Equilibrium modelling in developing energy systems

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Model CANOE — **C**apacity **A**nd **N**etwork **O**ptimization and **E**xpansion

Model parameters:

- Set of Generating Companies (GCs): $\mathcal{G}(g \in \mathcal{G})$
- Capacity types: $\mathcal{I}(i \in \mathcal{I} = \{hydro, pumped, thermal, nuclear, ...\}$
- Set of nodes: $\mathcal{J}(j \in \mathcal{J})$
- Set of seasons: $\mathcal{S}(s \in \mathcal{S} = \{winter, spring, summer, autumn\})$
- Set of seasons with peak loads: $\mathcal{S}^\sharp(s \in \mathcal{S}^\sharp)$
- Set of hours: $\mathcal{T}(t \in \mathcal{T} = \{1, 2, \ldots, 24\})$
- Set of hours with peak load: $\mathcal{T}^\sharp(t \in \mathcal{T}^\sharp)$
- Working and Holidays (weekends, days off) loads: $d^w_{jst}, d^h_{jst}, j \in \mathcal{J}, s \in \mathcal{S}, t \in \mathcal{T}$
- Numbers of working and weekend days in seasons: τ^w_s $(e.g., 65), \tau^h_s$ $(e.g., 25), s \in \mathcal{S}$
- Generating costs: $c_{gji}, g \in \mathcal{G}, j \in \mathcal{J}, i \in \mathcal{I}$
- Unit investments: $k_{gji}, g \in \mathcal{G}, j \in \mathcal{J}, i \in \mathcal{I}$
- Specific fixed costs: $b_{gji}, g \in \mathcal{G}, j \in \mathcal{J}, i \in \mathcal{I}$

Model parameters:

- Installed power capacity: $z^0_{gji}, g \in \mathcal{G}, j \in \mathcal{J}, i \in \mathcal{I}$
- Max available power capacity: $\overline{z}_{gji}, g \in \mathcal{G}, j \in \mathcal{J}, i \in \mathcal{I}$
- Installed lines capacity: $v^0_{jj'}, j \in \mathcal{J}, j' \in \mathcal{J}$
- Max available lines capacity: $\overline{v}_{jj'}, j \in \mathcal{J}, j' \in \mathcal{J}$
- Lines loses: $\delta_{jj'}, j \in \mathcal{J}, j' \in \mathcal{J}$
- Daily capacity i usage (e.g, pumped storage power plant): $\kappa_i, i \in \mathcal{I}$
- Seasonal capacity i usage (e.g., hydropower plant): $\omega_i, i \in \mathcal{I}$
- Year capacity i usage (e.g., hydropower plant): $\theta_i, i \in \mathcal{I}$
- Min capacity availability coefficients: α_{gji}
- Max capacity availability coefficients: β_{gji}

Model parameters:

- Working days generation: x^{w}_{gjist}
- Weekend days generation: x^{h}_{gjist}
- Power capacity: z_{gji}
- Flow: $y_{jj'}$
- Line capacity: $v_{jj'}$

Constraints:

- ($Generation_g$) constraints:

$$\alpha_{gji} z_{gji} \leq x^{w}_{gjist} \leq \beta_{gji} z_{gji},$$

$$\alpha_{gji} z_{gji} \leq x^{h}_{gjist} \leq \beta_{gji} z_{gji},$$

- ($Capacity_g$) constraints:

$$z^{0}_{gji} \leq z_{gji} \leq \overline{z}_{gji},$$

$$g \in \mathcal{G}, j \in \mathcal{J}, i \in \mathcal{I},$$

Constraints:

- (Day_g) generation constraints (pumped storage power plant – pspp):

$$\sum_t x^w_{gj(pspp)st} \leq \kappa_{(pspp)} z_{gj(pspp)}, \quad \sum_t x^h_{gj(pspp)st} \leq \kappa_{(pspp)} z_{gj(pspp)}$$

- ($Season_g$) generation constraints (hydropower plant – shp):

$$\sum_t (\tau^w_s \cdot x^w_{gj(shp)st} + \tau^w_s \cdot x^h_{gj(shp)st}) \leq \omega_{(shp)} z_{gji}$$

- ($Year_g$) generation constraints – yhp:

$$\sum_s \sum_t (\tau^w_s \cdot x^w_{gj(yhp)st} + \tau^w_s \cdot x^h_{gj(yhp)st}) \leq \theta_{(yhp)} z_{gji}$$

Constraints:

- (*Reserve*) constraints

$$\sum_g \sum_i z_{gji} + \sum_{j'} \mathrm{a}_{jj'} \left(1 - \delta_{jj'}\right) \widetilde{y}_{j'jst} - \sum_{j'} \mathrm{a}_{jj'} \widetilde{y}_{jj'st} \geq \mathrm{d}^{\mathrm{w}}_{jst} + \mathrm{r}_{jst},$$

$$j \in J,\ s \in S^{\sharp},\ t \in T^{\sharp},$$

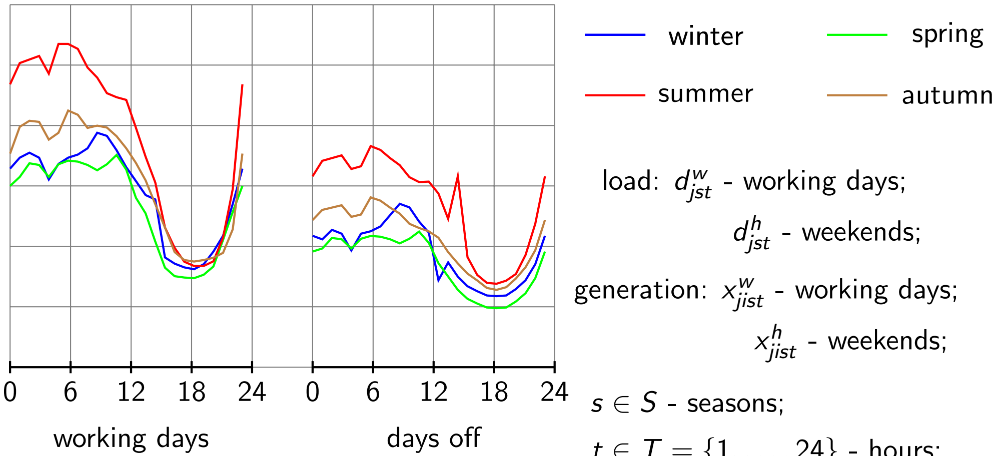
- (*Balance*) constraints

$$\sum_g \sum_i x^{\mathrm{w}}_{gjist} + \sum_{j'} \mathrm{a}_{jj'} \left(1 - \delta_{jj'}\right) y^{\mathrm{w}}_{j'jst} - \sum_{j'} \mathrm{a}_{jj'} y^{\mathrm{w}}_{jj'st} = \mathrm{d}^{\mathrm{w}}_{jst},$$

$$\sum_g \sum_i x^{\mathrm{h}}_{jist} + \sum_{j'} \mathrm{a}_{jj'} \left(1 - \delta_{jj'}\right) y^{\mathrm{h}}_{j'jst} - \sum_{j'} \mathrm{a}_{jj'} y^{\mathrm{h}}_{jj'st} = \mathrm{d}^{\mathrm{h}}_{jst},$$

$$j \in J,\ s \in S,\ t \in T,$$

Load



load: d^w_{jst} - working days;

d^h_{jst} - weekends;

generation: x^w_{jist} - working days;

x^h_{jist} - weekends;

$s \in S$ - seasons;

$t \in T = \{1, \ldots, 24\}$ - hours;

Costs=(generation)+(power)+(lines)

$$\sum_g \sum_j \sum_i \sum_s \sum_t \tau_s^{\mathrm{w}} \mathrm{c}_{gji} x_{gjist}^{w} + \sum_g \sum_j \sum_i \sum_s \sum_t \tau_s^{\mathrm{h}} \mathrm{c}_{gji} x_{gjist}^{h} +$$

$$+\mathrm{f} \sum_g \sum_j \sum_i \gamma_{gji} \left(z_{gji} - \mathrm{z}_{gji}^{0} \right) + \sum_g \sum_j \sum_i \kappa_{gji} z_{gji} +$$

$$+\mathrm{f} \sum_j \sum_{j'} \rho_{jj'} \left(v_{jj'} - \mathrm{v}_{jj'}^{0} \right) + \sum_j \sum_{j'} \mathrm{b}_{jj'} v_{jj'} \to \min_{(x,z,v,y)},$$

s.t. $(\mathit{Generation}_g)$, $(\mathit{Capacity}_g)$, (Day_g), (Season_g), (Year_g), $(\mathit{Reserve})$, $(\mathit{Balance})$.

Dual problem

$$-\mathrm{f}\sum_{j}\sum_{i}\gamma_{ji}\mathrm{z}^{0}_{ji}-\mathrm{f}\sum_{j}\sum_{j'}\rho_{jj'}\mathrm{v}^{0}_{ji}+\sum_{j}\sum_{s^\sharp}\sum_{t^\sharp}\left(\mathrm{d}^{\mathrm{w}}_{js^\sharp t^\sharp}+\mathrm{r}_{js^\sharp t^\sharp}\right)\nu_{js^\sharp t^\sharp}$$
$$+\sum_{j}\sum_{s}\sum_{t}\mathrm{d}^{\mathrm{w}}_{jst}\lambda^{\mathrm{w}}_{jst}+\sum_{j}\sum_{s}\sum_{t}\mathrm{d}^{\mathrm{h}}_{jst}\lambda^{\mathrm{h}}_{jst}+$$
$$+\sum_{j}\sum_{i}\mathrm{z}^{0}_{ji}\underline{\sigma}_{ji}-\sum_{j}\sum_{i}\overline{\mathrm{z}}_{ji}\overline{\sigma}_{ji}+\sum_{j}\sum_{j'}\mathrm{v}^{0}_{jj'}\underline{\omega}_{ji}-\sum_{j}\sum_{j'}\overline{\mathrm{v}}_{ji}\overline{\omega}_{ji}\to\max,$$

$$\mathrm{f}\gamma_{ji}+\kappa_{ji}-\sum_{s\in S^\sharp}\sum_{t\in T^\sharp_s}\nu_{jst}+\sum_{s\in S}\alpha^{\mathrm{w}}_{jis}\left(\sum_{t\in T}\underline{\mu}^{\mathrm{w}}_{jist}\right)-\sum_{s\in S}\beta^{\mathrm{w}}_{jis}\left(\sum_{t\in T}\overline{\mu}^{\mathrm{w}}_{jist}\right)+$$
$$+\sum_{s\in S}\alpha^{\mathrm{h}}_{jis}\left(\sum_{t\in T}\underline{\mu}^{\mathrm{h}}_{jist}\right)-\sum_{s\in S}\beta^{\mathrm{h}}_{jis}\left(\sum_{t\in T}\overline{\mu}^{\mathrm{h}}_{jist}\right)-\underline{\sigma}_{ji}+\overline{\sigma}_{ji}=0,\ j\in J, i\in I\setminus\{\text{HP},\text{PHPP}\}.$$

Dual problem

$$\mathrm{f}\gamma_{j(\mathrm{HP})} + \kappa_{j(\mathrm{HP})} - \sum_{s\in S^\sharp}\sum_{t\in T_s^\sharp}\nu_{jst} +$$

$$+\sum_{s\in S}\alpha^{\mathrm{w}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\underline{\mu}^{\mathrm{w}}_{j(\mathrm{HP})st}\right) - \sum_{s\in S}\beta^{\mathrm{w}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\overline{\mu}^{\mathrm{w}}_{j(\mathrm{HP})st}\right) +$$

$$+\sum_{s\in S}\alpha^{\mathrm{h}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\underline{\mu}^{\mathrm{h}}_{j(\mathrm{HP})st}\right) - \sum_{s\in S}\beta^{\mathrm{h}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\overline{\mu}^{\mathrm{h}}_{j(\mathrm{HP})st}\right) +$$

$$+\overline{\sigma}_{j(\mathrm{HP})} - \underline{\sigma}_{j(\mathrm{HP})} - \sum_{s\in S}\mathrm{H}^{S}_{j\mathrm{HP}_s}\xi_{js} = 0,\ j\in J^{S}_{\mathrm{HP}}.$$

$$\mathrm{f}\gamma_{j(\mathrm{HP})} + \kappa_{j(\mathrm{HP})} - \sum_{s\in S^\sharp}\sum_{t\in T_s^\sharp}\nu_{jst} +$$

$$+\sum_{s\in S}\alpha^{\mathrm{w}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\underline{\mu}^{\mathrm{w}}_{j(\mathrm{HP})st}\right) - \sum_{s\in S}\beta^{\mathrm{w}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\overline{\mu}^{\mathrm{w}}_{j(\mathrm{HP})st}\right) +$$

$$+\sum_{s\in S}\alpha^{\mathrm{h}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\underline{\mu}^{\mathrm{h}}_{j(\mathrm{HP})st}\right) - \sum_{s\in S}\beta^{\mathrm{h}}_{j(\mathrm{HP})s}\left(\sum_{t\in T}\overline{\mu}^{\mathrm{h}}_{j(\mathrm{HP})st}\right) +$$

$$+\overline{\sigma}_{j(\mathrm{HP})} - \underline{\sigma}_{j(\mathrm{HP})} - \mathrm{H}^{\mathrm{Y}}_{j\mathrm{HP}}\xi^{Y}_{j} = 0,\ j\in J^{Y}_{\mathrm{HP}}.$$

Dual problem

$$
\begin{aligned}
&\mathsf{f}\gamma_{j(\mathrm{PHPP})} + \kappa_{j(\mathrm{PHPP})} - \sum_{s\in S^\sharp}\sum_{t\in T_s^\sharp} \nu_{jst} + \\
&+ \sum_{s\in S} \alpha^{\mathrm{w}}_{j(\mathrm{PHPP})s} \left(\sum_{t\in T} \underline{\mu}^{\mathrm{w}}_{j(\mathrm{PHPP})st}\right) - \sum_{s\in S} \beta^{\mathrm{w}}_{j(\mathrm{PHPP})s} \left(\sum_{t\in T} \overline{\mu}^{\mathrm{w}}_{j(\mathrm{PHPP})st}\right) + \\
&+ \sum_{s\in S} \alpha^{\mathrm{h}}_{j(\mathrm{PHPP})s} \left(\sum_{t\in T} \underline{\mu}^{\mathrm{h}}_{j(\mathrm{PHPP})st}\right) - \sum_{s\in S} \beta^{\mathrm{h}}_{j(\mathrm{PHPP})s} \left(\sum_{t\in T} \overline{\mu}^{\mathrm{h}}_{j(\mathrm{PHPP})st}\right) + \\
&+ \overline{\sigma}_{j(\mathrm{HP})} - \underline{\sigma}_{j(\mathrm{PHPP})} - \sum_{s\in S} \mathrm{G}^{\mathrm{PHPP}}_{js} \left(\sum_{t\in T} \theta^{\mathrm{w}}_{jst}\right) - \sum_{s\in S} \mathrm{G}^{\mathrm{PHPP}}_{js} \left(\sum_{t\in T} \theta^{\mathrm{h}}_{jst}\right) - \\
&- \mathrm{H}^{\mathrm{PHPP}}_{j} \left(\sum_{s\in S} \varphi^{\mathrm{w}}_{js}\right) - \mathrm{H}^{\mathrm{PHPP}}_{j} \left(\sum_{s\in S} \varphi^{\mathrm{h}}_{js}\right) = 0, \; j \in J^{Y}_{\mathrm{PHPP}}.
\end{aligned}
$$

Dual problem

$$\tau_s^{\mathrm{w}} c_{ji} - \lambda_{jst}^{\mathrm{w}} - \underline{\mu}_{jist}^{\mathrm{w}} + \overline{\mu}_{jist}^{\mathrm{w}} = 0,\ j \in J,\ i \in I \setminus \{\mathrm{HP}, \mathrm{PHPP}\},\ s \in S,\ t \in T.$$

$$\tau_s^{\mathrm{w}} c_{j(\mathrm{HP})} - \lambda_{jst}^{\mathrm{w}} - \underline{\mu}_{j(\mathrm{HP})st}^{\mathrm{w}} + \overline{\mu}_{j(\mathrm{HP})st}^{\mathrm{w}} + \tau_s^{\mathrm{w}} \xi_{js} = 0,\ j \in J_{\mathrm{HP}}^{S},\ s \in S,\ t \in T.$$

$$\tau_s^{\mathrm{w}} c_{j(\mathrm{HP})} - \lambda_{jst}^{\mathrm{w}} - \underline{\mu}_{j(\mathrm{HP})st}^{\mathrm{w}} + \overline{\mu}_{j(\mathrm{HP})st}^{\mathrm{w}} + \tau_s^{\mathrm{w}} \xi_{j}^{Y} = 0,\ j \in J_{\mathrm{HP}}^{Y},\ s \in S,\ t \in T.$$

$$\tau_s^{\mathrm{w}} c_{j(\mathrm{PHPP})} - \lambda_{jst}^{\mathrm{w}} - \underline{\mu}_{j(\mathrm{PHPP})st}^{\mathrm{w}} + \overline{\mu}_{j(\mathrm{PHPP})st}^{\mathrm{w}} + \psi_{js}^{\mathrm{w}} + \varphi_{js}^{\mathrm{w}} = 0,\ j \in J,\ s \in S,\ t \in T.$$

$$\tau_s^{\mathrm{h}} c_{ji} - \lambda_{jst}^{\mathrm{h}} - \underline{\mu}_{jist}^{\mathrm{h}} + \overline{\mu}_{jist}^{\mathrm{h}} = 0,\ j \in J,\ i \in I \setminus \{\mathrm{HP}, \mathrm{PHPP}\},\ s \in S,\ t \in T.$$

$$\tau_s^{\mathrm{h}} c_{j(\mathrm{HP})} - \lambda_{jst}^{\mathrm{h}} - \underline{\mu}_{j(\mathrm{HP})st}^{\mathrm{h}} + \overline{\mu}_{j(\mathrm{HP})st}^{\mathrm{h}} + \tau_s^{\mathrm{h}} \xi_{js} = 0,\ j \in J_{\mathrm{HP}}^{S},\ s \in S,\ t \in T.$$

$$\tau_s^{\mathrm{h}} c_{j(\mathrm{HP})} - \lambda_{jst}^{\mathrm{h}} - \underline{\mu}_{j(\mathrm{HP})st}^{\mathrm{h}} + \overline{\mu}_{j(\mathrm{HP})st}^{\mathrm{h}} + \tau_s^{\mathrm{h}} \xi_{j}^{Y} = 0,\ j \in J_{\mathrm{HP}}^{Y},\ s \in S,\ t \in T.$$

$$\tau_s^{\mathrm{h}} c_{j(\mathrm{PHPP})} - \lambda_{jst}^{\mathrm{h}} - \underline{\mu}_{j(\mathrm{PHPP})st}^{\mathrm{h}} + \overline{\mu}_{j2st}^{\mathrm{h}} + \psi_{js}^{\mathrm{h}} + \varphi_{js}^{\mathrm{h}} = 0,\ j \in J,\ s \in S,\ t \in T.$$

$$\frac{1}{2} f \rho_{jj'} + \frac{1}{2} b_{jj'} - \sum_{s \in S^\sharp} \sum_{t \in T_s^\sharp} \widetilde{\eta}_{jj'st} - \sum_{s \in S} \sum_{t \in T} \eta_{jj'st}^{\mathrm{w}} -$$

$$- \sum_{s \in S} \sum_{t \in T} \eta_{jj'st}^{\mathrm{h}} + \overline{\omega}_{jj'} - \underline{\omega}_{jj'} + \pi_{jj'} - \pi_{j'j} = 0,\ j \in J,\ j' \in J.$$

Dual problem

$$\mathsf{a}_{jj'}\left(\nu_{jst} - (1-\delta_{jj'})\nu_{j'st}\right) + \widetilde{\eta}_{jj'st} \geqslant 0,\ j \in J,\ j' \in J,\ s \in S^{\sharp},\ t \in T^{\sharp}_{s}.$$

$$\mathsf{a}_{jj'}\left(\lambda^{\mathrm{w}}_{jst} - (1-\delta_{jj'})\lambda^{\mathrm{w}}_{j'st}\right) + \eta^{\mathrm{w}}_{jj'st} \geqslant 0,\ j \in J,\ j' \in J,\ s \in S,\ t \in T.$$

$$\mathsf{a}_{jj'}\left(\lambda^{\mathrm{h}}_{jst} - (1-\delta_{jj'})\lambda^{\mathrm{h}}_{j'st}\right) + \eta^{\mathrm{h}}_{jj'st} \geqslant 0,\ j \in J,\ j' \in J,\ s \in S,\ t \in T.$$

$$\lambda^{\mathrm{w}}_{jst} + \theta^{\mathrm{w}}_{jst} - \mathsf{q}_j\psi^{\mathrm{w}}_{js} \geqslant 0,\ j \in J,\ s \in S,\ t \in T.$$

$$\lambda^{\mathrm{h}}_{jst} + \theta^{\mathrm{h}}_{jst} - \mathsf{q}_j\psi^{\mathrm{h}}_{js} \geqslant 0,\ j \in J,\ s \in S,\ t \in T.$$

$$\nu_{js^{\sharp}t^{\sharp}} \geqslant 0,\ \underline{\mu}^{\mathrm{w}}_{jist} \geqslant 0,\ \overline{\mu}^{\mathrm{w}}_{jist} \geqslant 0,\ \underline{\mu}^{\mathrm{h}}_{jist} \geqslant 0,\ \overline{\mu}^{\mathrm{h}}_{jist} \geqslant 0,\ \underline{\sigma}_{ji} \geqslant 0,\ \overline{\sigma}_{ji} \geqslant 0,$$

$$\widetilde{\eta}_{jj'st} \geqslant 0,\ \eta^{\mathrm{w}}_{jj'st} \geqslant 0,\ \eta^{\mathrm{h}}_{jj'st} \geqslant 0,\ \overline{\omega}_{jj'} \geqslant 0,\ \underline{\omega}_{jj'} \geqslant 0,\ \xi_{js} \geqslant 0,\ \xi^{Y}_{j} \geqslant 0,$$

$$\theta^{\mathrm{w}}_{jst} \geqslant 0,\ \theta^{\mathrm{h}}_{jst} \geqslant 0,\ \psi^{\mathrm{w}}_{js} \geqslant 0,\ \psi^{\mathrm{h}}_{js} \geqslant 0,\ \varphi^{\mathrm{w}}_{js} \geqslant 0,\ \varphi^{\mathrm{h}}_{js} \geqslant 0,$$

$$s^{\sharp} \in S^{\sharp},\ t^{\sharp} \in T^{\sharp}_{s},\ j \in J,\ j' \in J,\ i \in I,\ s \in S,\ t \in T.$$

$$\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{w}}_{jst}x^{\mathrm{w}}_{jist}+\sum_{s\in S}\sum_{t\in T}\lambda^{\mathrm{h}}_{jst}x^{\mathrm{h}}_{jist}+\sum_{s\in S^{\sharp}}\sum_{t\in T^{\sharp}_{s}}\nu_{jst}z_{ji}=$$

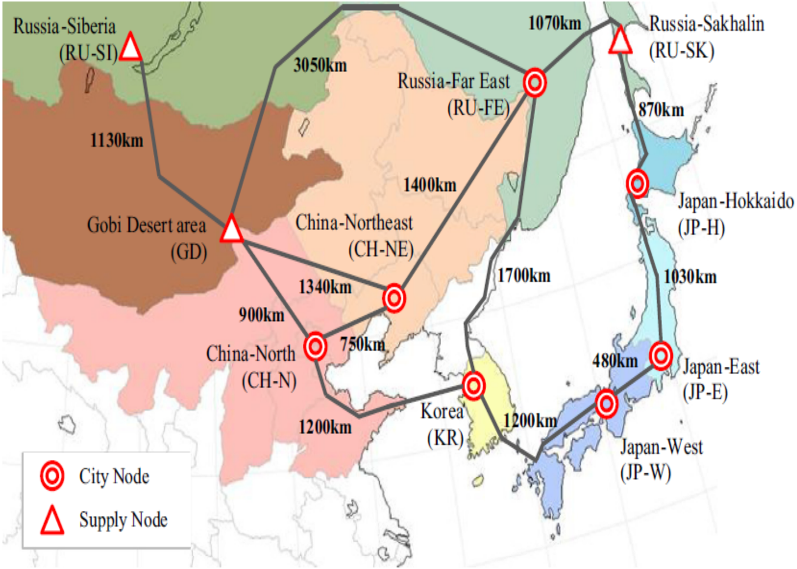
$$=\sum_{s\in S}\sum_{t\in T}\tau^{\mathrm{w}}_{s}\mathrm{c}_{ji}x^{\mathrm{w}}_{jist}+\sum_{s\in S}\sum_{t\in T}\tau^{\mathrm{h}}_{s}\mathrm{c}_{ji}x^{\mathrm{h}}_{jist}+$$

$$+\mathrm{f}\gamma_{ji}(z_{ji}-\mathrm{z}^{0}_{ji})+\kappa_{ji}z_{ji}+$$

$$+\mathrm{f}\gamma_{ji}\mathrm{z}^{0}_{ji}+\overline{\sigma}_{ji}z_{ji}-\underline{\sigma}_{ji}z_{ji}+$$

$$j\in J,\ i\in I.$$

$\underline{\text{PCI}:\overline{\sigma}_{ji}>0,\underline{\sigma}_{ji}=0;}$ $\underline{\text{PCII}:\overline{\sigma}_{ji}=0,\underline{\sigma}_{ji}=0;}$ $\underline{\text{PCIII}:\overline{\sigma}_{ji}=0,\underline{\sigma}_{ji}>0;}$



Russia-Siberia
(RU-SI)
3050km
1070km
Russia-Sakhalin
(RU-SK)
Russia-Far East
(RU-FE)
870km
1130km
1400km
Japan-Hokkaido
(JP-H)
Gobi Desert area
(GD)
China-Northeast
(CH-NE)
1700km
1030km
1340km
900km
750km
China-North
(CH-N)
480km
Japan-East
(JP-E)
Korea
(KR)
1200km
1200km
Japan-West
(JP-W)
City Node
Supply Node

Dual estimates (prices) at nodes of the model, cent/kWh

Country/ region		Winter			Spring			Summer			Autumn			Year
		max	min	aver.	max	min	aver.	max	min	aver.	max	min	aver.	aver.
Russia	Siberia	21.9	2.6	4.8	13.7	2.6	3.0	80.1	2.2	5.1	4.7	2.6	2.7	3.8
	East	21.4	3.8	9.2	13.7	2.6	4.2	84.5	3.3	9.2	13.2	3.8	4.9	6.7
	Sakhalin	21.6	3.6	12.5	13.4	2.5	6.6	21.9	3.4	8.1	13.7	3.6	8.7	9.1
Mongolia		23,0	2.4	4.8	14.4	2.4	3.0	84.5	2.1	8.5	4.5	2.4	2.6	4.4
China	North	21.7	2.2	4.2	14.8	2.0	2.9	120.0	2.0	10.5	2.5	2.0	2.3	5.1
	Northeast	22.4	2.3	5.1	14.4	2.3	3.3	116.4	2.1	10.5	13.4	2.1	3.7	5.6
DPRK		22.0	2.4	10.1	14.1	2.4	4.0	14.1	3.0	7.0	13.7	3.6	4.9	6.7
RoK		21.8	2.4	10.4	14.0	2.4	5.2	14.0	3.0	8.3	13.8	3.7	7.9	7.9
Japan		22.6	3.8	14.5	14.0	2.5	7.0	23.0	3.8	13.7	14.3	3.8	9.8	11.4

Power capacity ordering

1.	Korea	Thermal gas	-16451947734.4
2.	Japan	Thermal1 oil	-13085531842.1
3.	Japan	Thermal2 oil	-5009759517.2
	...	...	...
6.	Siberia PES	Thermal coal	-1210690958.4
	...	...	...
	Siberia PES	HP1	9302267741.0
	...	...	...
48.	Japan	Nuclear	23623204195.4
49.	North China	Thermal2 coal	24806784000.0
50.	North China	Thermal 3 coal	29410180388.6

Nodal prices

$$\frac{\lambda^{\mathrm{w}}_{jst}}{\tau^{\mathrm{w}}_{s}} = \mathsf{c}_{ji} - \frac{\underline{\mu}^{\mathrm{w}}_{jist}}{\tau^{\mathrm{w}}_{s}} + \frac{\overline{\mu}^{\mathrm{w}}_{jist}}{\tau^{\mathrm{w}}_{s}},\ j \in J,\ i \in I \setminus \{\text{HP}, \text{PHPP}\},\ s \in S,\ t \in T.$$

$$\frac{\lambda^{\mathrm{w}}_{jst}}{\tau^{\mathrm{w}}_{s}} = \frac{\underline{\mu}^{\mathrm{w}}_{j(\text{HP})st}}{\tau^{\mathrm{w}}_{s}} - \frac{\overline{\mu}^{\mathrm{w}}_{j(\text{HP})st}}{\tau^{\mathrm{w}}_{s}} + \xi_{js},\ j \in J^{S}_{\text{HP}},\ s \in S,\ t \in T.$$

ξ_{js} — dual variable $\sim$ water rent.

Line compensation

$$j \to j' : \; \lambda^{\mathrm{w}}_{jst} - \left(1 - \delta_{jj'}\right) \lambda^{\mathrm{w}}_{j'st} + \eta^{\mathrm{w}}_{jj'st} = 0$$
$$\left(1 - \delta_{jj'}\right) \lambda^{\mathrm{w}}_{j'st} = \lambda^{\mathrm{w}}_{jst} + \eta^{\mathrm{w}}_{jj'st}$$

Buyer compensates line and loses,
$\eta^{\mathrm{w}}_{jj'st}$ — unit profit of line $j \to j'$.

Line capacity expansion

$$
\begin{aligned}
\mathrm{f}\rho_{jj'} v_{jj'} + \mathrm{b}_{jj'} v_{jj'} = & \sum_{s\in S^\sharp}\sum_{t\in T_s^\sharp} \widetilde{\eta}_{jj'st}\widetilde{y}_{jj'st} + \sum_{s\in S}\sum_{t\in T} \eta^{\mathrm{w}}_{jj'st} y^{\mathrm{w}}_{jj'st} + \sum_{s\in S}\sum_{t\in T} \eta^{\mathrm{h}}_{jj'st} y^{\mathrm{h}}_{jj'st} + \\
& + \sum_{s\in S^\sharp}\sum_{t\in T_s^\sharp} \widetilde{\eta}_{j'jst}\widetilde{y}_{j'jst} + \sum_{s\in S}\sum_{t\in T} \eta^{\mathrm{w}}_{j'jst} y^{\mathrm{w}}_{j'jst} + \sum_{s\in S}\sum_{t\in T} \eta^{\mathrm{h}}_{j'jst} y^{\mathrm{h}}_{j'jst} + \\
& + \overline{\omega}_{jj'} v_{jj'} - \underline{\omega}_{jj'} v_{jj'} + \overline{\omega}_{j'j} v_{j'j} - \underline{\omega}_{j'j} v_{j'j}.
\end{aligned}
$$

e.g.: new line Siberia PES — Mongolia,
payment of Siberia PES — 3%
payment of Mongolia — 97%

Object

Power grid interconnections in Northeast Asia:
Russia, **C**hina, **M**ongolia, **N**orth Korea, **S**outh Korea, **J**apan.
Countries exchange the power in the case of power shortage.
Target year 2030

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Target year 2030

Cooperative games (basic definitions)

$N = \{1, 2, \ldots, n\}$	is the set of players
$K \subseteq N$	is a coalition
N	is the grand coalition

Characteristic function $v\colon \{K \mid K \subseteq N\} \to \mathbb{R}$ is a function that assigns to every coalition $K \subseteq N$ an attainable payoff $v(K)$ such that $v(\varnothing) = 0$. Normally, v satisfies superadditivity condition

$$v(K) + v(T) \leqslant v(K \cup T) \quad \text{for every } K \subseteq N, T \subseteq N, K \cap T = \varnothing.$$

Cooperative game (in characteristic function form) is a pair (N, v).

Imputation is a vector $x \in \mathbb{R}^n$ satisfying the efficiency condition

$$\sum_{i \in N} x_i = v(N),$$

and the individual rationality condition

$$x_i \geqslant v(\{i\}) \quad \text{for every } i \in N.$$

Cooperative games (basic definitions)

An imputation x is an element of *the Core* if

$$\sum_{i \in K} x_i \geqslant v(K) \quad \text{for every } K \subset N.$$

The Shapley value is an imputation x such that

$$x_i = \sum_{K:\, i \in K \subseteq N} \frac{(|K|-1)!(n-|K|)!}{n!}\,(v(K) - v(K \setminus \{i\}))\,, \quad i \in N.$$

From ORIRES to Cooperative game

Pessimistic (maximin) view: a coalition K's payoff $v(K)$ is the maximal benefit guaranteed to K (the case of the worst behaviour of $N \setminus K$).

Introducing new generation capacities is much more expensive than transmitting power from another country:
New generation capacity 800–8500 \$/kW **vs.** New line 180–950 \$/kW

$$\Downarrow$$

The worst condition for any coalition is isolated work.

$v(K)$ is the minimal costs of K in the case of isolated work.

Number of coalitions: $2^6 - 1 = 63$, incl. N.

Various partitions $(K, N \setminus K)$*:* 32, incl. $(N, \varnothing)$.

To define characteristic function v one need to solve 32 LP problems with 42000 variables and 56000 constraints.

The Core

is a nonempty set with infinite number of elements (in our case)

Chebyshev center of the Core

	Imputation (costs)		*Isolated*	*Integration effect*	
	mill. doll.	%	mill. doll.	%	mill. doll.
Russia	7353	2.21	7591	−3.14	−239
Mongolia	670	0.20	909	−26.26	−239
China	135301	40.63	147757	−8.43	−12455
North Korea	0	0	5047	−100.00	−5047
South Korea	54239	16.29	57153	−5.10	−2915
Japan	135438	40.67	138441	−2.17	−3004
Total	333001	100.00	356899	−6.70	−23898

Another element of the Core

	Imputation (costs)		*Isolated*	*Integration effect*	
	mill. doll.	%	mill. doll.	%	mill. doll.
Russia	6853	2.06	7591	−9.72	−738
Mongolia	837	0.25	909	−7.95	−72
China	135801	40.78	147757	−8.09	−11956
North Korea	4662	1.40	5047	−7.62	−385
South Korea	52574	15.79	57153	−8.01	−4580
Japan	132274	39.72	138441	−4.45	−6167
Total	333001	100.00	356899	−6.70	−23898

Shapley value
always exists

Shapley value turns out to be an element of the Core.

Shapley value

	Imputation (costs)		*Isolated*	*Integration effect*	
	mill. doll.	%	mill. doll.	%	mill. doll.
Russia	3867	1.16	7591	−49.07	−3725
Mongolia	350	0.11	909	−61.46	−559
China	141328	42.44	147757	−4.35	−6429
North Korea	527	0.16	5047	−89.56	−4520
South Korea	52578	15.79	57153	−8.01	−4575
Japan	134351	40.35	138441	−2.95	−4090
Total	333001	100.00	356899	−6.70	−23898

Effects of cooperation

Basic effects

	Investments, bill. doll.			*New gen. capacities, GW*
	Plants	Lines	***Total***	
Isolated	404.5	—	**404.5**	368.8
Cooperation	290.7	40.6	**331.3**	301.7

Power exchange between countries, TWh per year

	Export	*Import*	*Net export*
Russia	54.6	37.2	17.4
Mongolia	13.8	21.0	−7.2
China	162.7	21.0	141.7
North Korea	89.9	117.9	−28.0
South Korea	87.2	87.3	−0.1
Japan	0	112.1	−112.1

Perfect market model (price p — const.)
Equations:

- GCs $g \in \mathcal{G}$ profit:

$$\Pi_g = p \cdot \sum_j \sum_i \sum_s \left(\tau_s^w \cdot \sum_t x_{gjist} + \tau_s^h \cdot \sum_t y_{gjist} \right) -$$

$$- \sum_j \sum_i \sum_s \left(\tau_s^w \cdot \sum_t c_{gji} x_{gjist} + \tau_s^h \cdot \sum_t c_{gji} y_{gjist} \right) -$$

$$-f \cdot \sum_j \sum_i k_{gji} (z_{gji} - z_{gji}^0) - \sum_j \sum_i b_{gji} z_{gji}$$

Perfect market model (price p – const.)

$$p$$

$$\Downarrow$$

(Disconnected) System of LP_g ($g \in \mathcal{G}$):

$$\Pi_g \to \max,$$

s.t. $(Generation_g)$, $(Capacity_g)$, (Day_g), $(Season_g)$, $(Year_g)$.

$$\Downarrow$$

solution: $x^*_{gjist}(p), y^*_{gjist}(p), z^*_{gji}(p)$

solution: $x^*_{gjist}(p), y^*_{gjist}(p), z^*_{gji}(p)$

$$\Downarrow$$

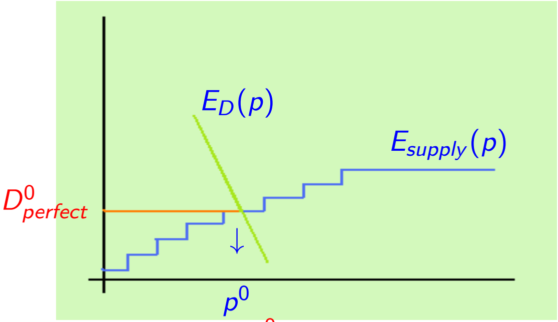
Year electricity supply:

$$E_{supply}(p) = \sum_g \sum_j \sum_i \sum_s \left(\tau_s^w \cdot \sum_t x^*_{gjist}(p) + \tau_s^h \cdot \sum_t y^*_{gjist}(p) \right)$$

Capacity supply:

$$C_{supply}(p) = \sum_g \sum_j \sum_i z^*_{gji}(p)$$

Constructing the demand function: $E_D(p) = d - a \cdot p$



- Given demand $D^0_{perfect}$
- Calculate p^0 : $E_{supply}(p^0) = D^0_{perfect}$
- Given elasticity $\varepsilon^0 = \varepsilon(p^0) = \frac{E'(p^0)}{E(p^0)} p^0$
- Calculate coefficients: $d = (1 + \varepsilon) D^0_{perfect}$, $a = \varepsilon \frac{D^0_{perfect}}{p^0}$

From perfect market to oligopoly

- *Price p is now variable*
- Step I. *Model Perfect is substituted by model Cournot*
- Step II. *Network-Load distribution model*

Cournot model

System of connected linear programming problems:

$$\Pi_g \to \max,$$

s.t. $(Generation_g)$, $(Capacity_g)$, (Day_g), $(Season_g)$, $(Year_g)$,

$$g \in \mathcal{G}$$

$+$ (connecting) equations:

$$E_T = d - a \cdot p,$$

$$E_T = \sum_{g \in \mathcal{G}} E_g.$$

Cournot model is a potential game

$$\Downarrow$$

from system of connected ! problems to one problem

$$-\frac{1}{2a}E^T QE + d\sum_g E_g - (\text{costs}) \to \max,$$

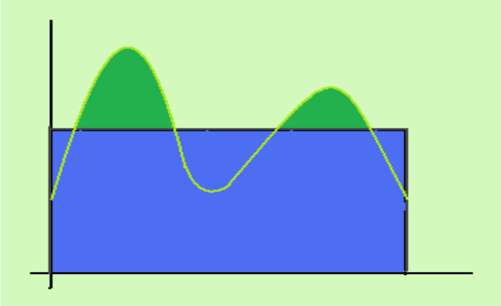
subject to $(Generation_g)$, $(Capacity_g)$, (Day_g), $(Season_g)$, $(Year_g)$,$(Price_g)$,$g \in \mathcal{G}$,

$$E = (E_1, \dots, E_{|\mathcal{G}|}),$$

$$Q = \begin{pmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{pmatrix} \succ 0.$$

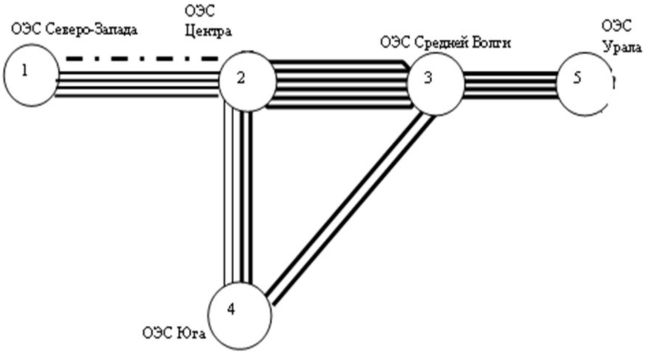
Model Cournot does not consider

- Load curves



- Network constraints

Scheme of central Russia



- 50 GCs (UGC1,...,UGK6,TGC1,...,TGC16, Rosatom, Rusgidro, InterRao,...)
- 91 power plants
- 7 capacity types: coal, gas ...
- 4 seasons, 24 hours
- Dimension $\sim$ 200 000

- Models Cournot and Distribution in GAMS
- Time $\sim$ 5 minutes.

- Price $p^{perfect}$=0.068$
- Price $p^{cournot}$=0.105$
- Perfect electricity supply 422.6 TW
- Oligopoly electricity supply 374.2 TW
- Perfect capacity expansion 28.3 GW
- Oligopoly capacity expansion 15 GW

In progress.

1. Inverse demand function for each node.
2. Network capacity constraints.

Thank you for attention!