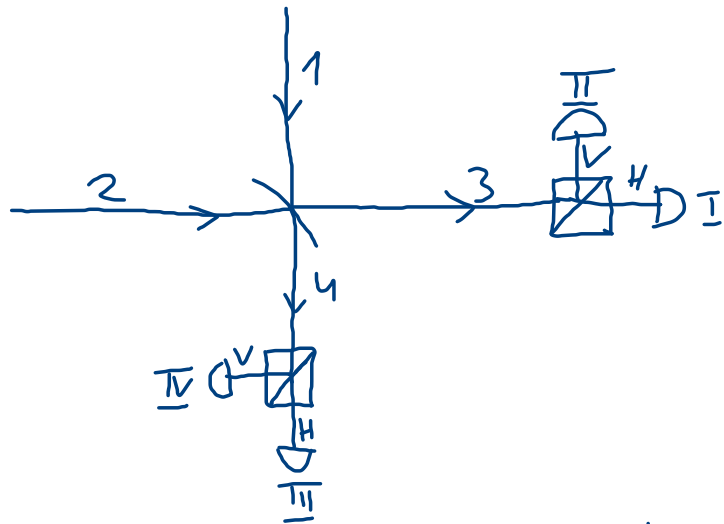


$$|\phi^+\rangle \otimes |\phi^+\rangle$$

Оптическое измерение в базисе Белла



$$|HH\rangle = \frac{1}{2} [(a_{3H}^+) ^2 - (a_{4H}^+) ^2] |\Omega\rangle \oplus$$

$$\frac{(a_{3H}^+)^n}{\sqrt{n!}} |\Omega\rangle = |\underbrace{H \dots H}_n\rangle$$

$$\oplus \frac{1}{\sqrt{2}} (|HH\rangle_3 + |HH\rangle_4)$$

$$|VV\rangle = \frac{1}{2} [(a_{3V}^+) ^2 - (a_{4V}^+) ^2] |\Omega\rangle$$

$$|HV\rangle = \frac{1}{2} [a_{3H}^+ a_{3V}^+ - a_{4H}^+ a_{4V}^+ - a_{3H}^+ a_{4V}^+ + a_{4H}^+ a_{3V}^+] |\Omega\rangle$$

$$|VH\rangle = \frac{1}{2} [a_{3H}^+ a_{3V}^+ - a_{4H}^+ a_{4V}^+ - a_{3V}^+ a_{4H}^+ + a_{4V}^+ a_{3H}^+] |\Omega\rangle$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}} (|HH\rangle_{00} \pm |VV\rangle_{11}) =$$

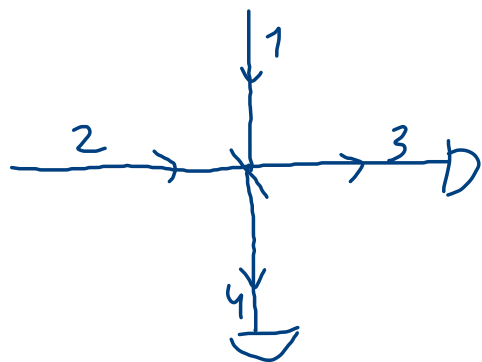
$$= \frac{1}{\sqrt{2}} \frac{1}{2} \left[\underbrace{(a_{3H}^+)^2}_{\text{I}} - \underbrace{(a_{4H}^+)^2}_{\text{II}} \pm \{ \underbrace{(a_{3V}^+)^2}_{\text{III}} - \underbrace{(a_{4V}^+)^2}_{\text{IV}} \} \right] |\Omega\rangle$$

Оба фотона — в 1 детекторе

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}} (|HV\rangle + |VH\rangle) = \frac{1}{\sqrt{2}} (a_{3H}^+ a_{3V}^+ - a_{4H}^+ a_{4V}^+) |\Omega\rangle \quad - \quad (\text{I u II}) \text{ или } (\text{III u IV})$$

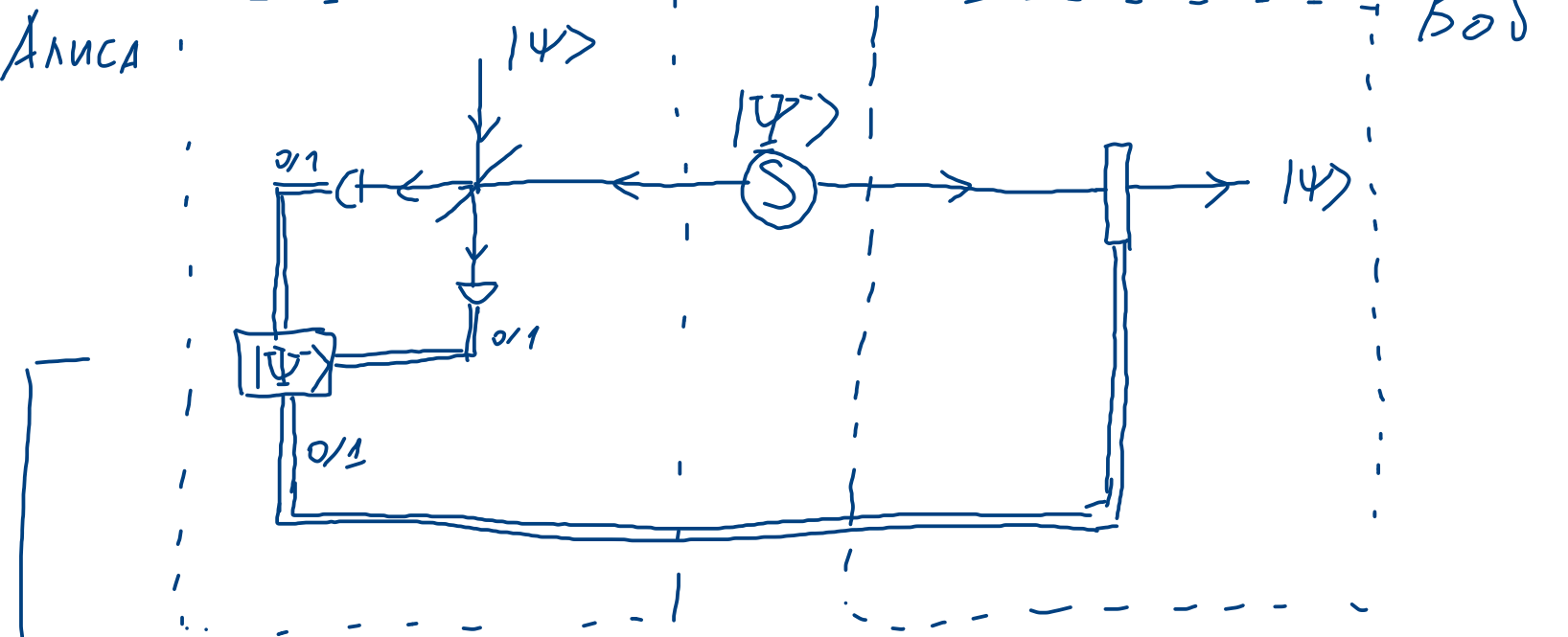
$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|HV\rangle - |VH\rangle) = \frac{1}{\sqrt{2}} (a_{3V}^+ a_{4H}^+ - a_{3H}^+ a_{4V}^+) |\Omega\rangle \quad - \quad (\text{II u III}) \text{ или } (\text{I u IV})$$

Упрощенная схема:



$|\Psi^-\rangle$: кликают оба детектора

Телепортация



$$|\psi\rangle = \alpha|H\rangle_1 + \beta|V\rangle_1$$

$$|\psi\rangle_1 \otimes |\Psi^-\rangle_{23} = |\gamma\rangle = (\alpha|H\rangle_1 + \beta|V\rangle_1) \otimes \frac{1}{\sqrt{2}}(|HV\rangle_{23} - |VH\rangle_{23})$$

$$\begin{aligned} \underbrace{(|\Psi^-\rangle_{12} \langle\Psi^-| \otimes I_3)}_{I = |0\rangle\langle 0| + |1\rangle\langle 1|} |\gamma\rangle &= \underbrace{(|\Psi^-\rangle_{12} \langle\Psi^-| \otimes (|H\rangle\langle H| + |V\rangle\langle V|))}_{\substack{(1\ 0) \\ (0\ 1) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\ 1\ 0 \times 0\ 1 & \quad 1\ 1 \times 1\ 1}} |\gamma\rangle \stackrel{?}{=} \\ &\sim \underline{|\Psi^-\rangle_{12} \otimes |\psi\rangle_3} \quad \checkmark \end{aligned}$$

$$(\langle\Psi^-| \otimes \langle H|) |\gamma\rangle = \frac{1}{\sqrt{2}} (\langle HV|H\rangle - \langle VH|H\rangle) |\gamma\rangle = \frac{1}{2} \alpha$$

$$(\langle\Psi^-| \otimes \langle V|) |\gamma\rangle = \frac{1}{\sqrt{2}} (\langle HV|V\rangle - \langle VH|V\rangle) |\gamma\rangle = \frac{1}{2} \beta$$

$$\Rightarrow \frac{1}{2} |\Psi^-\rangle_{12} \otimes (\alpha|H\rangle + \beta|V\rangle) = \frac{1}{2} |\Psi^-\rangle \otimes |\psi\rangle, \text{ т.т.т.}$$

Простейшие квантовые алгоритмы

1. Представление функции на квантовой компьютере

$$f: \{0,1\}^n \rightarrow \{0,1\} \quad f(x_1, \dots, x_n) = y, \quad x_1, \dots, x_n, y \in \{0,1\}$$

$$\begin{array}{c} |x_1\rangle \\ \vdots \\ |x_n\rangle \\ |z\rangle \end{array} \xrightarrow{U_f} \begin{array}{c} |x_1\rangle \\ \vdots \\ |x_n\rangle \\ |z \oplus f(x_1, \dots, x_n)\rangle \end{array} \quad U_f^2 = I \quad \begin{array}{c} |x_1\rangle \\ \vdots \\ |x_n\rangle \\ |0\rangle \end{array} \xrightarrow{U_f} \begin{array}{c} |x_1\rangle \\ \vdots \\ |x_n\rangle \\ |f(x_1, \dots, x_n)\rangle \end{array}$$

$$\begin{array}{c} |x_1\rangle \\ \vdots \\ |x_n\rangle \\ |-\rangle \end{array} \xrightarrow{U_f}$$

$$|x_1, \dots, x_n\rangle |-\rangle = \frac{1}{\sqrt{2}} (|x_1, \dots, x_n\rangle |0\rangle - |x_1, \dots, x_n\rangle |1\rangle) \xrightarrow{U_f}$$

$$\rightarrow \frac{1}{\sqrt{2}} (|x_1, \dots, x_n\rangle |y\rangle - |x_1, \dots, x_n\rangle |\bar{y}\rangle) = |x_1, \dots, x_n\rangle \otimes \frac{1}{\sqrt{2}} (|y\rangle - |\bar{y}\rangle) \equiv$$

$$\equiv (-1)^y |x_1, \dots, x_n\rangle |-\rangle \equiv (-1)^{f(x_1, \dots, x_n)} |x_1, \dots, x_n\rangle |-\rangle \quad \begin{cases} y=0: \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle \\ y=1: \frac{1}{\sqrt{2}} (|1\rangle - |0\rangle) = -|-\rangle \end{cases}$$

$$|x_1, \dots, x_n\rangle \rightarrow (-1)^{f(x_1, \dots, x_n)} |x_1, \dots, x_n\rangle$$

$$\left(\sum_{x_1, \dots, x_n} \chi_{x_1, \dots, x_n} |x_1, \dots, x_n\rangle \right) |-\rangle \rightarrow \left[\sum_{x_1, \dots, x_n} (-1)^{f(x_1, \dots, x_n)} \chi_{x_1, \dots, x_n} |x_1, \dots, x_n\rangle \right] |-\rangle$$

2. Задача Дайна

$$f: \{0,1\} \rightarrow \{0,1\}$$

K₁:

$$x \rightarrow \neg f(x)$$

K₂:

$$\begin{array}{c} |x\rangle \\ |z\rangle \end{array} \xrightarrow{U_f} \begin{array}{c} |x\rangle \\ |z \oplus f(x)\rangle \end{array}$$

Задача: $f(0) \stackrel{?}{=} f(1)$ за наименьшее число обращений к оракулу Дайны

K₁: 2 обращения

K₂: 1 обращение

Алгоритм Дайны

$$\begin{array}{c} |0\rangle \\ |-\rangle \end{array} \xrightarrow{H} \begin{array}{c} |0\rangle \\ |-\rangle \end{array} \xrightarrow{U_f} \begin{array}{c} |0\rangle \\ |-\rangle \end{array} \xrightarrow{H} \begin{array}{c} |0\rangle \\ |-\rangle \end{array} \quad \begin{array}{l} 0: f(0) = f(1) \\ 1: f(0) \neq f(1) \end{array}$$

$$|0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$|0\rangle |-\rangle \xrightarrow{H} \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |-\rangle \xrightarrow{U_f} \frac{1}{\sqrt{2}} ((-1)^{f(0)} |0\rangle + (-1)^{f(1)} |1\rangle) |-\rangle =$$

$$= \frac{(-1)^{f(0)}}{\sqrt{2}} [|0\rangle + (-1)^{f(1)-f(0)} |1\rangle] |-\rangle$$

$$f(0) = f(1): \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle \quad \begin{array}{c} H \\ \downarrow \\ |+\rangle \end{array} \quad H|+\rangle \sim H(|0\rangle + |1\rangle) \sim |0\rangle + |1\rangle + |0\rangle + |1\rangle \sim |0\rangle$$

$$f(0) \neq f(1): \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) = |-\rangle \quad \begin{array}{c} H \\ \downarrow \\ |-\rangle \end{array} \quad H|-\rangle \sim H(|0\rangle - |1\rangle) \sim |0\rangle - |1\rangle - |0\rangle + |1\rangle \sim |1\rangle$$

Ср:

$$\begin{array}{c} |0\rangle \\ |0\rangle \end{array} \xrightarrow{H} \begin{array}{c} |0\rangle \\ |0\rangle \end{array} \xrightarrow{U_f} \begin{array}{c} |0\rangle \\ |0\rangle \end{array} \quad |0\rangle |0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) |0\rangle \xrightarrow{U_f} \frac{1}{\sqrt{2}} (|0\rangle |f(0)\rangle + |1\rangle |f(1)\rangle)$$

1HV 1HP