The probability of reaching a receding boundary by a random walk on branching process with fading branching and heavy-tailed jump distribution

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Let Z_n be a branching process in varying environment with the set of offspring distributions $\widehat{\mathcal{P}} = (\mathcal{P}_0, \mathcal{P}_1, \ldots)$ and let $\mathcal{T}$ be a genealogical tree for Z_n. Define a *random walk on* $\mathcal{T}$ as follows:

$$S(\pi) = \sum_{e \in \pi} \xi_{n(e), j(e)},$$

where π is an arbitrary path in $\mathcal{T}$ starting in root, $n(e)$ is the number of generation in which e ends, $j(e)$ is the number of particle in generation $n(e)$ in which e ends and $\{\xi_{n,j}\}_{n,j \geq 1}$ is the sequence of independent and identically distributed random variables that does not depend on genealogical tree $\mathcal{T}$.

We are interested in studying tail asymptotics for the

$$R^g_\mu = \sup_{\pi: |\pi| \leq \mu} \left(S(\pi) - g(|\pi|) \right),$$

that is the rightmost point of g-shifted random walk on $\mathcal{T}$, where $\mu \leq \infty$ is an arbitrary counting random variable and g is an arbitrary function on $\{0, 1, 2, \ldots\}$).

We obtain conditions under which

$$\mathbb{P}\left(R^g_\mu > x \right) = (1 + o(1)) H^g_\mu(x; \widehat{\mathcal{P}}) \qquad \text{as } x \to \infty,$$

uniformly over all suitable classes of time moments μ and functions g, where

$$H^g_\mu(x; \widehat{\mathcal{P}}) = \sum_{n=1}^{\infty} \mathbb{E}\left[Z_n \mathbb{I}(\mu \geq n) \right] \overline{F}(x + g(n)).$$

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