Some functional limit theorems for branching processes with dependent immigration

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KEY WORDS: Branching process, immigration, regularly varying functions, $m-$dependence, $\psi-$mixing, a fluctuation limit theorem

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80, 62F12

Let for each $n \geq 1$, $\left\{\xi_{k,j}^{(n)},\, k,j \geq 1\right\}$ and $\left\{\varepsilon_k^{(n)},\, k \geq 1\right\}$ be two independent families of independent identically distributed random variables with nonnegative integer values which are defined on a fixed probability space $(\Omega, \mathcal{F}, \mathbf{P})$. The sequence of branching processes with immigration $\left\{X_k^{(n)},\, k \geq 0\right\}$, $n \geq 1$ is defined by recursion:

$$X_0^{(n)} = 0, \quad X_k^{(n)} = \sum_{j=1}^{X_{k-1}^{(n)}} \xi_{k,j}^{(n)} + \varepsilon_k^{(n)}, \quad k,n \geq 1. \tag{11}$$

We discuss conditions on validity of weak convergence of properly normalized process (1) to the deterministic function under assumption that immigration is a rowwise $\psi-$mixing and the offspring mean tends to its critical value 1, moreover, immigration mean and variance controlled by regularly varying functions. Furthermore, we obtain a fluctuation limit theorem for branching process with immigration when immigration is $m-$dependent where m may tend to infinity with the row index at a certain rate. In this case the limiting process is a time-changed Wiener process. Our results extend and improve the results in [1] and [2].

References

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