Branching processes in random environment with cooling

Ivan KORSHUNOV *Steklov Mathematical Institute of Russian Academy of Science, Russia*, E-mail: IDKorshunov@mail.ru

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Abstract: It is well known that a branching process in random environment can be described by the associated random walk

$$S_n = \xi_1 + \ldots + \xi_n,$$

where $\xi_k = \ln \varphi'_{\eta_k}(1)$, $\varphi_x(t)$ and η_k are the generating functions of the number of descendants and the random environment respectively. The talk will address the issue of degeneration of a branching process in random environment with cooling with $\mathbf{E}\xi_1 > 0$ which differs from the classic BPRE in that each environment lasts for several generations. It turns out that this varianant of BPRE is also closely related to random walk

$$S_n = \tau_1\xi_1 + \ldots + \tau_n\xi_n,$$

where $\xi_k = \ln \varphi'_{\eta_k}(1)$ and $\varphi_x(t)$ and η_k are generating functions of the number of descendants and the random environment ewspectively and τ_k is a duration of the k-th cooling.

In this talk we will show that if for any $\varepsilon > 0$

$$\sum_{n=1}^{\infty} \mathbf{P}\left(\varepsilon\xi_1 < -\frac{\tau_1 + \ldots + \tau_n}{\tau_n}\right)$$

is divergent then the process degenerates with probability 1. Also we will show that if $0 < \mathbf{D}\xi_1 < \infty$ and

$$\sum_{n=1}^{\infty} \frac{\tau_n^2}{(\tau_1 + \ldots + \tau_n)^2} < \infty$$

then the process degenerates with probability less than 1.

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