Asymptotic regimes in near-Hamiltonian systems with damped oscillatory perturbations

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Consider the system of two differential equations

$$\frac{dx}{dt} = \partial_y H(x,y) + F(x,y,S(t),t), \qquad \frac{dy}{dt} = -\partial_x H(x,y) + G(x,y,S(t),t),$$

where $H(x,y)$, $F(x,y,S,t)$, and $G(x,y,S,t)$ are smooth functions defined for all $\mathbf{z} = (x,y) \in \mathbb{R}^2$, $S \in \mathbb{R}$, $t > 0$ and are 2π-periodic with respect to S. It is assumed that $H(x,y) = |\mathbf{z}|^2/2 + \mathcal{O}(|\mathbf{z}|^3)$ as $\mathbf{z} \to 0$, $F(x,y,S,t) \to 0$ and $G(x,y,S,t) \to 0$ as $t \to \infty$ for any fixed values of (x,y,S), and $S'(t) \to \varkappa \in \mathbb{Z}_+$ as $t \to \infty$. The influence of the damped perturbations $F(x,y,S(t),t)$ and $G(x,y,S(t),t)$ on the long-term behaviour of solutions of the limiting system is investigated. In particular, possible asymptotic regimes for solutions are described and stability of the equilibrium is analysed. It is shown that depending on the parameters and structure of perturbations there can be a phase-locking mode with a phase difference tending to a constant at infinity, and a phase-drifting mode with an unboundedly growing phase difference. In both regimes, the equilibrium can remain stable or become asymptotically stable or unstable in the perturbed system. In the case of loss of stability, the trajectories of the perturbed system that start near the equilibrium can have an unboundedly growing amplitude at infinity. The proposed technique is based on a combination of the averaging method and the construction of suitable Lyapunov functions.

References

1. *Sultanov O.A.* Bifurcations in asymptotically autonomous Hamiltonian systems under oscillatory perturbations // Discrete Contin. Dyn. Syst. 2021. V. 41. P. 5943–5978.