

Exponential ergodicity of branching processes with immigration and competition

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Abstract: We study the ergodic property of a continuous-state branching process with immigration and competition, which is an extension of the models studied by Pardoux (2016, Springer) and Berestycki et al. (Probab. Theory Related Fields, 2018) with an additional immigration structure. The exponential ergodicity in a weighted total variation distance is proved under natural assumptions. The result applies to general branching mechanism including all stable types. The proof is based on a Markov coupling process and a nonsymmetric control function for the distance, which are designed to identify and to take the advantage of the dominating factor among the branching, immigration and competition mechanisms in different parts of the state space. The approach provides a way of finding explicitly the ergodicity rate.

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