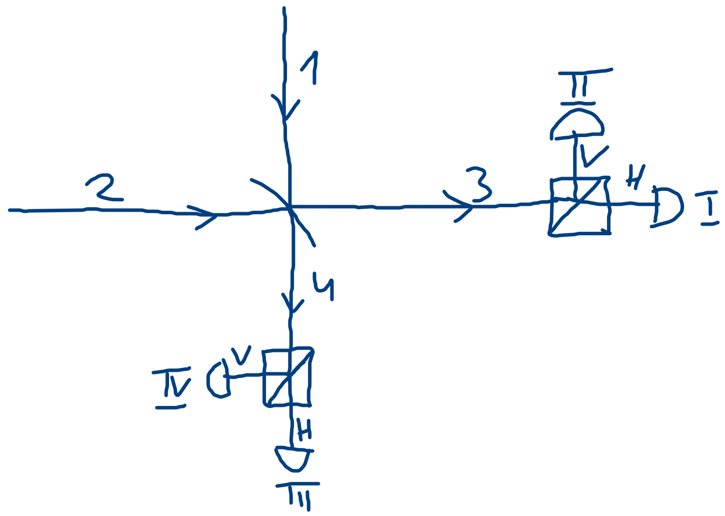
$|\Phi^+\rangle \otimes |\Phi^+\rangle$

Оптическое измерение в базисе Белла



$$|HH\rangle = \frac{1}{2}\left[(a^+_{3H})^2 - (a^+_{4H})^2\right]|\Omega\rangle =$$

$$\frac{(a^+_{3H})^n}{\sqrt{n!}}|\Omega\rangle = |\underbrace{H\ldots H}_{n}\rangle_3$$

$$= \frac{1}{\sqrt{2}}\left(|HH\rangle_3 + |HH\rangle_4\right)$$

$$|VV\rangle = \frac{1}{2}\left[(a^+_{3V})^2 - (a^+_{4V})^2\right]|\Omega\rangle$$

$$|HV\rangle = \frac{1}{2}\left[a^+_{3H}a^+_{3V} - a^+_{4H}a^+_{4V} - a^+_{3H}a^+_{4V} + a^+_{4H}a^+_{3V}\right]|\Omega\rangle$$

$$|VH\rangle = \frac{1}{2}\left[a^+_{3H}a^+_{3V} - a^+_{4H}a^+_{4V} - a^+_{3V}a^+_{4H} + a^+_{4V}a^+_{3H}\right]|\Omega\rangle$$

$$|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}\left(\underset{00}{|HH\rangle} \pm \underset{11}{|VV\rangle}\right) =$$

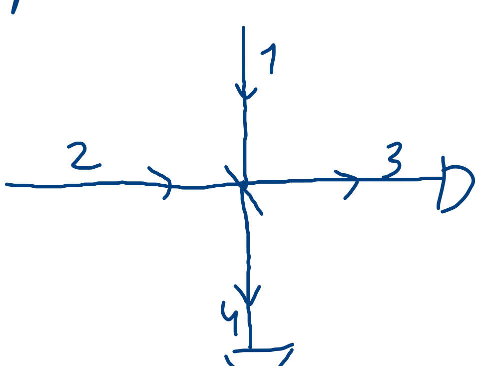
$$= \frac{1}{\sqrt{2}}\frac{1}{2}\left[\underbrace{(a^+_{3H})^2}_{I} - \underbrace{(a^+_{4H})^2}_{III} \pm \left\{\underbrace{(a^+_{3V})^2}_{II} - \underbrace{(a^+_{4V})^2}_{IV}\right\}\right]|\Omega\rangle$$

Оба фотона — в 1 детектор

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right) = \frac{1}{\sqrt{2}}\left(a^+_{3H}a^+_{3V} - a^+_{4H}a^+_{4V}\right)|\Omega\rangle$$ — (I и II) или (III и IV)

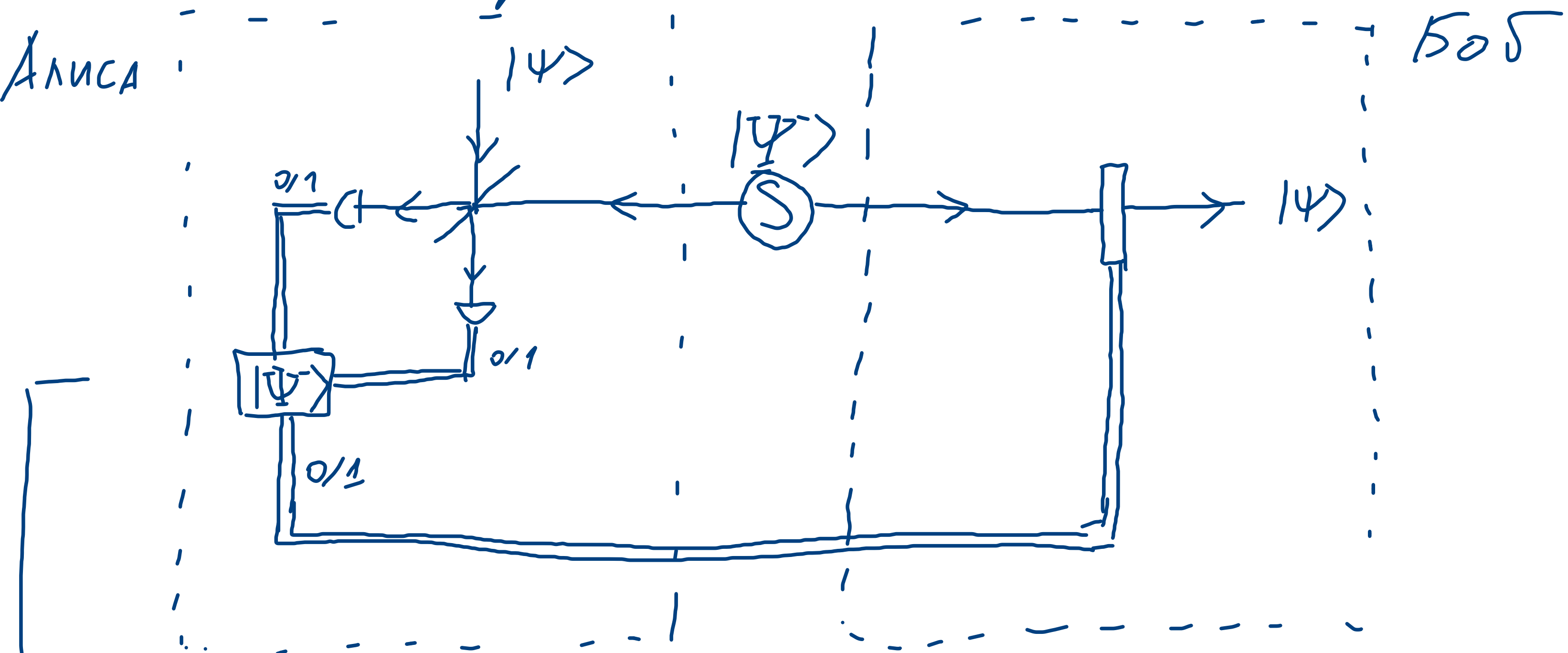
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right) = \frac{1}{\sqrt{2}}\left(a^+_{3V}a^+_{4H} - a^+_{3H}a^+_{4V}\right)|\Omega\rangle$$ — (II и III) или (I и IV)

Упрощённая схема:



$|\Psi^-\rangle$: кликают оба детектора

Телепортация



$$|\psi\rangle = \alpha|H\rangle_1 + \beta|V\rangle_1$$

$$|\psi\rangle_1 \otimes |\Psi^-\rangle_{23} = |\Upsilon\rangle = (\alpha|H\rangle_1 + \beta|V\rangle_1) \otimes \frac{1}{\sqrt{2}}(|HV\rangle_{23} - |VH\rangle_{23})$$

$$\left(|\Psi^-\rangle\langle\Psi^-|_{12} \otimes I_3\right)|\Upsilon\rangle = \left[|\Psi^-\rangle\langle\Psi^-|_{12} \otimes (|H\rangle\langle H| + |V\rangle\langle V|)\right]|\Upsilon\rangle \overset{?}{\sim} (=)$$

$$\sim |\Psi^-\rangle_{12} \otimes |\psi\rangle_3 \quad ✓$$

$$I = |0\rangle\langle 0| + |1\rangle\langle 1|$$

$$\begin{pmatrix}1&0\\0&1\end{pmatrix} = \underbrace{\begin{pmatrix}1&0\\0&0\end{pmatrix}}_{|0\rangle\langle 0|} + \underbrace{\begin{pmatrix}0&0\\0&1\end{pmatrix}}_{|1\rangle\langle 1|}$$

$$\left(\langle\Psi^-| \otimes \langle H|\right)|\Upsilon\rangle = \frac{1}{\sqrt{2}}\left(\langle HVH| - \langle VHH|\right)|\Upsilon\rangle = \frac{1}{2}\alpha$$

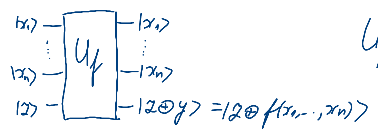
$$\left(\langle\Psi^-| \otimes \langle V|\right)|\Upsilon\rangle = \frac{1}{\sqrt{2}}\left(\langle HVV| - \langle VHV|\right)|\Upsilon\rangle = \frac{1}{2}\beta$$

$$(=)\ \frac{1}{2}|\Psi^-\rangle_{12} \otimes (\alpha|H\rangle + \beta|V\rangle) = \frac{1}{2}|\Psi^-\rangle \otimes |\psi\rangle, \quad \text{ч.т.д.}$$

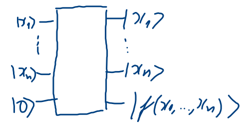
Простейшие квантовые алгоритмы

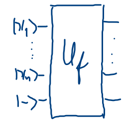
1. Представление функций на квантовом компьютере

$f: \{0,1\}^n \to \{0,1\}$ $\quad f(x_1,\dots,x_n) = y$, $\quad x_1,\dots,x_n, y \in \{0,1\}$



$U_f^2 = I$





$|x_1\dots x_n\rangle\underline{\underline{|-\rangle}} = \frac{1}{\sqrt2}\left(|x_1\dots x_n\rangle|0\rangle - |x_1\dots x_n\rangle|1\rangle\right) \xrightarrow{U_f}$

$\to \frac{1}{\sqrt2}\left(|x_1\dots x_n\rangle|y\rangle - |x_1\dots x_n\rangle|\bar y\rangle\right) = |x_1\dots x_n\rangle\otimes\frac{1}{\sqrt2}(|y\rangle - |\bar y\rangle) \;=$

$= (-1)^y|x_1\dots x_n\rangle|-\rangle \equiv \underline{(-1)^{f(x_1,\dots,x_n)}}|x_1\dots x_n\rangle\underline{\underline{|-\rangle}}$

$y=0$: $\frac{1}{\sqrt2}(|0\rangle - |1\rangle) = |-\rangle$

$y=1$: $\frac{1}{\sqrt2}(|1\rangle - |0\rangle) = -|-\rangle$

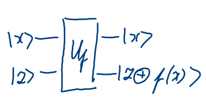
$|x_1\dots x_n\rangle \to (-1)^{f(x_1\dots x_n)}|x_1\dots x_n\rangle$

$\left(\sum_{x_1\dots x_n}\alpha_{x_1\dots x_n}|x_1\dots x_n\rangle\right)\underline{\underline{|-\rangle}} \to \left[\sum_{x_1\dots x_n}(-1)^{f(x_1\dots x_n)}\alpha_{x_1\dots x_n}|x_1\dots x_n\rangle\right]\underline{\underline{|-\rangle}}$

2. Задача Дойча

$f: \{0,1\}\to\{0,1\}$

Кл. $\qquad$ Кв:

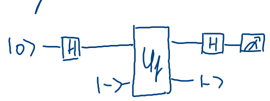


Задача: $\underline{f(0) \stackrel{?}{=} f(1)}$ за наименьшее число обращений к чёрному ящику

Кл: 2 обращения

Кв: 1 обращение

Алгоритм Дойча



0: $f(0)=f(1)$

1: $f(0)\neq f(1)$

$|0\rangle \xrightarrow{H} \frac{1}{\sqrt2}(|0\rangle + |1\rangle)$

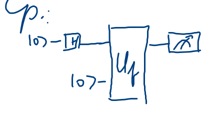
$|0\rangle|-\rangle \xrightarrow{H} \frac{1}{\sqrt2}(|0\rangle+|1\rangle)|-\rangle \xrightarrow{U_f} \frac{1}{\sqrt2}\left((-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right)|-\rangle =$

$= \frac{(-1)^{f(0)}}{\sqrt2}\left[|0\rangle + (-1)^{f(1)-f(0)}|1\rangle\right]|-\rangle$

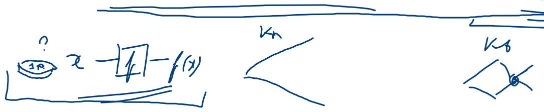
$f(0)=f(1)$: $\frac{1}{\sqrt2}(|0\rangle + |1\rangle) = |+\rangle \xrightarrow{H} |0\rangle$ $\qquad H|+\rangle \sim H(|0\rangle+|1\rangle) \sim |0\rangle + \cancel{|1\rangle} + |0\rangle - \cancel{|1\rangle} \sim |0\rangle$

$f(0)\neq f(1)$: $\frac{1}{\sqrt2}(|0\rangle - |1\rangle) = |-\rangle \xrightarrow{H} |1\rangle$ $\qquad H|-\rangle \sim H(|0\rangle - |1\rangle) \sim \cancel{|0\rangle} + |1\rangle - \cancel{|0\rangle} + |1\rangle \sim |1\rangle$

Ср.:



$|0\rangle|0\rangle \xrightarrow{H} \frac{1}{\sqrt2}(|0\rangle+|1\rangle)|0\rangle \xrightarrow{U_f} \frac{1}{\sqrt2}\left(|0\rangle|f(0)\rangle + |1\rangle|f(1)\rangle\right)$



$|HV\rangle$ $|HP\rangle$