G.W. Leibniz (1646-1716)

From the history of combinatorial analysis: from idea to scientific schools

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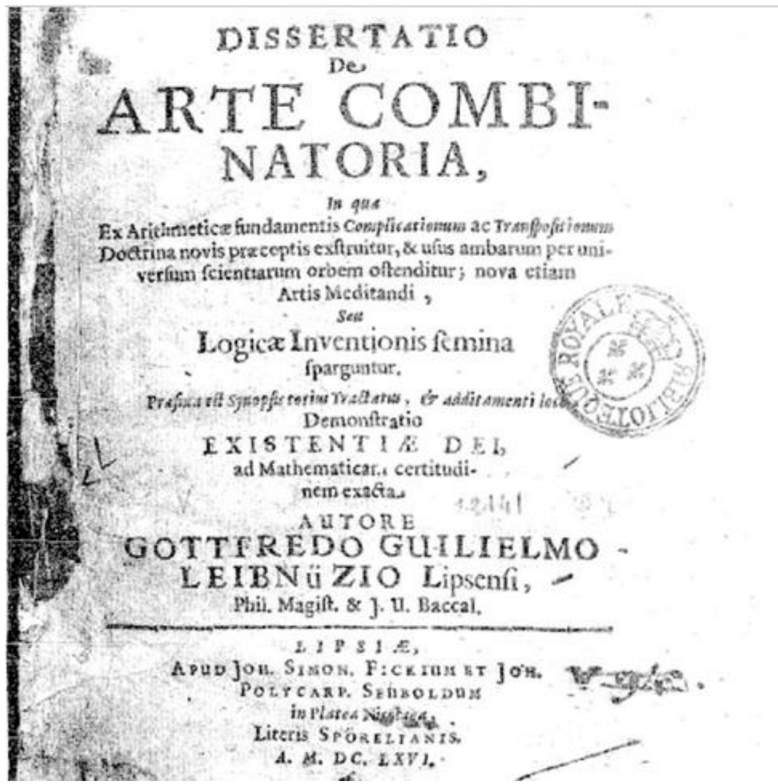
G.W. Leibniz (1646-1716)

- Leibniz has rightly been called the father of combinatorics

Dissertatio de Arte Combinatoria



DISSERTATIO
De
ARTE COMBI-
NATORIA,
In qua
Ex Arithmeticæ fundamentis Complicationum ac Transpositionum
Doctrina novis præceptis exstruitur, & usus ambarum per uni-
versum scientiarum orbem ostenditur; nova etiam
Artis Meditandi,
Seu
Logicæ Inventionis semina
sparguntur.
Præfixa est Synopsis totius Tractatus, & additamenti loco
Demonstratio
EXISTENTIÆ DEI,
ad Mathematicam certitudi-
nem exacta.
AUTORE
GOTTFREDO GUILIELMO
LEIBNüZIO Lipsensi,
Phil. Magist. & J. U. Baccal.

LIPSIÆ,
APUD JOH. SIMON. FICKIUM ET JOH.
POLYCARP. SEUBOLDUM
in Platea Nicolæa,
Literis SPÖRELIANIS.
A. M. DC. LXVI.

Ars combinatoria

Leibniz himself spoke of the "ars combinatoria", a philosophical term to wich he attached different meanings in the course of his life, but which always embraced more than what we call combinatorics today and was not even restricted to mathematical problems.

As Leibniz understood, the "combinatorial science" included not only algebra and theory of numbers, but also affected all fields of mathematics known in his time

E. Knobloch - researcher of unpublished manuscripts of Leibniz

- *Knobloch E*. Die mathematischen Studien von G.W. Leibniz zur Kombinatorik, Auf Grund fast ausschließlich handschriftlicher Aufzeichnungen dargelegt und kommentiert. Wiesbaden **1973**. **277** S.
- *Knobloch E.* Die mathematischen Studien von G.W. Leibniz zur Kombinatorik, Textband, im Anschluß an den gleichnamigen Abhandlungsband zum ersten Mal nach den Originalhandschriften herausgegeben. Wiesbaden **1976**.
- *Knobloch* E. Der Beginn der Determinantentheorie: Leibnizens nachgelassene Studien zum Determinantenkalkul. Textband. **1980. 332 S.**

Quotes from the works of E. Knobloch

- 1. Contrary to his plans Leibniz never published any further mathematical contributions to the *Ars Combinatoria* except for a short essay on the theory of probability (1690), but hundreds of mostly uncollated manuscripts among the more 7300 pages of mathematical material he left behind bear witness of the numerous studies in this field.

Quotes from the works of E. Knobloch

- 2.Disregarding those studies exclusively concerned with the theory of numbers or with algebraic problems, the relevant notes may be roughly grouped under five headings: 1. Combinatorial theory in a narrower sense (basis combinatorial operations). 2. Symmetric functions (together with the theory of equations). 3. Partitions (a part of additive theory of numbers). 4. Determinants (elimination of unknowns in systems of equations of higher degree). 5. Theory of probability and related fields (theory of games, calculation of rents and interest).

Quotes from the works of E. Knobloch

- 3. Leibniz was in possession of many results not published by other mathematicians until many decades later. These include a recursion formula for partitions of n into k parts (first published by Euler in 1751), the Stirling numbers of second kind (first published in 1730), and several special cases of the general formula for partitions that was published only in 1840 by Stern.

Analysis situs

Leibniz's idea of ~~an~~ analysis situs refers to a new understanding of "geometric algebra". It ~~She~~ played an exceptional role in the development of geometry and ~~all~~ mathematics in general. In a letter to X. Huygens dated ~~on~~ September 8, 1679, Leibniz wrote: "... I am still dissatisfied with Algebra in the sense that it does not deliver the shortest paths or the most beautiful constructions in the field of geometry. Therefore ... I believe, that we need another, purely geometric or linear, analysis directly expressing for us the situation (**situm**), as Algebra expresses the magnitude (magnitudinem). I think that I have such a means and that figures and even cars and movements could be represented by signs (en caracteres), as Algebra represents numbers and quantities; And I am sending you an etude about it, which, in my opinion, is of significant importance ..."

Analysis situs to geometry on a chessboard

- In a letter to R. de Montmort written on 17th January 1718 G. W. Leibnitz wrote : "The game called Solitaire pleases me much. I take it in reverse order. That is to say, instead of making a configuration according to the rules of the game, which is to jump to an empty place and remove the piece over which one has jumped, I thought it was better to reconstruct what had been demolished by filling an empty hole over which one has leaped."

Analysis situs to geometry on a chessboard

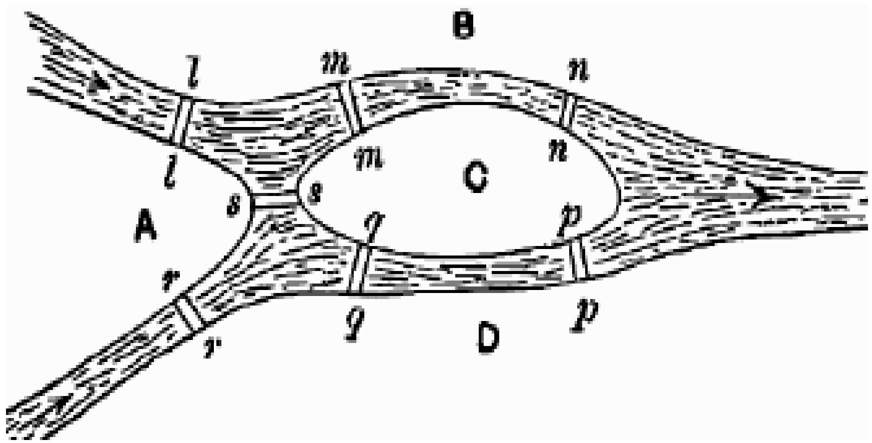
- Following Leibniz, L. Euler (1758) and Sh. A. Vandermonde (1771) were engaged in geometry on the chessboard. Euler solved the problem of bypassing all the cells of the chess board, in which the knight ~~horse~~ must visit each cell exactly once. Euler considered square, cross-shaped and rectangular boards. Vandermonde (Charles Auguste Vandermonde, 1735-1796) generalized the problem of the knight's move to the three-dimensional case

L. Euler. 1707-1783

L. Euler, Solutio Problematis ad Geometriam Situs Pertinentis

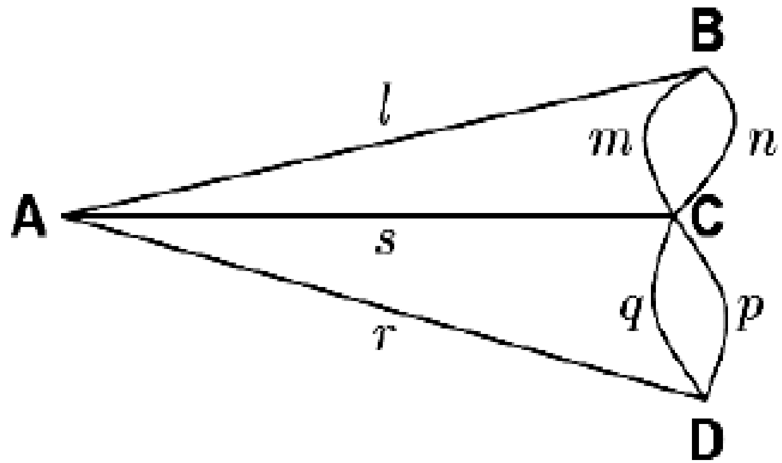
1. Praeter illam geometriae partem, quae circa quantitates versatur et omni tempore summo studio est exculta, alterius partis etiamnum admodum ignotae primus mentionem fecit LEIBNITZIUS[1]), quam *Geometriam situs* vocavit. Ista pars ab ipso in solo situ determinando situsque proprietatibus eruendis occupata esse statuitur; in quo negotio neque ad quantitates respiciendum neque calculo quantitatum utendum sit. Cuiusmodi autem problemata ad hanc situs geometriam pertineant et quali methodo in iis resolvendis uti oporteat, non satis est definitum. Quamobrem, cum nuper problematis cuiusdam mentio esset facta, quod quidem ad geometriam pertinere videbatur, at ita erat comparatum, ut neque determinationem quantitatum requireret neque solutionem calculi quantitatum ope admitteret, id ad geometriam situs referre haud dubitavi, praesertim quod in eius solutione solus situs in considerationem veniat, calculus vero nullius prorsus sit usus. Methodum ergo meam, quam ad huius generis problemata solvenda inveni, tanquam specimen Geometriae situs hic exponere constitui.

Mathematical recreations



Ball W.W. Rouse Mathematical recreations and problems of past and present times. 1892

Ball W.W.R.



A figure is described *unicursally* when the whole of it traversed in one route

L. Euler. Recherches sur une nouvelle espece de quarres magiques, 1782

- Euler takes the concept of Latin square (an $n \times n$ square containing the numbers 1 through n, each of which appears exactly once in each row and in each column of the square) and generalizes it to a Graeco-Latin square (essentially, two Latin squares laid over each other in a special way). The primary question the paper addresses is: what sizes of Graeco-Latin squares are possible to construct? Euler gives hundreds of examples of Latin and Graeco-Latin squares and takes many lengthy detours through this paper, asking questions about Latin squares in which the diagonals also satisfy the "Latin square" property. In the end, he argues (but fails to prove rigorously) that no Graeco-Latin square of size $4k+2$ can ever be constructed.

Graeco-Latin square

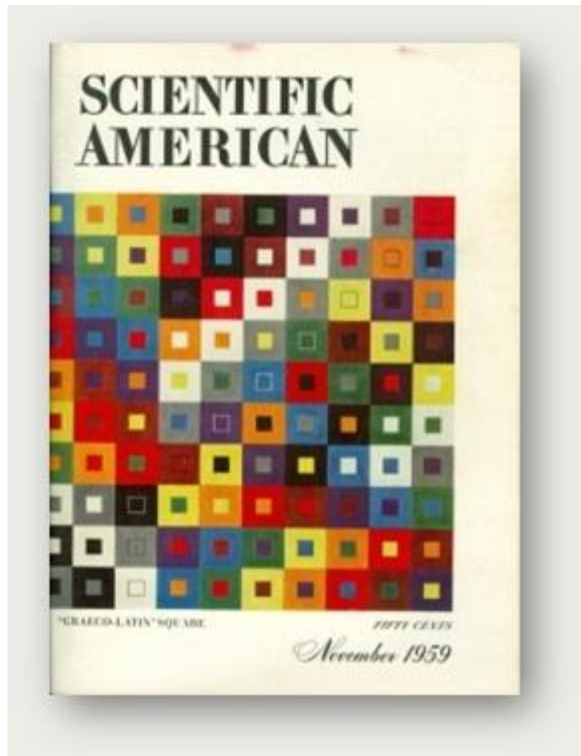
$a\alpha$	$a\beta$	$a\gamma$	$a\delta$	$a\varepsilon$	$a\xi$
$b\alpha$	$b\beta$	$b\gamma$	$b\delta$	$b\varepsilon$	$b\xi$
$c\alpha$	$c\beta$	$c\gamma$	$c\delta$	$c\varepsilon$	$c\xi$
$d\alpha$	$d\beta$	$d\gamma$	$d\delta$	$d\varepsilon$	$d\xi$
$e\alpha$	$e\beta$	$e\gamma$	$e\delta$	$e\varepsilon$	$e\xi$
$f\alpha$	$f\beta$	$f\gamma$	$f\delta$	$f\varepsilon$	$f\xi$

Refutation of the Euler's conjecture

00	47	18	76	29	93	85	34	61	52
86	11	57	28	70	39	94	45	02	63
95	80	22	67	38	71	49	56	13	04
59	96	81	33	07	48	72	60	24	15
73	69	90	82	44	17	58	01	35	26
68	74	09	91	83	55	27	12	46	30
37	08	75	19	92	84	66	23	50	41
14	25	36	40	51	62	03	77	88	99
21	32	43	54	65	06	10	89	97	78
42	53	64	05	16	20	31	98	79	87

Bose R.C., Shrikhande S.S.,Parker E.T.

Graek-Latin square of order 10 in colors



Carl Friedrich Hindenburg. 1721- 1808

- Hindenburg published a series of works on combinatorial mathematics, in particular probability, series and formulae for higher differentials. Hindenburg hoped for combinatorial operations to have the same importance as those of arithmetic, algebra and analysis but his expectations were not realised. He is recognised, however, as starting:-

... the first scientific school of combinatorial mathematics.

Sylvester J.J. 1814-1897

- I have elsewhere given the general name of Tactic to the third pure mathematical science , of which order is the proper sphere, as is number and space of the order two. Syntax and Groups are each of them only special branches of Tactic.

A. Cayley, 1821-1895

- On the notion and boundaries of algebra:

  «Although it may not be possible absolutely to separate the tactical and logistical operations; for in a series of logistical operations, there is always something that is tactical, and in many tactical operations (in the Partition of Numbers) there is something which is logistical, yet two great divisions of Algebra are Tactic and Logistic».

Geometric configurations

- *Reye K.T.* Das Problem der Configurationen, 1882.
- *Kantor S.* ber die Configurationen (3, 3) mit der Indieces 8, 9 und ihren Zusammenhang mit den Curven dritten Ordnung, 1881.
- *Steinitz E.* Konfigurationen der proiektiven Geometrie // Encyklopdie der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen.
- *Moore E.H.* Tactical memoranda I-III, 1896.
- *Levi F.* Geometrische Konfigurationen mit einer Einfrung in die Kombinatorische Flchentopologie. 1929.

E.G. Gonin. 1910-1983

Founder of the research combinatorial school in Perm

K.A. Rybnikov. 1913-2004

Founder of the Moscow research school of combinatorial analysis