On the hitting time of a growing level by catalytic branching walk

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Abstract: Branching random walks (BRWs) are probabilistic models allowing particles to move randomly (on a lattice or in the space) and occasionally produce offspring. We analyze catalytic branching random walk (CBRW) on an integer line $\mathbf{Z}$. The main feature of the CBRW is that the particles may produce offspring at the presence of a finite collection of catalysts located arbitrarily at fixed integer points. For a supercritical BRW, an interesting problem is the study of asymptotic behavior of its maximum, that is the coordinate of the right-most particle at time t, as t tends to infinity. Such a problem for a CBRW with light tails of the walk jump is solved in [1] and [2]. Here we go further and, for the CBRW, establish the limit theorem describing almost sure behavior of the time of first hitting a linearly growing level. We consider constant growth rate for the increasing level to guarantee the non-trivial limit. The new problem is more complicated than the mentioned above since we have to take into account not only the population maximum at time t, but also its dynamics before t, as t grows unboundedly. However, the new result and the previous one in [1] turn out to be close and involve the same constant in asymptotic formula. The proof is based on a (rather intricate) system of non-linear integral equations, large deviations theory for random walks, renewal theory and other techniques.

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