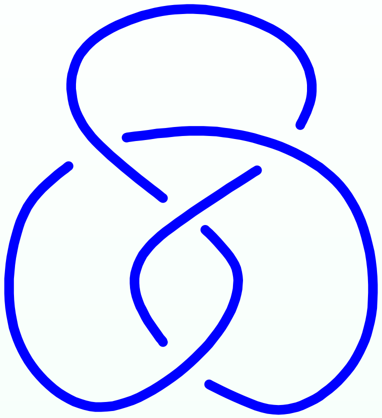
Relative Khovanov Homology

Oleg Viro

June 11, 2009

Introduction

What knot theory is about

Knots: 

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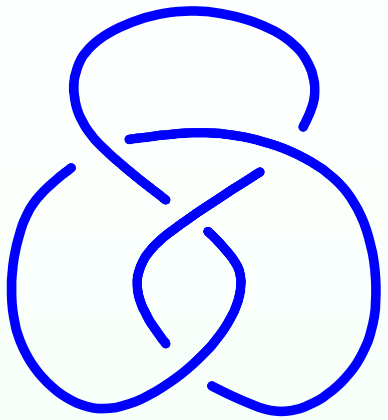
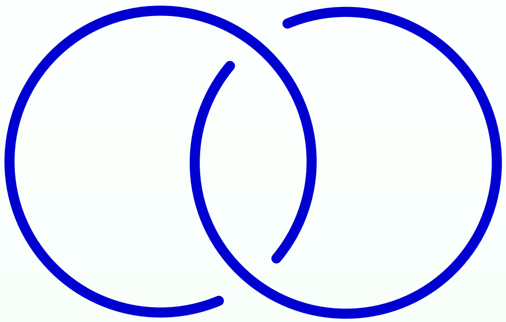
Knots:  and links 

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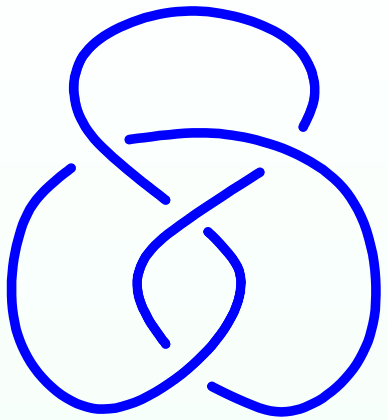
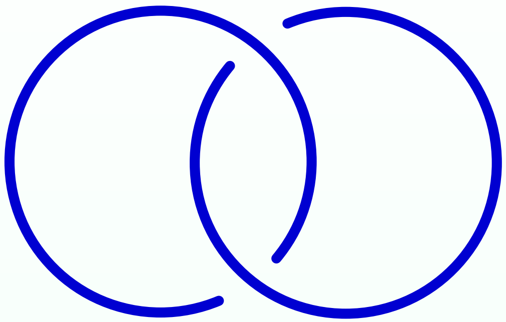
Knots:  and links 

Link diagrams considered up to moves.

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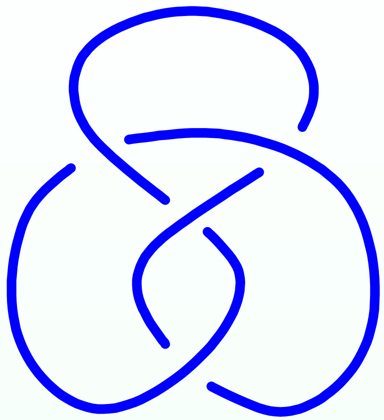
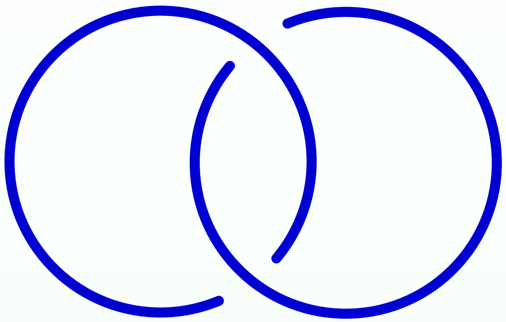
Knots:  and links 

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What happens to a link diagram, when the link moves?

What knot theory is about

Link diagram moves, too.

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Reidemeister moves:

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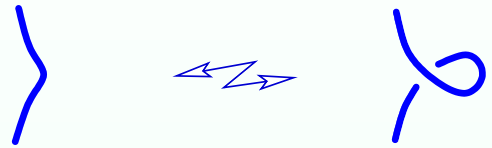
(R1): 

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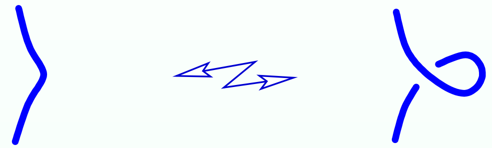
(R1): 

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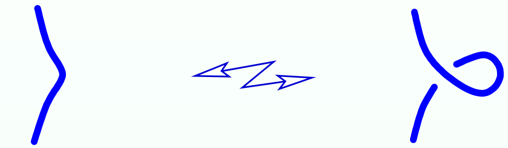
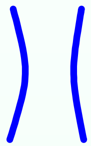
(R1): 

(R2): 

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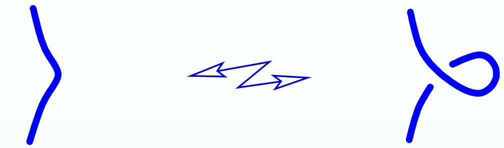
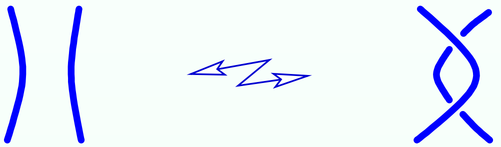
(R1): 

(R2): 

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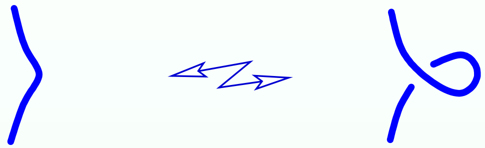
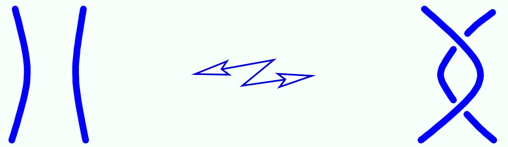
(R1): 

(R2): 

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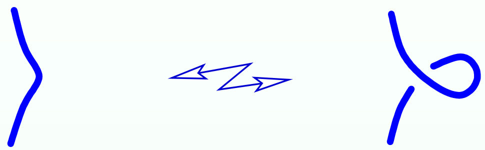
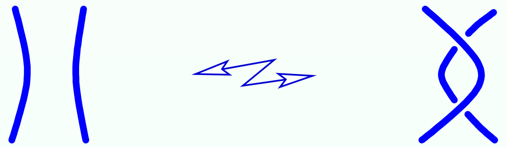
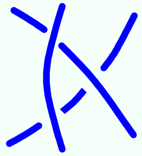
(R1): 

(R2): 

(R3): 

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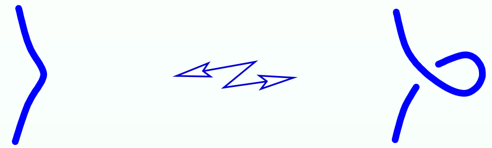
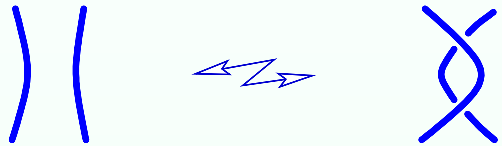
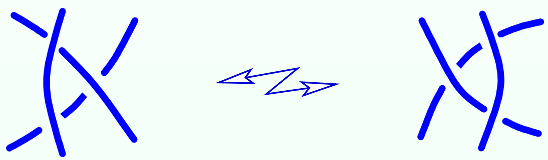
(R1): 

(R2): 

(R3): 

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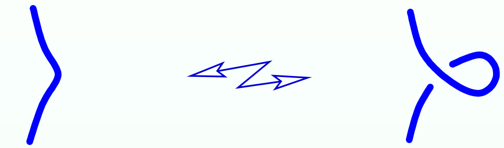
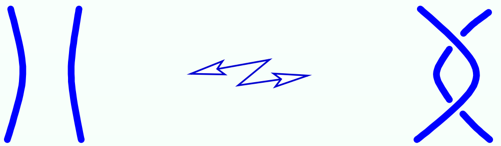
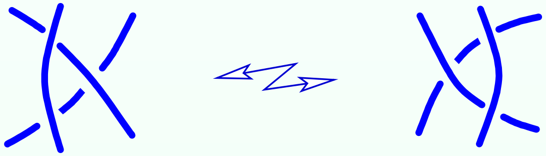
(R1): 

(R2): 

(R3): 

To speak about links, one needs terms invariant under the moves.

Types of invariants

Geometric and algebraic invariants.

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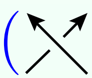
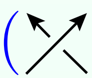
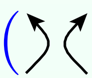
The Conway polynomial $\nabla(L)$ defined by two axioms:
normalization $\nabla(\bigcirc) = 1$,
skein relation $\nabla($  $) - \nabla($  $) + z\nabla($  $) = 0$.

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The Conway polynomial $\nabla(L)$ defined by two axioms:
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skein relation $\nabla(L_+) - \nabla(L_-) + z\nabla(L_0) = 0$.
the Jones polynomial $V(L)$ defined by two axioms:
normalization $V(\bigcirc) = 1$,
skein relation $t^{-1}V(L_+) - tV(L_-) + (t^{-1/2} - t^{1/2})V(L_0)$.

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Link homology: upgrade of quantum invariant.
Homology of a link diagram. Bi- or tri-graded.
The Euler characteristic is a quantum polynomial invariant.

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Khovanov homology

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$$\langle \text{Link diagram} \rangle \in \mathbb{Z}[A, A^{-1}]$$

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(a Laurent polynomial in A with integer coefficients).

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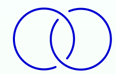
$\langle \text{Hopf link} \rangle =$ $\langle$  $\rangle =$

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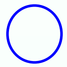
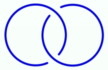
⟨unknot⟩ $=$ ⟨  ⟩ $= -A^2 - A^{-2}$

⟨Hopf link⟩ $=$ ⟨  ⟩ $= A^6 + A^2 + A^{-2} + A^{-6}$

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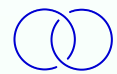
$\langle$Hopf link$\rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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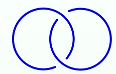
$\langle \text{Hopf link} \rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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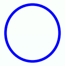
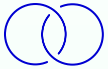
$\langle \text{unknot} \rangle =$ $\langle$  $\rangle = -A^2 - A^{-2}$

$\langle \text{Hopf link} \rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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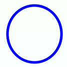
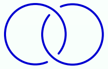
$\langle \text{unknot} \rangle =$ $\langle$  $\rangle = -A^2 - A^{-2}$

$\langle \text{Hopf link} \rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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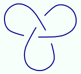
$\langle \text{trefoil} \rangle =$ $\langle$  $\rangle =$

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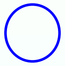
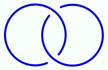
⟨unknot⟩ = ⟨⟩ $= -A^2 - A^{-2}$

⟨Hopf link⟩ = ⟨⟩ $= A^6 + A^2 + A^{-2} + A^{-6}$

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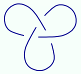
⟨trefoil⟩ = ⟨⟩ $= A^7 + A^3 + A^{-1} - A^{-9}$

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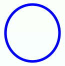
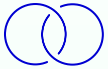
$\langle \text{unknot} \rangle =$ $\langle$  $\rangle = -A^2 - A^{-2}$

$\langle \text{Hopf link} \rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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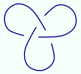
$\langle \text{trefoil} \rangle =$ $\langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle \text{figure-eight knot} \rangle =$

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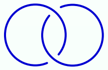
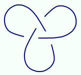
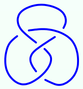
$\langle$Hopf link$\rangle$ = $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

$\langle$empty link$\rangle$ = $\langle\ \rangle = 1$

$\langle$trefoil$\rangle$ = $\langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle$figure-eight knot$\rangle$ = $\langle$  $\rangle =$

Kauffman bracket

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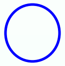
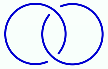
$\langle$unknot$\rangle =$ $\langle$  $\rangle = -A^2 - A^{-2}$

$\langle$Hopf link$\rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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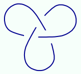
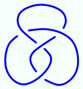
$\langle$trefoil$\rangle =$ $\langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle$figure-eight knot$\rangle =$ $\langle$  $\rangle = -A^{10} - A^{-10}$

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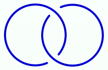
$\langle \text{Hopf link} \rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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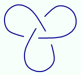
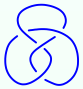
$\langle \text{trefoil} \rangle =$ $\langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle \text{figure-eight knot} \rangle =$ $\langle$  $\rangle = -A^{10} - A^{-10}$

Kauffman bracket is defined by the following properties:

Kauffman bracket

$$\langle \text{Link diagram} \rangle \in \mathbb{Z}[A, A^{-1}]$$

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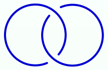
$\langle \text{Hopf link} \rangle = \quad \langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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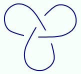
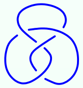
$\langle \text{trefoil} \rangle = \quad \langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle \text{figure-eight knot} \rangle = \quad \langle$  $\rangle = -A^{10} - A^{-10}$

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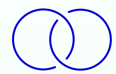
$\langle$Hopf link$\rangle$ = $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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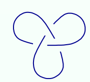
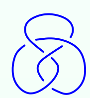
$\langle$trefoil$\rangle$ = $\langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle$figure-eight knot$\rangle$ = $\langle$  $\rangle = -A^{10} - A^{-10}$

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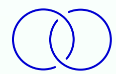
$\langle \text{Hopf link} \rangle =$ $\langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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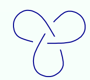
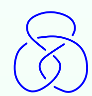
$\langle \text{trefoil} \rangle =$ $\langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle \text{figure-eight knot} \rangle =$ $\langle$  $\rangle = -A^{10} - A^{-10}$

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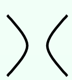
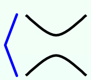
3. $\langle$  $\rangle = A \langle$  $\rangle + A^{-1} \langle$  $\rangle$ (*Kauffman Skein Relation*) .

Kauffman bracket

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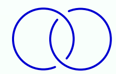
$\langle \text{Hopf link} \rangle = \langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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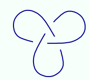
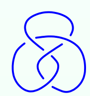
$\langle \text{trefoil} \rangle = \langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle \text{figure-eight knot} \rangle = \langle$  $\rangle = -A^{10} - A^{-10}$

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3. $\langle \times \rangle = A\langle)(\rangle + A^{-1}\langle \asymp \rangle$ (*Kauffman Skein Relation*) .

Uniqueness is obvious.

Kauffman bracket

$$\langle \text{Link diagram} \rangle \in \mathbb{Z}[A, A^{-1}]$$

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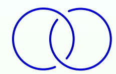
$\langle \text{Hopf link} \rangle = \langle$  $\rangle = A^6 + A^2 + A^{-2} + A^{-6}$

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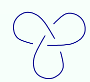
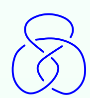
$\langle \text{trefoil} \rangle = \langle$  $\rangle = A^7 + A^3 + A^{-1} - A^{-9}$

$\langle \text{figure-eight knot} \rangle = \langle$  $\rangle = -A^{10} - A^{-10}$

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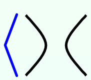
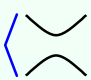
3. $\langle$  $\rangle = A\langle$  $\rangle + A^{-1}\langle$  $\rangle$ (*Kauffman Skein Relation*) .

Uniqueness is obvious.

Invariant under R2 and R3, under R1 multiplies by $-A^{\pm 3}$.

Kauffman state sum

A *state* of diagram is a distribution of *markers* over all crossings.

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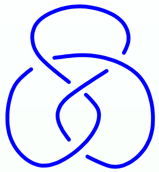
Knot diagram: 

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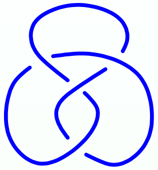
Knot diagram:  and its states:

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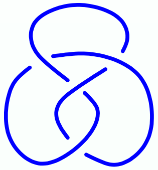
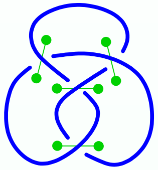
Knot diagram:  and its states: ,

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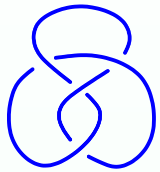
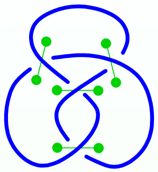
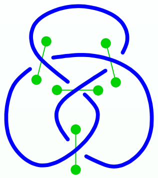
Knot diagram:  and its states: , ,

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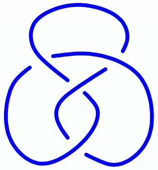
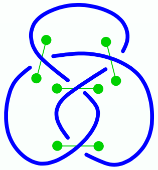
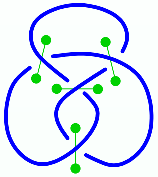
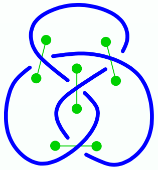
Knot diagram:  and its states: , , ,

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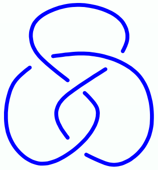
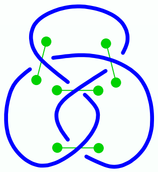
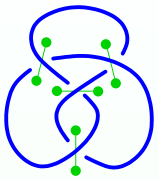
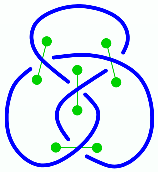
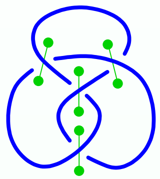
Knot diagram:  and its states: , , , ,

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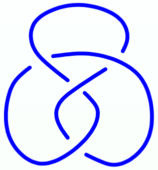
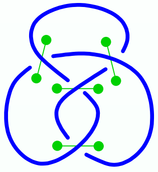
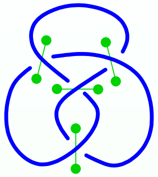
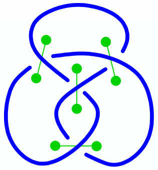
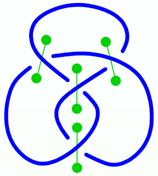
Knot diagram:  and its states: , , , , ...

Kauffman state sum

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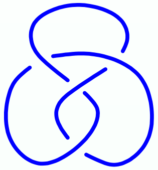
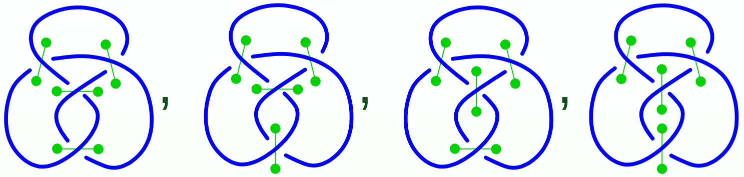
Knot diagram:  and its states: , . . .

Totally 2^c states, where c is the number of crossings.

Kauffman state sum

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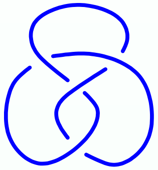
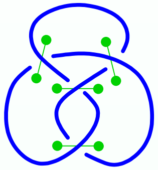
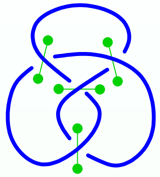
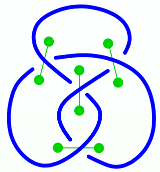
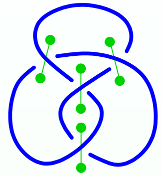
Knot diagram:  and its states: , , , , ...

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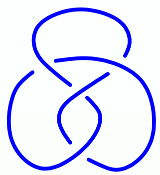
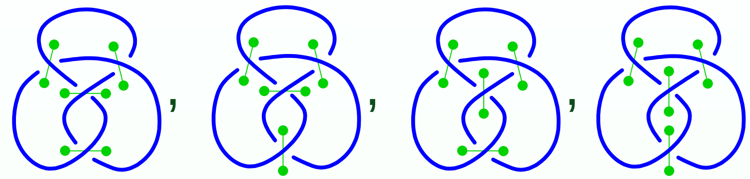
Knot diagram:  and its states:  , . . .

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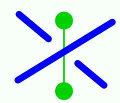
1. the number $a(s)$ of *positive* markers  ,

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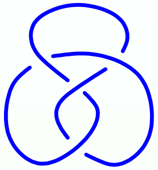
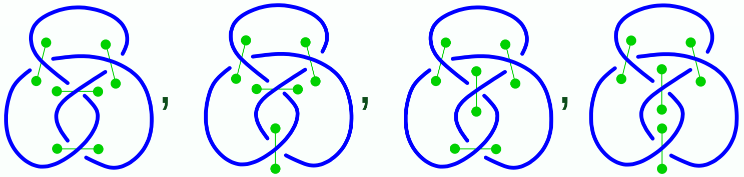
Knot diagram:  and its states: , . . .

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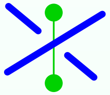
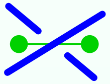
1. the number $a(s)$ of *positive* markers ,
2. the number $b(s)$ of *negative* markers ,

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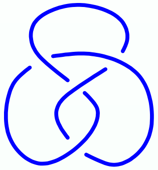
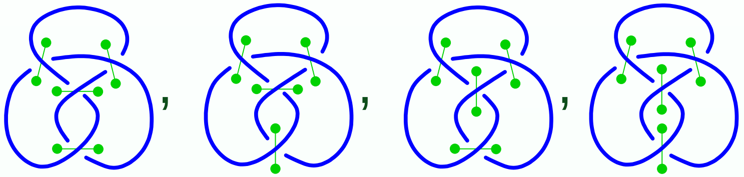
Knot diagram:  and its states:  , . . .

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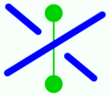
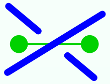
1. the number $a(s)$ of *positive* markers ,
2. the number $b(s)$ of *negative* markers ,
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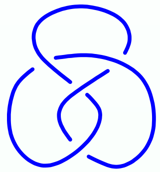
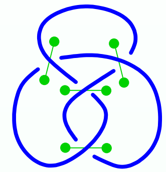
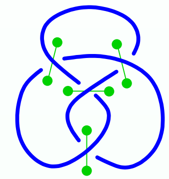
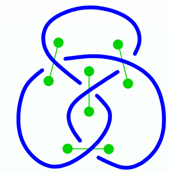
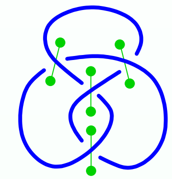
Knot diagram:  and its states: , , , , ...

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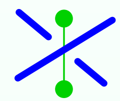
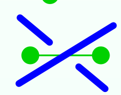
1. the number $a(s)$ of *positive* markers ,
2. the number $b(s)$ of *negative* markers ,
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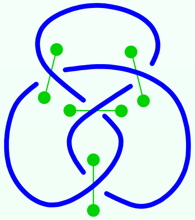
$s =$  $\mapsto$

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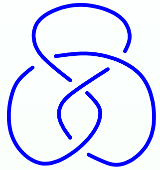
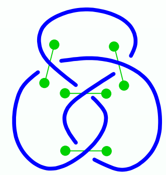
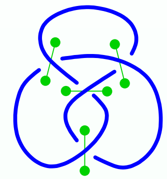
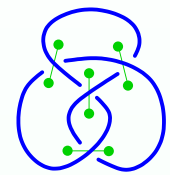
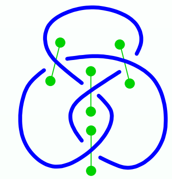
Knot diagram:  and its states: , , , , . . .

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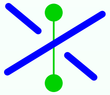
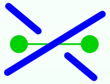
1. the number $a(s)$ of *positive* markers ,
2. the number $b(s)$ of *negative* markers ,
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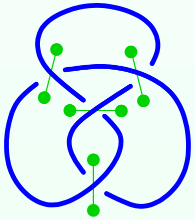
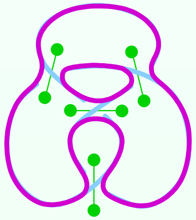
$s =$  $\mapsto$ $D_s =$ 

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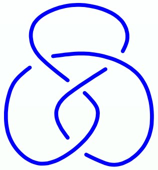
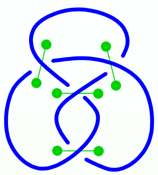
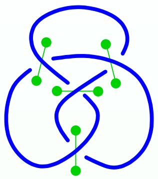
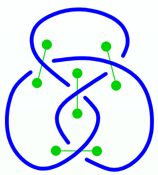
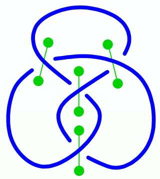
Knot diagram:  and its states: , , , , ...

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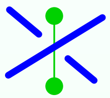
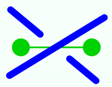
1. the number $a(s)$ of *positive* markers ,
2. the number $b(s)$ of *negative* markers ,
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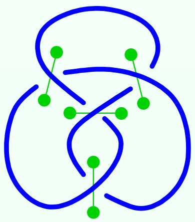
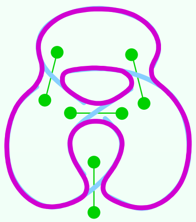
$s =$  $\mapsto \quad D_s =$  $\quad |s| = 2$

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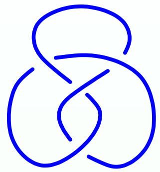
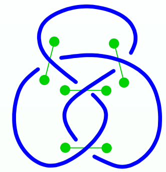
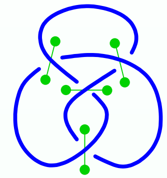
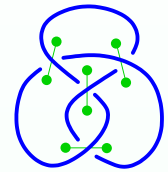
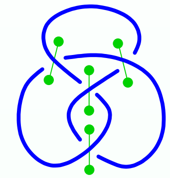
Knot diagram:  and its states: , , , , ...

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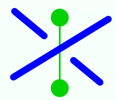
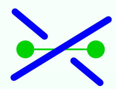
1. the number $a(s)$ of *positive* markers ,
2. the number $b(s)$ of *negative* markers ,
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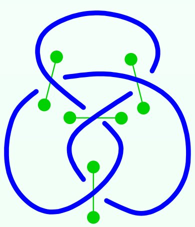
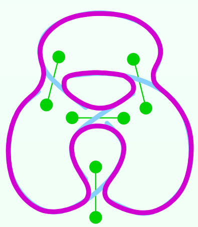
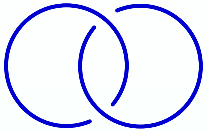
$s =$  $\mapsto$ $D_s =$  $|s| = 2$

State Sum: $\langle D \rangle = \sum_{s \text{ state of } D} A^{a(s)-b(s)}(-A^2 - A^{-2})^{|s|}$

Example

Hopf link, 

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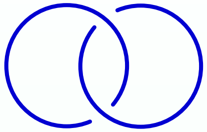
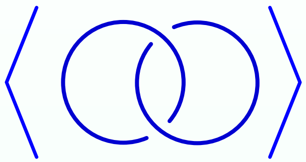
Hopf link, 

 $=$

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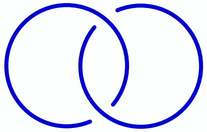
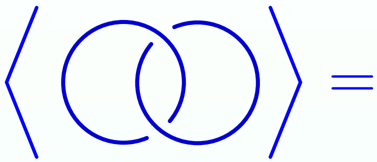
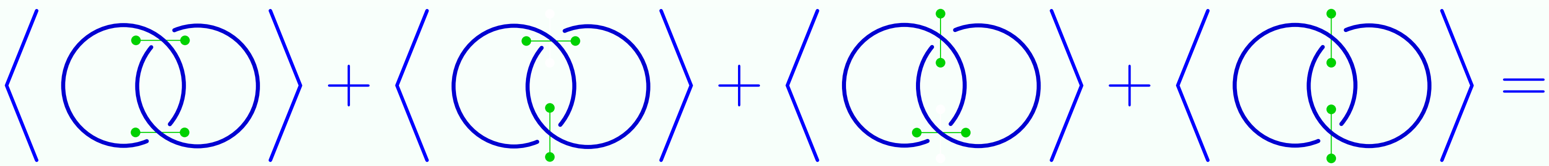
Hopf link, 





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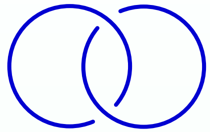
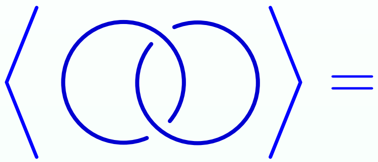
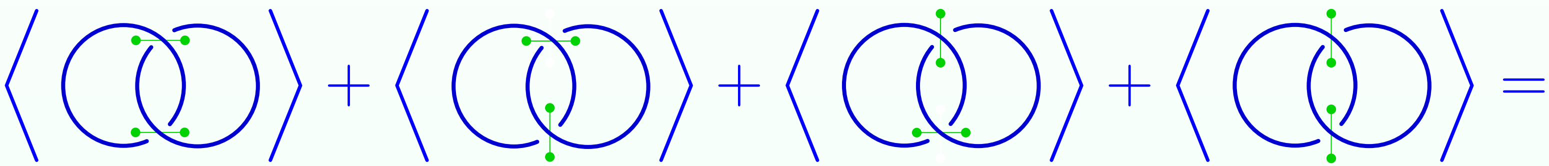
Hopf link, 





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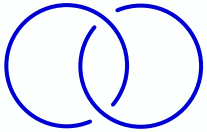
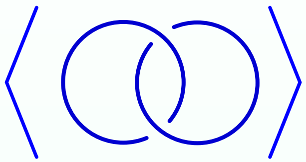
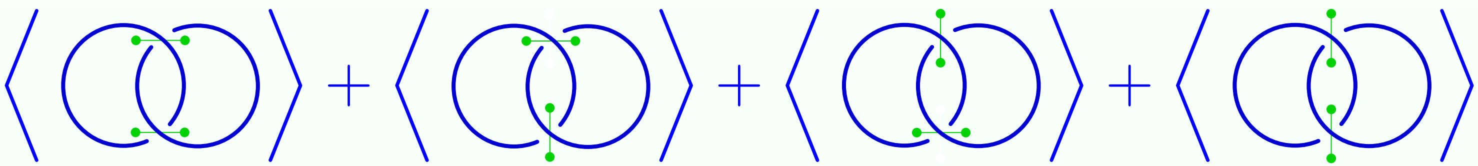
Hopf link, 

 $=$

 $=$

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$$A^6 + A^2 + A^{-2} + A^{-6}.$$

Categorifying Kauffman state sum. Chains

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Put on each component C of D_s

a graded group $V_C \cong \mathbb{Z} \oplus \mathbb{Z}$ with the summands of grades 1 and -1

V_C corresponds to $-A^2 - A^{-2}$.

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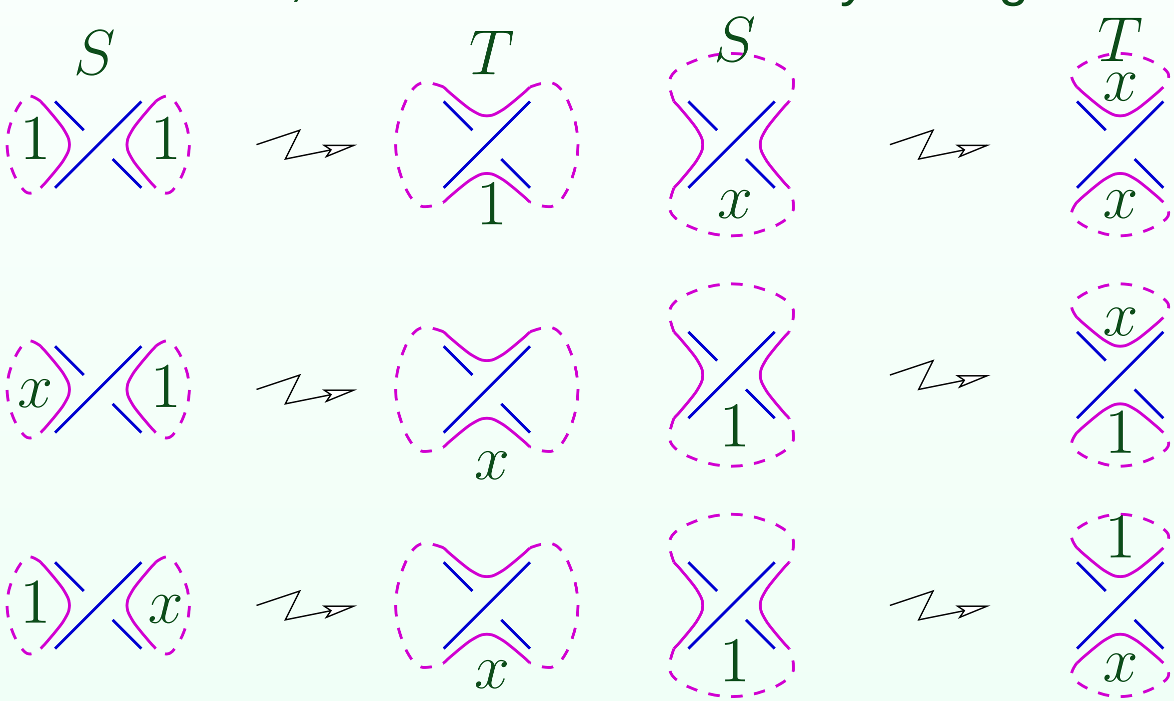

$V = \mathbb{Z} \oplus \mathbb{Z}$ is a Frobenius algebra with unity $1,$ relation $x^2 = 0$ and comultiplication $\Delta : V \to V \otimes V :$ $\Delta(1) = (1 \otimes x) + (x \otimes 1)$, $\Delta(x) = x \otimes x$.

Khovanov homology of tangles

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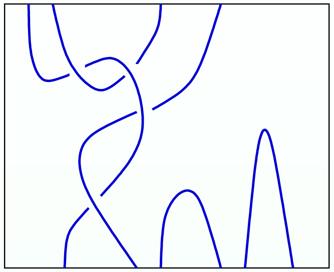

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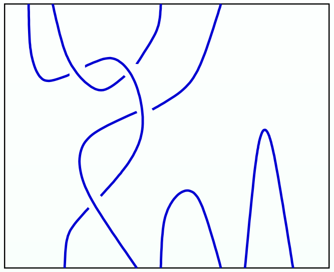

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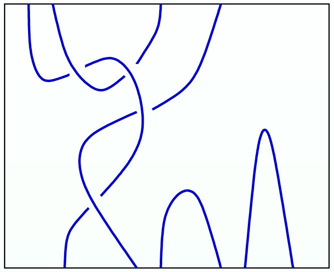

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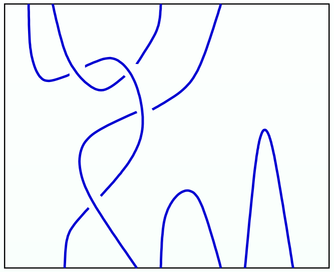

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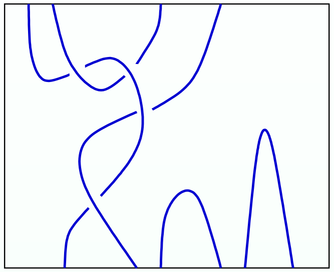

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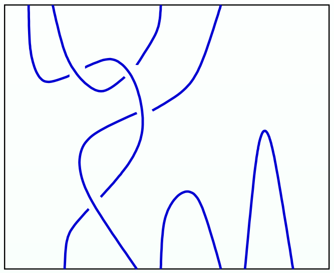

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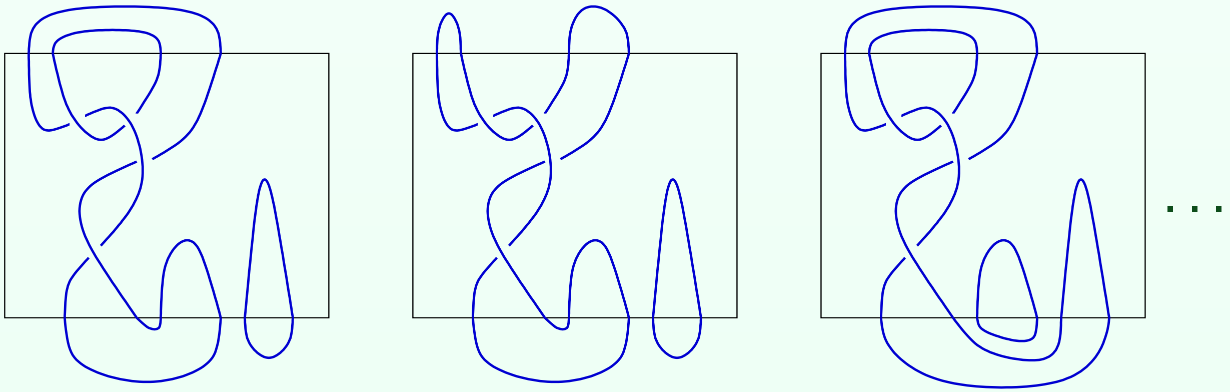

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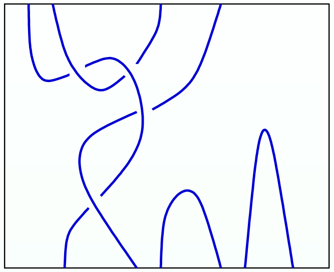

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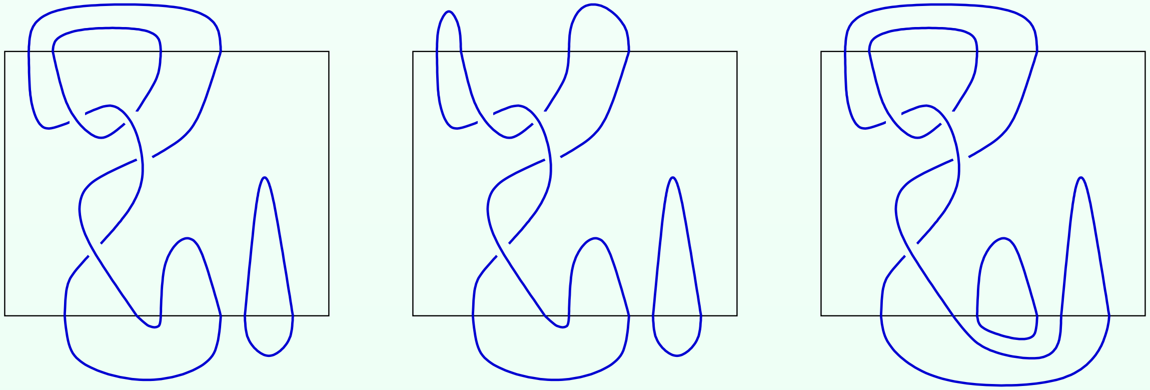

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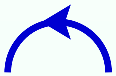	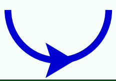	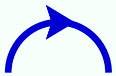	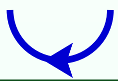
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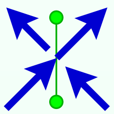	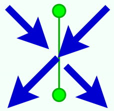	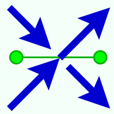	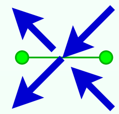	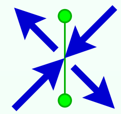	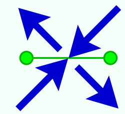	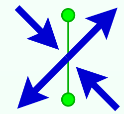	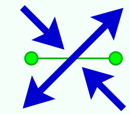
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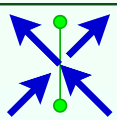	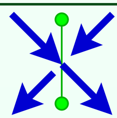	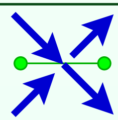	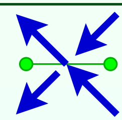	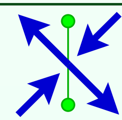	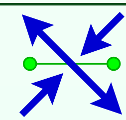	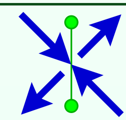	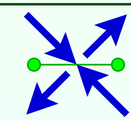
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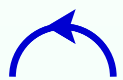	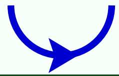	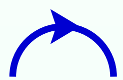	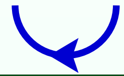
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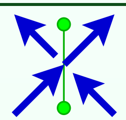
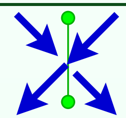
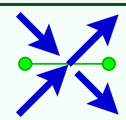
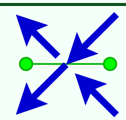
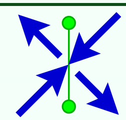
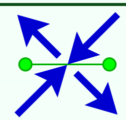
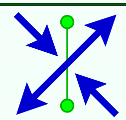
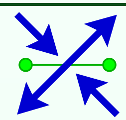
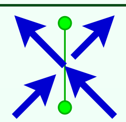
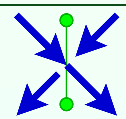
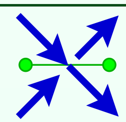
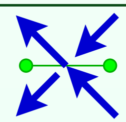
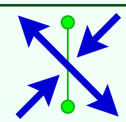
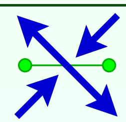
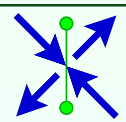
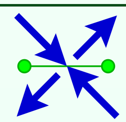
							
A	A	A^{-1}	A^{-1}	A	$-A^{-3}$	A	$-A$

							
A^{-1}	A^{-1}	A	A	A^{-1}	$-A^{-1}$	A^{-1}	$-A^{3}$

The Kauffman state sum turns into R-matrix state sum.

Arcs with oriented end points

A matrix element of the Reshetikhin-Turaev homomorphism is defined by the tangle with end-points equipped with orientations.

Arcs with oriented end points

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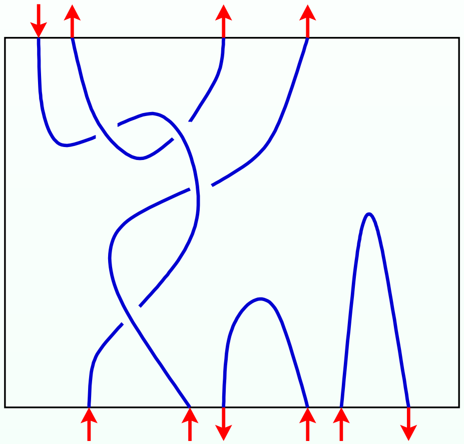

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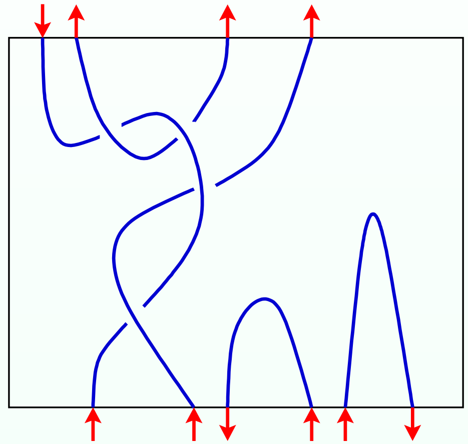

It can be computed as a state sum over distributions of markers at crossings and orientations of the corresponding smoothing.

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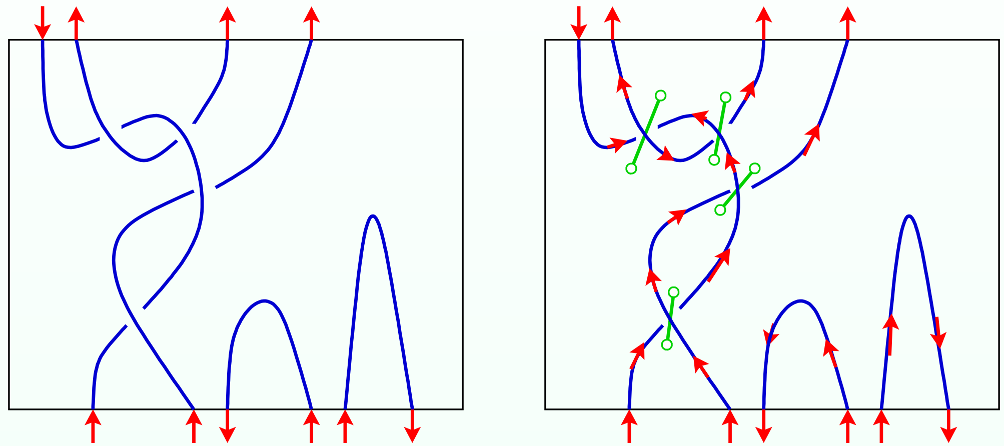

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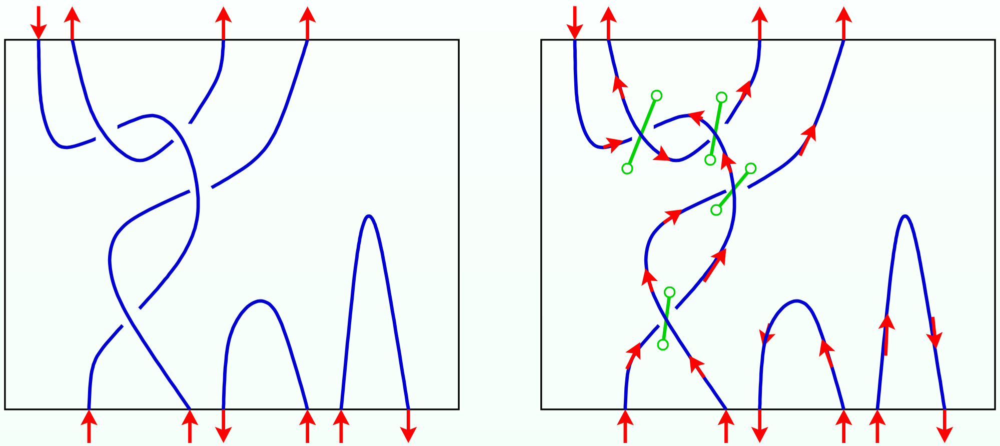

It can be computed as a state sum over distributions of markers at crossings and orientations of the corresponding smoothing.

The homology grading of a state is the degree of Gauss map of D_s evaluated as the average of local degrees at $\pm 1 \in S^1$

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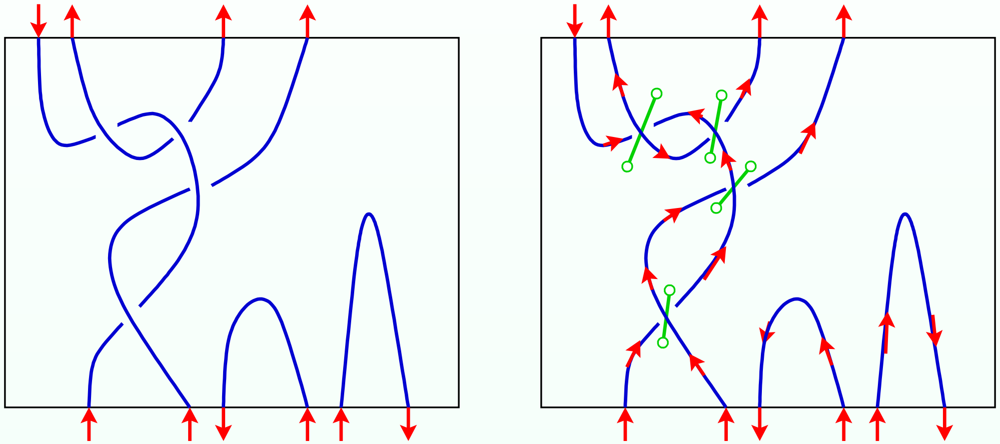

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$+3/2$ in the picture above.

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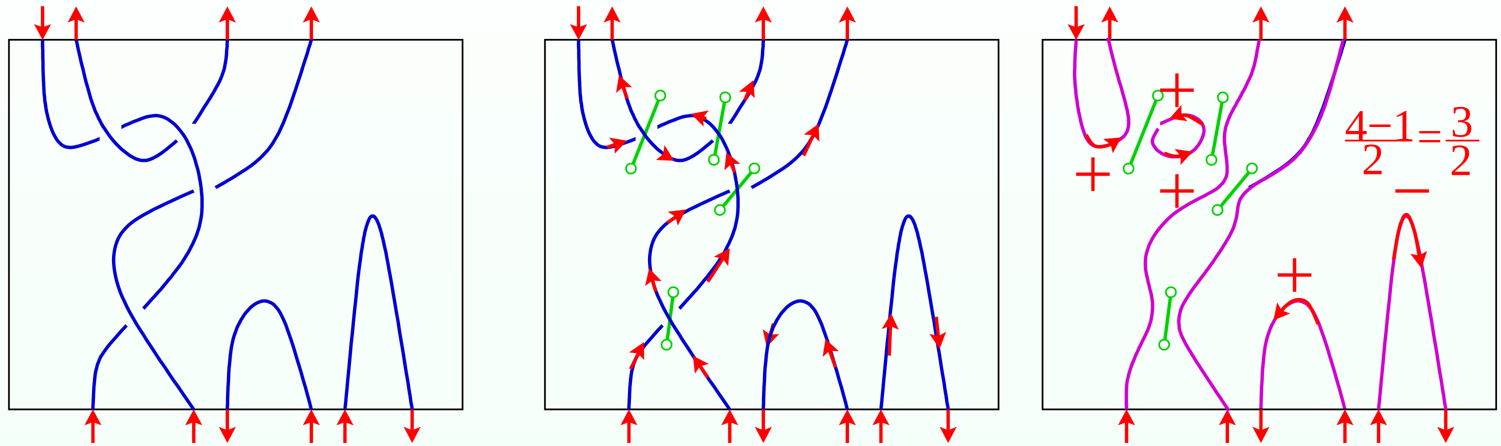

It can be computed as a state sum over distributions of markers at crossings and orientations of the corresponding smoothing.
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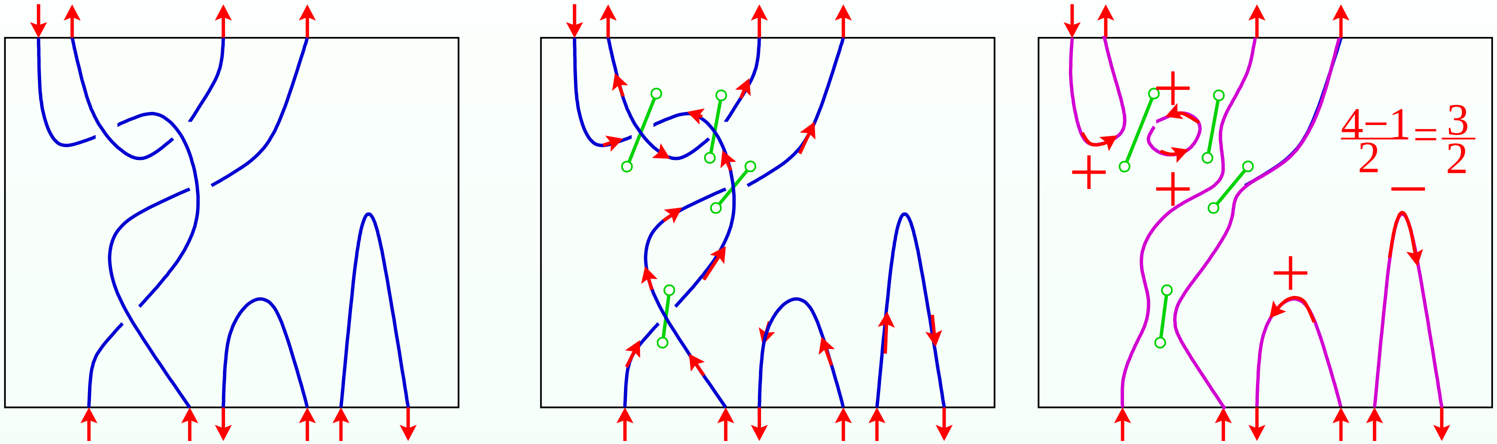

It can be computed as a state sum over distributions of markers at crossings and orientations of the corresponding smoothing.
The homology grading of a state is the degree of Gauss map of D_s evaluated as the average of local degrees at $\pm 1 \in S^1$

$+3/2$ in the picture above.
Differential: change of positive marker to a negative and change of adjacent orientation preserving A-grading, decreasing the homology grading by 1 and preserving the orientations of end points.