Perturbative expansion of thermodynamical effective Hamiltonian

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Introduction. We consider a quantum system with the Hamiltonian $H(\lambda) = H_0 + \lambda H_I$, where H_0 is a free Hamiltonian and H_I is an interaction Hamiltonian. We consider the time-dependent observables of the form $A(t) = e^{-iH_0 t} A e^{iH_0 t}$ (in the Shroedinger picture), i.e. they depend on time in a very special way such that in the interaction picture they become constant. Such observables could be implemented by dipole interaction with external an electromagnetic field which is in resonance with the free Hamiltonian.

The behaviour of such obervables at long time scales can (see [1]) be described by the effective Gibbs state $\mathcal{P}\rho_\beta(\lambda)$, where $\mathcal{P}$ is the averaging projector with respect to free evolution

$$\mathcal{P}\rho = \lim_{T\to+\infty} \frac{1}{T} \int_0^T e^{iH_0 t} \rho e^{-iH_0 t} dt$$

and $\rho_\beta(\lambda)$ is a Gibbs state with inverse temperature β for the Hamiltonian $H(\lambda)$. If one represents $\mathcal{P}\rho_\beta(\lambda)$ in the Gibbs-like form itself

$$\mathcal{P}\rho_\beta(\lambda) = \frac{1}{Z} e^{-\beta \tilde{H}_\beta(\lambda)}, \qquad Z = \operatorname{Tr} e^{-\beta \tilde{H}_\beta(\lambda)},$$

where $\tilde{H}_\beta(\lambda)$ is some effective Hamiltonian, then the problem of asymptotic expansion of $\tilde{H}_\beta(\lambda)$ in the powers of λ at $\lambda \to 0$ naturally arises.

Main result. For the case when both H_0 and H_I are bounded operators, the following theorem holds.

Theorem 1. The coefficients of the asymptotic expansion $\tilde{H}_\beta(\lambda) = H_0 + \sum_{n=1}^{\infty} \lambda^n \kappa_n$ could be defined by the formulae (if the integrals above converge)

$$\kappa_n \equiv -\beta^{-1} \sum_{k_0+\dots+k_m=n} \frac{(-1)^m}{m+1} \mathcal{M}_{k_0}(\beta)\mathcal{M}_{k_1}(\beta)\dots\mathcal{M}_{k_m}(\beta),$$

$$\mathcal{M}_k(\beta) \equiv (-1)^k \int_0^\beta d\beta_1 \dots \int_0^{\beta_{k-1}} d\beta_k \mathcal{P} H_I(\beta_1)\dots H_I(\beta_k),$$

$$H_I(\beta) \equiv e^{\beta H_0} H_I e^{-\beta H_0}.$$

Some corollaries of this results and applications to the quantum thermodynamics are discussed in the talk.

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References:

[1] A. E. Teretenkov. Effective Gibbs state for averaged observables // arXiv:2110.14407 [quant-ph] (2021)

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