Functional Limit Theorems for Continuous-Time Critical Recurrent Branching Random Walks

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Abstract: We consider continuous-time critical symmetric branching random walks on a multidimensional lattice $\mathbb{Z}^d$, $d \geq 1$, with the source of particle generation at the origin. We assume that the underlying random walk is symmetric, spatially homogeneous, and irreducible, and that the birth and death of particles at the source is described by a Markov branching process. One of the main problems is to study the exact form of the limiting distribution of the particle population at the source. This problem has been solved so far only for some relations between the parameters specifying walking and branching of particles. Based on limit theorems about the distribution of the sojourn time of the underlying recurrent stochastic random walk at the origin (see Aparin, Popov, and Yarovaya, 2021), we obtain limit theorems for the distribution of the particle population at the source with finite variance of the jumps of the random walk. Currently, stochastic walks with infinite variance of jumps have been much less studied than those with finite variance. In this context, the theorems for such stochastic walks deserve special attention. For $d = 1$, the limiting distribution of the particle population at the source under normalization on the Green's function of the transition probabilities depends on the parameters of the system and may take the form of the Mittag-Leffler or the exponential distribution for a recurrent random walk.

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References

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