Embeddability of matrices into real and positive semigroups

Tanja Eisner[1], **Agnes Radl**[2]

Keywords: embedding problem; real and positive operators; finite and infinite matrices; strongly continuous semigroups

MSC2010 codes: 15B48, 47B65, 47B37, 47B99, 47D06

It is a well-known problem in probability theory whether a Markov matrix is embeddable into a Markov semigroup. Even today it is an active field of research, see e. g. the recent survey [1]. We consider a related problem: Given a (finite or infinite) matrix T, is it embeddable into a real/positive C_0-semigroup, i. e., is there a real/positive C_0-semigroup $(T(t))_{t\geq 0}$ such that $T(1) = T$?

We will give necessary and sufficient conditions for embeddability of a real matrix into a real C_0-semigroup. In the case that T is positive we will present necessary conditions for embeddability of T into a positive C_0-semigroup. Moreover, we will give a full description in the case that T is a 2×2 matrix.

This presentation is based on [2].

References

[1] M. Baake, J. Sumner. *Notes on Markov embedding.* // Linear Algebra Appl. 2020. Vol. 594. P. 262–299

[2] T. Eisner, A. Radl. *Embeddability of real and positive operators.* // To appear in Linear and Multilinear Algebra.

[2] C. Bonatti, V. Grines, O. Pochinka. *Topological classification of Morse-Smale diffeomorphisms on 3-manifolds.* // Duke Math. J. 2019. Vol. 168. No. 13. P. 2507–2558.

[3] A.V. Vedenin, V.S. Voevodkin, V.D. Galkin, E.Yu. Karatetskaya, I.D. Remizov. *Speed of convergence of Chernoff approximations to solutions of evolution equations.* // arXiv:1910.09440 [math.FA] (2020).

[4] I. Remizov. *Quasi-Feynman formulas – a method of obtaining the evolution operator for the Schrödinger equation.* // J. Funct. Anal. 2016. Vol. 270. No. 12. P. 4540–4557

[1]University of Leipzig, Institute of Mathematics, Germany, Leipzig. Email: eisner@math.uni-leipzig.de

[2]Technische Universität Berlin, Insitute of Mathematics, Germany, Berlin. Email: radl@math.tu-berlin.de