Asymptotic behaviour of the survival probability of almost critical branching processes in a random environment with geometric distribution

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Abstract:

We consider the branching process in random environment, given by the sequence of independent probability generating functions

$$f_{i-1,n}(s) := \frac{1-p_{i,n}}{1-p_{i,n}s}, \quad p_{i,n} := \frac{1}{1+e^{-X_i-b_{i,n}}},\ i \in \{1,\dots,n\},$$

where X_i – independent identically distributed random variables with $\mathsf{E}X_1 = 0$, $\mathsf{D}X_1 \in (0,\infty)$, $b_{i,n}$ is some sequence of real numbers. Let $Z_{k,n}$ be the population size at moment k, $Z_{0,n} = 1$. Set

$$\widehat{X}_{i,n} := \ln f'_{i-1,n}(1) = X_i + b_{i,n}, \quad \widehat{S}_{0,n} := 0, \quad \widehat{S}_{k,n} := \widehat{X}_{1,n} + \ldots + \widehat{X}_{k,n}.$$

We will call the sequence $\widehat{S}_{k,n}$, $k \geq 0$, *the associated random walk* for $Z_{k,n}$. In the case $b_{i,n} \equiv 0$, the associated random walk is random walk with finite variance and zero drift. In this case we denote the population size at moment k by Z_k^0.

Our main result is the following theorem.

Theorem 1 *Assume that there exists $\delta \in (0, 1/2)$ such that,*

$$\max_{k\leq n} k^{\delta-1/2} \left| \sum_{i=1}^{k} b_{i,n} \right| \to 0,\ n \to \infty.$$

Then

$$\mathsf{P}(Z_{n,n} > 0) \sim \mathsf{P}(Z_n^0 > 0),\ n \to \infty.$$

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