Π-STRATEGY FOR INERTIALLY MOVING OBJECTS UNDER INTEGRAL-GEOMETRIC CONSTRAINTS

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We have considered the pursuit–evasion problems in a differential game with one pursuer and one evader such that each of them has inertial motion. The control of the pursuer is subject to an integral constraint, and that of the evader is subject to a geometric constraint. In the pursuit problem, we propose the parallel approach strategy, which is known as the Π-strategy, for the pursuer, and we define a sufficient solvability condition of pursuit. In the evasion problem, a special admissible strategy is implemented for the evader, and a sufficient solvability condition of evasion is obtained. Besides, the lower boundary of the distance between the pursuer and evader is shown.

Suppose that in $\mathbb{R}^n$ a controlled player P, called the pursuer, chases another player E, called the evader. Denote by x the position of the pursuer and by y the position of the evader in $\mathbb{R}^n$. In the present work, we consider the pursuit–evasion problems when the players move according to the differential equations with initial values

$$P\colon\ \ddot{x} = u, \quad x(0) = x_0, \quad \dot{x}(0) = x_1, \tag{1}$$

$$E\colon\ \ddot{y} = v, \quad y(0) = y_0, \quad \dot{y}(0) = y_1, \tag{2}$$

respectively, where $x, y, u, v \in \mathbb{R}^n$, $n \geq 2$; x_0, y_0 are the initial positions of the players, and x_1, y_1 are their initial velocities, respectively. It is assumed that $x_0 \neq y_0$, $x_1 = y_1$. The control parameter u is the acceleration vector, and it is considered to be a measurable function $u(\cdot)\colon \mathbb{R}_+ \to \mathbb{R}^n$ subject to the following integral constraint (in short, the I-constraint) [2, 4, 6, 7]:

$$\int_0^t (t-s)|u(s)|^2\, ds \leq \rho_0 \qquad \text{for almost every} \ \ t \geq 0, \tag{3}$$

where ρ_0 is a given positive number which expresses the amount of the pursuer's energy at time $t = 0$. We denote by $\mathbb{U}_I$ the class of all control functions $u(\cdot)$ that satisfy the constraint (3).

Similarly, the control parameter v is the acceleration vector, and it is a measurable function $v(\cdot)\colon \mathbb{R}_+ \to \mathbb{R}^n$ subject to the geometric constraint (in short, the G-constraint) of the form [1, 3]

$$|v(t)| \leq \beta \qquad \text{for almost every} \ \ t \geq 0, \tag{4}$$

where β is a given positive number which expresses the maximal value of the evader's acceleration. We denote by $\mathbb{V}_G$ the class of all control functions $v(\cdot)$ that satisfy the constraint (4).

Definition 1. By equations (1), (2), any triplets $(x_0, x_1, u(\cdot))$, where $u(\cdot) \in \mathbb{U}_I$, and $(y_0, y_1, v(\cdot))$ with $v(\cdot) \in \mathbb{V}_G$ generate the trajectories

$$x(t) = x_0 + x_1 t + \int_0^t (t-s)u(s)\,ds, \qquad y(t) = y_0 + y_1 t + \int_0^t (t-s)v(s)\,ds,$$

respectively. In this case, $x(t)$ is called the pursuer's motion trajectory and $y(t)$ is called the evader's motion trajectory.

Definition 2. For each pair $(\rho_0, u(\cdot))$, $u(\cdot) \in \mathbb{U}_I$, we call the scalar function [5, 6]

$$\rho(t) = \rho_0 - \int_0^t (t-s)|u(s)|^2\,ds, \qquad \rho(0) = \rho_0,$$

a residual resource of the pursuer at the time t.

Definition 3. In the pursuit game (1)–(4), the function

$$\mathbf{u}(v) = v - \lambda(v)\xi_0 \tag{5}$$

is called the Π_{IG}-strategy of the pursuer, where

$$\lambda(v) = \langle v, \xi_0\rangle + \frac{\rho_0}{2|z_0|} + \sqrt{\left(\langle v, \xi_0\rangle + \frac{\rho_0}{2|z_0|}\right)^2 - |v|^2}, \qquad \xi_0 = \frac{z_0}{|z_0|},$$

and $\langle v, \xi_0\rangle$ denotes the inner product of the vectors v and ξ_0 in $\mathbb{R}^n$. Here $\lambda_{IG}(v)$ is usually called the resolving function.

Proposition. *If $\rho_0 \geq 4\beta|z_0|$, then the function $\lambda_{IG}(v)$ is defined, continuous and non-negative for all control functions $v(\cdot)$ that satisfy* (4).

Definition 4. We say that the strategy $\mathbf{u}(v)$ guarantees capture at time $T(\mathbf{u})$ if at some time $t_* \in [0, T(\mathbf{u})]$ the equality $x(t_*) = y(t_*)$ is satisfied for any control $v(\cdot) \in \mathbb{V}_G$ of the evader, where $x(t)$ and $y(t)$ are the solutions of the initial value problems

$$\ddot{x} = \mathbf{u}(v(t)), \qquad x(0) = x_0, \quad \dot{x}(0) = x_1,$$
$$\ddot{y} = v(t), \qquad y(0) = y_0, \quad \dot{y}(0) = y_1,$$

where $t \geq 0$.

Theorem 1. *If $\rho_0 \geq 4\beta|z_0|$ is satisfied in the pursuit game* (1)–(4), *then the Π_{IG}-strategy* (5) *guarantees capture in the time interval $[0, T_{IG}]$, where*

$$T_{IG} = \sqrt{\frac{2|z_0|}{\theta}}, \qquad \theta = \frac{\rho_0}{2|z_0|} - \beta + \sqrt{\frac{\rho_0^2}{4|z_0|^2} - \frac{\rho_0\beta}{|z_0|}}.$$

Definition 5. In the evasion game (1)–(4), we call the function

$$\mathbf{v}(t) = -\beta\xi_0 \tag{6}$$

the strategy of the evader.

Definition 6. We say that the strategy $\mathbf{v}(t)$ guarantees evasion on $[0, +\infty)$ if for any control $u(\cdot) \in \mathbb{U}_I$ of the pursuer the condition $x(t) \neq y(t)$ holds for all $t \in [0, +\infty)$, where $x(t)$ and $y(t)$ are the solutions of the initial value problems

$$\ddot{x} = u(t), \qquad x(0) = x_0, \quad \dot{x}(0) = x_1,$$

$$\ddot{y} = \mathbf{v}(t), \qquad y(0) = y_0, \quad \dot{y}(0) = y_1,$$

where $t \geq 0$.

Theorem 2. *If* $\rho_0 < 4\beta|z_0|$ *is valid in the evasion game* (1)–(4), *then the strategy* (6) *guarantees evasion on the time interval* $[0, +\infty)$ *and the following estimation holds for the distance between the players*:

$$|z(t)| \geq |z_0| - \frac{\rho_0}{4\beta}.$$

References

1. *Isaacs R.* Differential games. New York: J. Wiley and Sons, 1965.
2. *Pontryagin L.S.* Selected Works. Moscow: MAKS Press, 2004 (in Russian).
3. *Petrosjan L.A.* Differential games of pursuit. Singapore: World Scientific, 1993. (Series on optimization).
4. *Grigorenko N.L.* Mathematical methods of control for several dynamic processes. Moscow: Moscow State Univ., 1990 (in Russian).
5. *Azamov A.A., Samatov B.T.* The Π-strategy: Analogies and applications // Fourth Int. Conf. Game Theory and Management. St. Petersburg, 2010. P. 33–47.
6. *Samatov B.T.* The pursuit–evasion problem under integral-geometric constraints on pursuer controls // Autom. Remote Control. 2013. V. 74, No. 7. P. 1072–1081.
7. *Ibragimov G.I., Salimi M.* Pursuit–evasion differential game with many inertial players // Math. Probl. Eng. 2009. V. 65, No. 6. P. 1–15.