Large Deviations of Subcritical Branching Processes in Random Environment with and without Immigration.

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Abstract: We consider a strongly subcritical branching process $\{Z_n, n > 0\}$ in a random environment (BPRE). We assume that $\mathbf{E}Z_1^h < +\infty$ for some $h > 1$ and consider large deviations probabilities in integral $\mathbf{P}(\ln Z_n \geq x)$ and integro-local $\mathbf{P}(\ln Z_n \in [x, x + \Delta))$ form, $x/n \in (0, \gamma)$, where γ is some constant. D. Buraczewski and P. Dyszewski ([1]), A. Shklyaev ([2]) considered the supercritical BPRE for $x/n \in (\mu, m^+)$, critical, weakly and intermediatly subcritical BPREs for $x/n \in (0, m^+)$ and the strongly subcritical BPRE for $x/n \in (\gamma, m^+)$, where m^+ is some positive constant. E. Prokopenko, M. Struleva ([3]) considered large deviations for the supercritical case. It's known that in the strongly subcritical case for $x/n \in (0, \gamma)$ the asymptotical behaviour of $\mathbf{P}(\ln Z_n \geq x)$ has another form. It was proved by Kozlov ([4]) in the case of geometric conditional distribution and in LDP form by C. Bounghoff and G. Kersting ([5]). We'll discuss the results of A. Shklyaev ([6]) about precise asymptotics of large deviation probabilities in that case.

After that we consider branching process Z_n^* with immigration in random environment (BPIRE). We assume that $\mathbf{E}Z_1^h < +\infty$ for some $h > 1$ (including the immigration). Large deviations for BPIRE were considered by D. Dmitrusenkov and A. Shklyaev ([7]) in the geometric case and A. Shklyaev ([2]) for the general case. Both works deal with the supercritical and critical case for $x/n \in (\mu, m^+)$ and for subcritical case for $x/n \in (\gamma^*, m^+)$, where γ^* is some constant. The situation of $x/n \in (0, \gamma^*)$ was never studied, but there was a hypothesis that the behaviour of the process is close to those of strongly subcritical BPRE. We obtain the precise asymptotics of large deviation probabilities in that case. We'll discuss the difference between large deviations of BPRE and BPIRE.

References

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