The initial stage of the evolution for intermediately subcritical branching processes in random environment

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KEY WORDS: Branching process, random environment, random walk

MATHEMATICAL SUBJECT CLASSIFICATION: 60J80

Abstract: We consider a Galton-Watson branching process $Z = (Z_0, Z_1, \ldots)$ evolving in i.i.d. random environment $\{f_0, f_1, ...\}$, where $f_n = f_n(s)$ is the generating function of the reproduction law of particles of the n-th generation. Let $X_n = \log f_n'(1)$. We assume that the process Z is intermediately subcritical, i.e.

$$\mathbf{E}X_0 = 0,\ \mathbf{E}[X_0 e^{X_0}] = 0. \tag{1}$$

Let $\mathbf{N} = \{1, 2, ...\}$. Introduce the so-called associated random walk $S = \{S_n\}_{n\geq 0}$

$$S_n = X_0 + ... + X_n,\ n > 0,\ S_0 = 0.$$

Let

$$\tau_n = \min\{k \leq n \mid S_k \leq S_0, S_1, \ldots, S_n\}$$

be the moment, when S takes its minimum for the first time on the interval $[0, n]$. Let $r_n \in \mathbf{N}, n > 0$, and $r_n \to \infty, n \to \infty$. For brevity we will use the notation $r = r_n,\ \tau = \tau_r$. Let the symbol $\Rightarrow$ denotes weak convergence.

We show that if (1) is valid and $r = r_n = o(n)$ as $n \to \infty$, then under some mild technical conditions

1) there is a random variable ξ with values in $\mathbf{N}$ such that as $n \to \infty$

$$\big(Z_{\tau_r} \mid Z_n > 0\big) \Rightarrow \xi; \tag{2}$$

2) there is a positive random variable η such that as $n \to \infty$

$$\Big(\frac{Z_r}{e^{S_r - S_{\tau_r}}} \mid Z_n > 0\Big) \Rightarrow \eta. \tag{3}$$

Note also that the distribution of the number of particles at the initial period of the evolution for critical and weakly subcritical BPRE given their survival up to a distant moment were investigated in [2] and [3].

Acknowledgement This work was supported by the Russian Science Foundation under grant no.19-11-00111, https://rscf.ru/en/project/19-11-00111/.

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