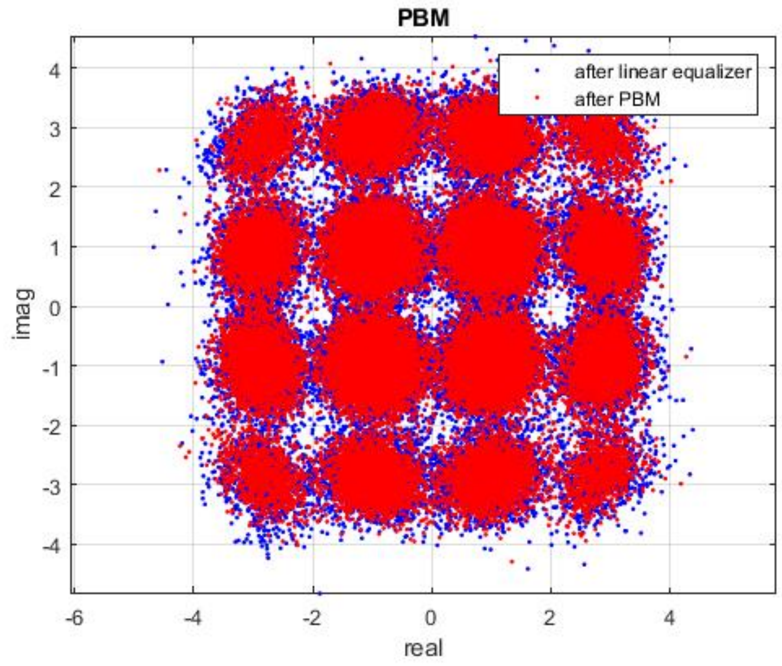


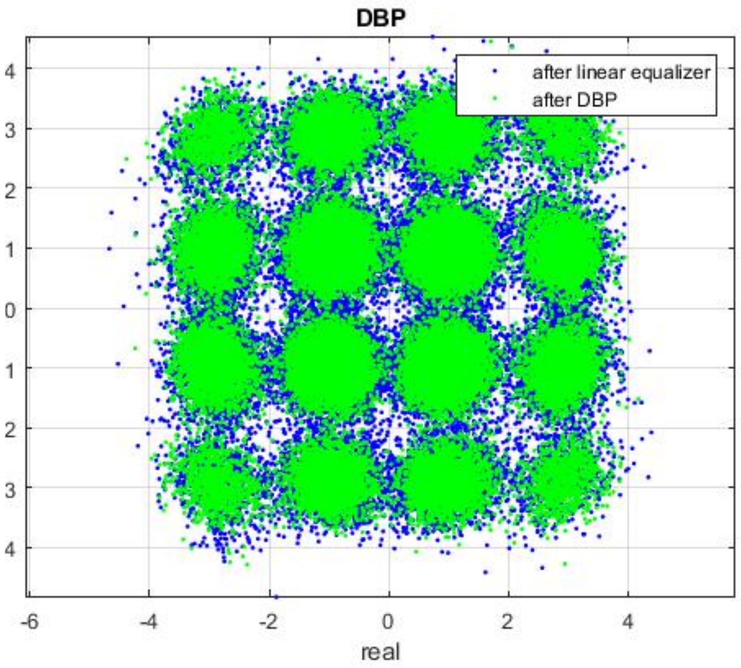


PBM: single step model, quadratic complexity vs memory size, limited order of nonlinearity, optimal for medium distances
DBP: multistep model, linear complexity vs number of spans, optimal for long distances

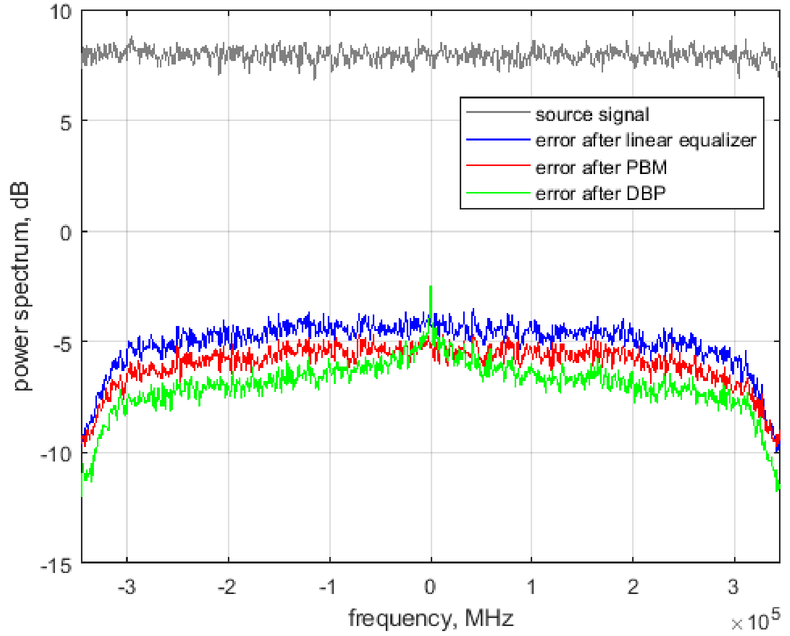
Example of simulation results

Signal constellation after PBM and DBP





Spectrum of error after PBM and DBP:



after linear equalizer:

NMSE: -12.866 dB, SER: 3.089e-02, BER: 7.068e-03, MI: 3.5498 bit/sym

after PBM:

NMSE: -13.849 dB, SER: 1.256e-02, BER: 3.160e-03, MI: 3.6147 bit/sym

after DBP:

NMSE: -14.739 dB, SER: 6.058e-03, BER: 1.519e-03, MI: 3.6421 bit/sym

NMSE criteria of algorithm performance:

$$e_{X|Y}(k) = x_{X|Y}(k) - z_{X|Y}(k)$$

$$NMSE = 10\lg\left(\frac{1}{2}\frac{\sum_{k=0}^{K-1}|e_X(k)|^2}{\sum_{k=0}^{K-1}|x_X(k)|^2} + \frac{1}{2}\frac{\sum_{k=0}^{K-1}|e_Y(k)|^2}{\sum_{k=0}^{K-1}|x_Y(k)|^2}\right)$$

$$\delta_{X|Y}(n) = \begin{cases} 0, if\ u_{X|Y}(n) = v_{X|Y}(n) \\ 1, if\ u_{X|Y}(n) \neq v_{X|Y}(n) \end{cases}$$

$$BER = \frac{1}{2N}\sum_{n=0}^{N-1}\left(\delta_X(n) + \delta_Y(n)\right)$$

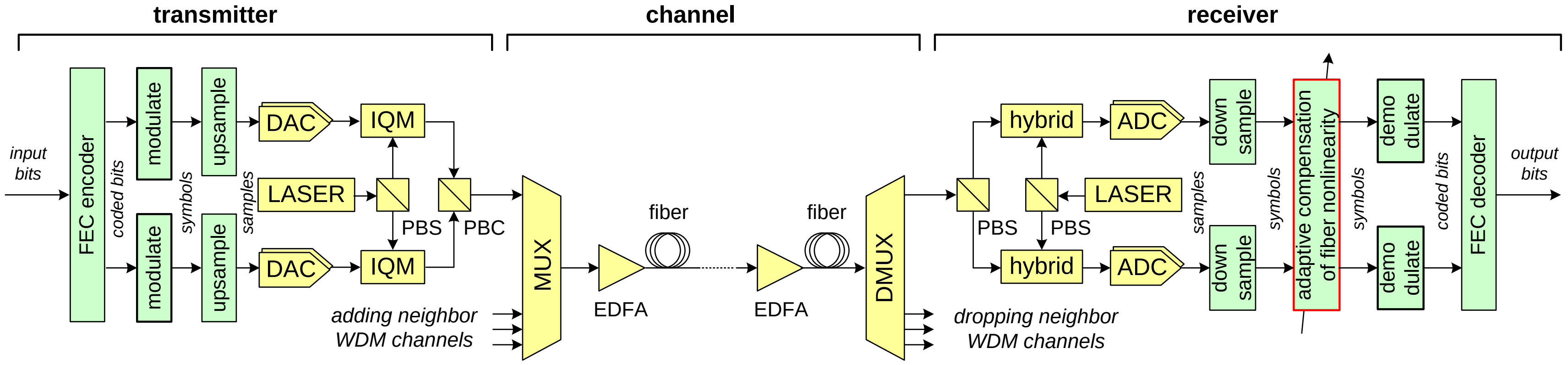
Content

- Introduction
- Channel capacity and Nonlinear Shannon limit
- Models of fiber nonlinearity
- Task formulation
- References

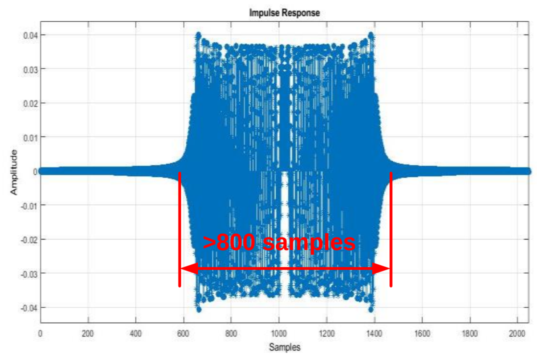
Compensation of fiber nonlinearity in receiver

Requirements: single channel algorithm for compensation of fiber optic nonlinearity on receiver side, nonlinear adaptive model
Challenges: compensation of nonlinear crosstalks from neighbor channels (not known in receiver), large memory of nonlinear model due to convolution of chromatic dispersion and Kerr nonlinearity

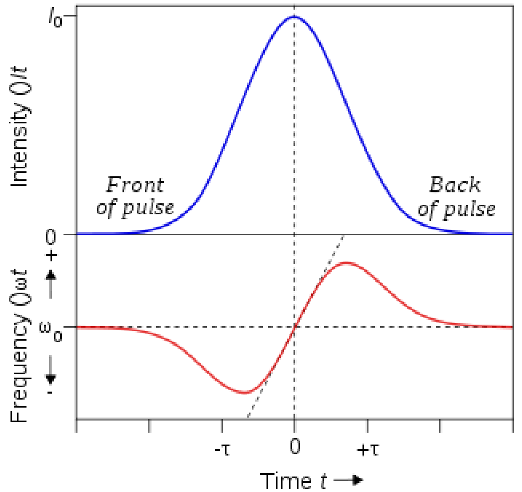
System structure:



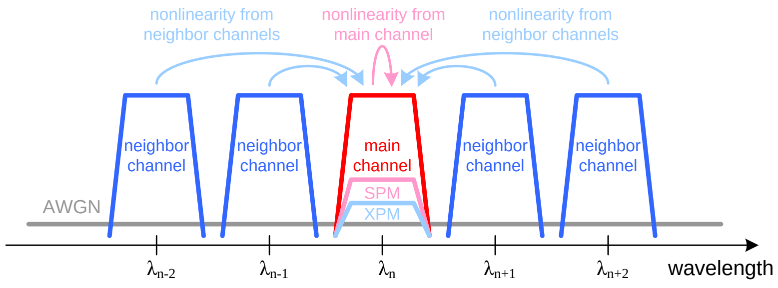
Impulse response of CD filter (example for 960 km):



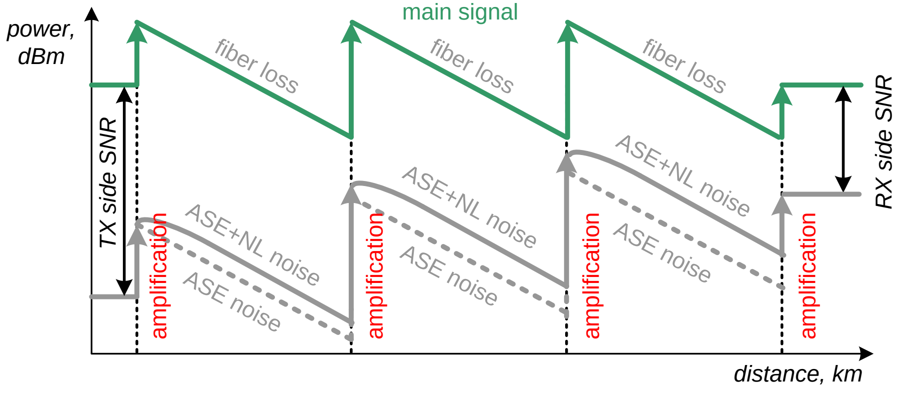
Kerr nonlinearity (example for pulse):



Nonlinear distortions for DWDM signal:



Signal and noise power profile:



PBM nonlinear model and principle of adaptation

Equation for PBM model:

$$y_{X|Y}(k) = x_{X|Y}(k) + \Delta x_{X|Y}(k);$$

$$\Delta x_{X|Y}(k) = \sum_{m=-M}^{M} \sum_{n=-M}^{M} c_{X|Y,0,m,n} x_{X|Y}(k+m) x_{X|Y}(k+n) x^{*}_{X|Y}(k+m+n) +$$

$$+ \sum_{m=-M}^{M} \sum_{n=-M}^{M} c_{X|Y,1,m,n} x_{X|Y}(k+m) x_{Y|X}(k+n) x^{*}_{Y|X}(k+m+n)$$

$X \mid Y$ - index of polarization (X or Y); k - number of sample;

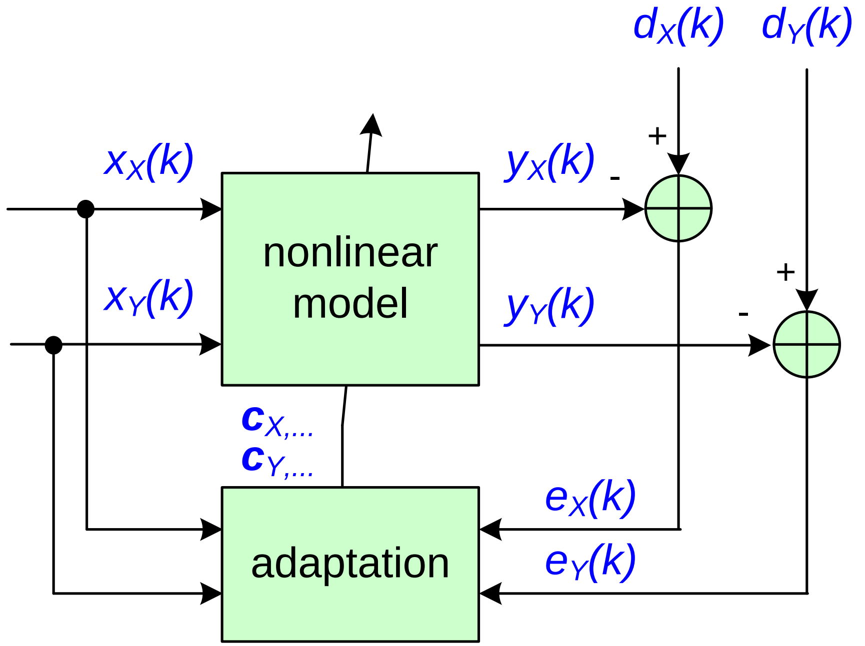
$c_{X,0},\ldots, c_{X,1},\ldots, c_{Y,0},\ldots, c_{Y,1},\ldots$ - coefficients of nonlinear model;

$x_X(k), x_Y(k)$ - input signals for X and Y polarizations;

$y_X(k), y_Y(k)$ - output signals for X and Y polarizations;

$d_X(k), d_Y(k)$ - desired signals for X and Y polarizations (we assume that it is available in receiver);

Structure of adaptation, based on NMSE criteria:



model with 2 complex-valued inputs and 2 complex-valued outputs

Adaptation of nonlinear model, based on NMSE criteria

For block of *K* data samples we can define matrix and vectors:

$$\mathbf{U}_{X|Y} = \begin{pmatrix} u_{X|Y,0,\dots}(0),\dots u_{X|Y,1,\dots}(0),\dots \\ \dots \\ u_{X|Y,0,\dots}(K-1),\dots u_{X|Y,1,\dots}(K-1),\dots \end{pmatrix}$$

- matrix of nonlinear terms;

with elements, filled with triplets from input signal

$$u_{X|Y,0,m,n}(k) = x_{X|Y}(k+m)x_{X|Y}(k+n)x^*_{X|Y}(k+m+n);$$

$$u_{X|Y,1,m,n}(k) = x_{X|Y}(k+m)x_{Y|X}(k+n)x^*_{Y|X}(k+m+n)$$

$$\mathbf{d}_{X|Y} = \left(d_{X|Y}(0),\dots,d_{X|Y}(K-1)\right)^T$$ - vector of desired signal;

$$\mathbf{y}_{X|Y} = \left(y_{X|Y}(0),\dots,y_{X|Y}(K-1)\right)^T$$ - vector of output signal;

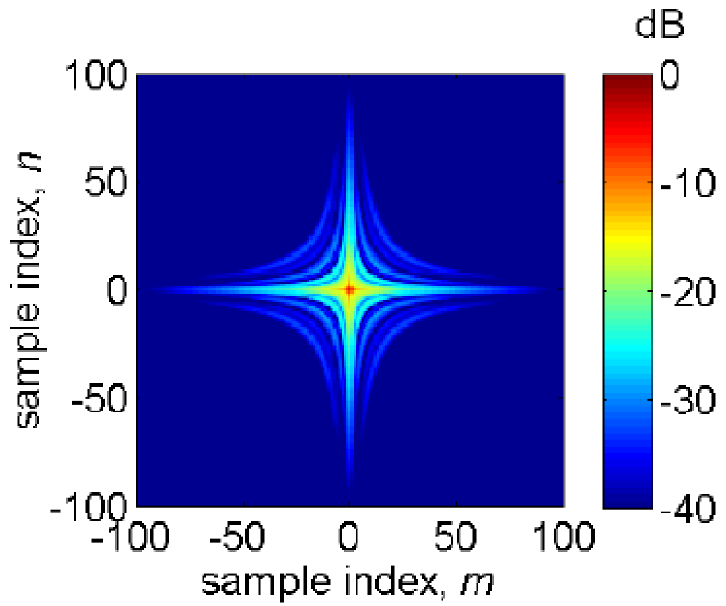
$$\mathbf{c}_{X|Y} = \left(c_{X|Y,0,\dots},\dots,c_{X|Y,1,\dots},\dots\right)^T$$ - vector of coefficients;

NMSE (normalized mean square error) criteria of algorithm performance:

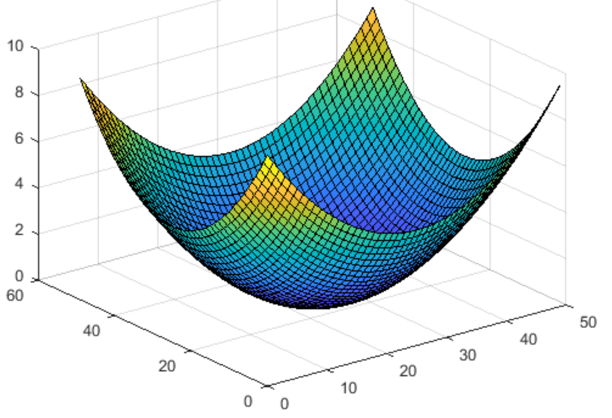
$$e_{X|Y}(k) = d_{X|Y}(k) - y_{X|Y}(k)$$

$$NMSE_{X|Y} = 10\lg\left(\sum_{k=0}^{K-1}\left|e_{X|Y}(k)\right|^2\right) \Big/ \left(\sum_{k=0}^{K-1}\left|x_{X|Y}(k)\right|^2\right)$$

Typical coefficients of PBM model:



Surface of goal function in the space of coefficients:



We can rewrite model's equation in matrix-vector form,

$$\mathbf{y}_{X|Y} = \mathbf{U}_{X|Y}\mathbf{c}_{X|Y}$$

formulate optimization problem as

$$\mathbf{c}_{X|Y} = \arg\min\left\|\mathbf{d}_{X|Y} - \mathbf{y}_{X|Y}\right\|_2^2 = \arg\min\left\|\mathbf{d}_{X|Y} - \mathbf{U}_{X|Y}\mathbf{c}_{X|Y}\right\|_2^2$$

and apply least squares (LS) method for calculation of coefficients

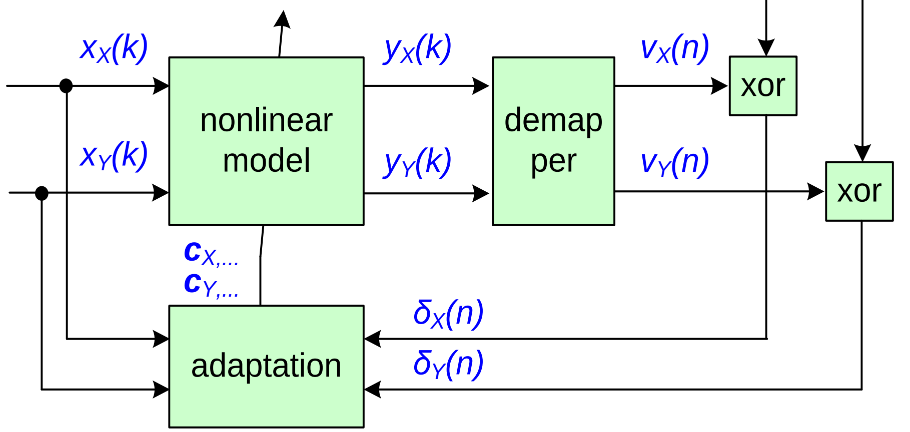
$$\mathbf{c}_{X|Y} = \left(\mathbf{U}^H_{X|Y}\mathbf{U}_{X|Y}\right)^{-1}\mathbf{U}^H_{X|Y}\mathbf{d}_{X|Y}$$

Adaptation of nonlinear model, based on BER and MI criteria

Problem: NMSE (normalized mean square error) criteria is simple and convenient for adaptation of coefficients, but doesn't guarantee best detection of signal in the presence of nonlinear distortions. If we want achieve better performance, we should use BER (bit error ratio) or MI (mutual information) criteria for adaptation. Extremums of NMSE, BER and MI are located in different points in the space of coefficients.

Task: Propose efficient adaptation algorithm for coefficients of PBM model, based on BER or MI criteria.

Structure of adaptation, based on BER criteria:



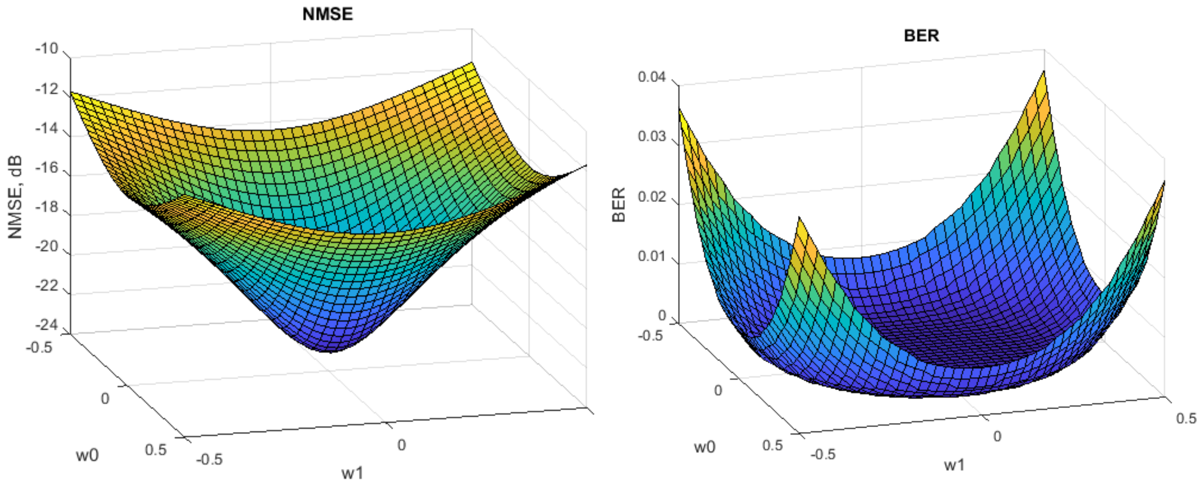
BER criteria:

$$\delta_{X|Y}(n) = \begin{cases} 0, if\ u_{X|Y}(n) = v_{X|Y}(n) \\ 1, if\ u_{X|Y}(n) \neq v_{X|Y}(n) \end{cases}; \qquad BER_{X|Y} = \frac{1}{N}\sum_{n=0}^{N-1}\delta_{X|Y}(n);$$

where n - number of bit; $u_{X|Y}(n)$ - source (desired) bits;

$v_{X|Y}(n)$ - bits from demapper after applying of PBM model

Example of NMSE and BER performance (z axis) versus coefficients (x and y axis):



MI criteria:

$$I(X;Y) \geq \frac{1}{K}\sum_{k=0}^{K-1}\log_2\frac{q_{Y|S}(y(k)|\,s(k))}{q_Y(y(k))}; \qquad q_Y(y(k)) = \sum_{i=0}^{M-1}P_S(s_i)q_{Y|S}(y(k)|\,s_i);$$

$$q_{Y|S}(y(k)|\,s(k)=c_j) = \frac{1}{\sqrt{2\pi\det(\Sigma_j)}}\exp\left(-\frac{1}{2}\begin{pmatrix} y_I(k)-\mu_{I,j} \\ y_Q(k)-\mu_{Q,j}\end{pmatrix}^T \Sigma_j^{-1}\begin{pmatrix} y_I(k)-\mu_{I,j} \\ y_Q(k)-\mu_{Q,j}\end{pmatrix}\right)$$

additional details how to calculate BER and MI can be provided by request

Call for proposals

Task 1: Constant Matrix Multiplications (was announced on previous seminar)

Task 2: Adaptation of PBM model, based on BER or MI criteria

- Propose efficient algorithms for adaptation of PBM model, based on BER or MI criteria (minimum for NMSE can be taken as initial point)

Steps:

1) Huawei Optic Algorithm Lab announces and publishes task description;
2) If you are interesting, please write to contacts below and we will provide additional information (parameters of model, type of signal, reference performance) and examples of data;
3) Huawei is collecting research proposals with basic description of algorithms and simulation results until 28 February 2022;
4) Huawei arranges internal review of proposals, select the best one and pay award up to 5000$

Contacts:

pavel.plotnikov@huawei.com
Pavel Plotnikov, PhD., Principal Engineer, Director of Optic Algorithm Lab

dolgikh.dmitry@huawei.com
Dmitry Dolgikh, PhD., Principal Engineer, Team Leader of Optic Algorithm Lab

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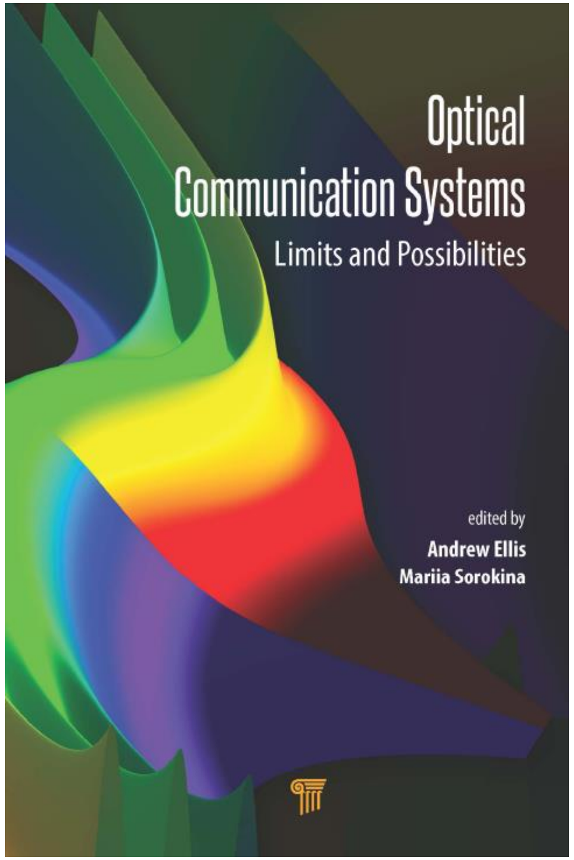
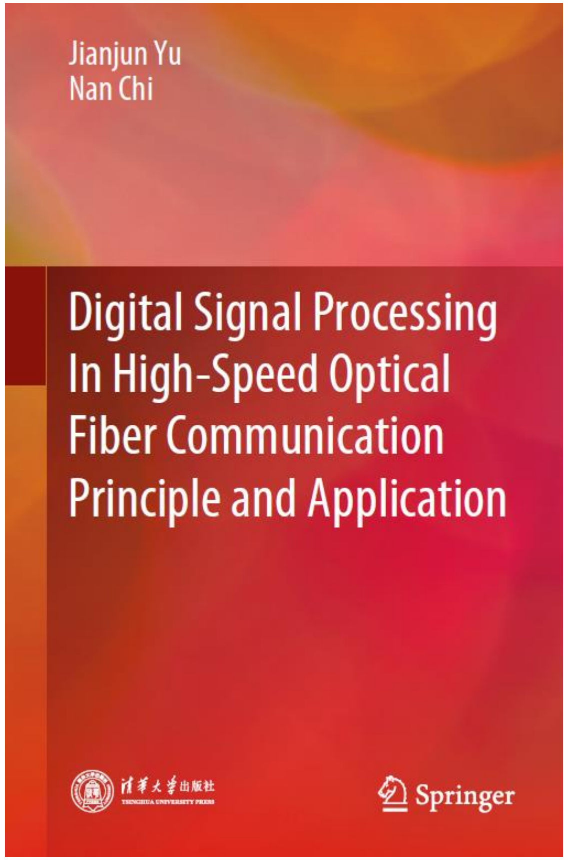
References

1. Andrew Ellis, Mariia Sorokina, Optical Communication Systems: Limits and Possibilities, Stanford Publishing Pte. Ltd. 2020

2. Amari Abdelkerim and others, A Survey on Fiber Nonlinearity Compensation for 400 Gb/s and Beyond Optical Communication Systems, 2017

3. Ali Bakhshali, Nonlinearity Compensation for Next Generation Coherent Optical Fiber Communication Systems, 2017





Thank you for attention!

Any questions?