On structural equivalence of S-tuples in Markov chains

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Abstract: The paper presents the main results of [3]. Let H be the permutation group on the set $\{1,\dots,N\}$. Typles $(a_1,\dots,a_s)$, $(b_1,\dots,b_s)$ of elements sets $\{1,\dots,N\}$ are called *H-equivalent* if there is a permutation $h\in H$ such that $b=h(a)$, i.e.

$$b_i = h(a_i),\quad i=1,\dots,s.$$

For H-equivalent tuples $a=(a_1,\dots,a_s)$ and $b=(b_1,\dots,b_s)$ we will use the notation aHb. If the tuples a and b are not H-equivalent, then we use the notation $\overline{H}$.

Let $x_1,x_2,\dots$ be the sequence elements of the set $\{1,\dots,N\}$. We will say that the tuple $z=(x_j,\dots,x_{j+s-1})$ is the H-repetition of the tuple $y=(x_i,\dots,x_{i+s-1})$, $j>i$, if yHz.

Further as a sequence $x_1,x_2,\dots$ consider a nonperiodic homogeneous Markov chain $\mathbf{X}=\{X_0,X_1,\dots,X_n,\dots\}$ with outcomes $1,\dots,N$, indecomposable matrix transition probabilities $\mathbb{P}=\|p_{k,l}\|$ and arbitrary initial distribution. Denote $\pi=(\pi_1,\dots,\pi_N)$, where $\pi_k>0,\ k=1,...,N$, stationary distribution of the chain $\mathbf{X}$.

We are interested in events $\left\{Y_{i_1-1}\overline{H}Y_{i_2-1},\ Y_{i_1}(s)HY_{i_2}(s)\right\}$, consisting in the fact that at the moments i_1 and i_2 the series begins H-repetitions of s-tuples. We study the asymptotic behavior of the distribution of the number of series of H-repetitions s-tuples starting up to the moment n:

$$\widetilde{\xi}_2(n,s,H)=\sum_{1\le i_1<i_2\le n} I\Big\{Y_{i_1-1}\overline{H}Y_{i_2-1},\ Y_{i_1}(s)HY_{i_2}(s)\Big\}.$$

The problem of the number of equivalent tuples in random discrete sequences was first considered in [1]. In this paper, sufficient

conditions for the Poisson approximation were obtained for the number of pairs of equivalent tuples in a sequence independent random variables distributed uniformly on set $\{1,\dots,N\}$. Further development of this direction is reflected in the review paper [2], which describes the results of works that appeared before 2003 year, and also announced a number of results published a little later.

Theorem 1. *Let the matrix $\mathbb{P}$ be indecomposable, $p^2 < \rho$, $n \to \infty$, and $s = s(n) \to \infty$ so that the condition holds $n^2\rho^s = O(1)$. Then*

$$\mathbf{P}\Big\{\tilde{\xi}_2(n,s,H) = \tilde{\xi}_2(n,s,H_{\mathbb{P}})\Big\} \to 1.$$

Let us introduce the notation $R^2_{H_{\mathbb{P}}} = \rho^{s-2}(1-\rho)|H_{\mathbb{P}}| \sum\limits_{a,b\in\{1,\dots,N\}} \pi_a^2 p_{a,b}^2$.

Theorem 2. *Let the matrix $\mathbb{P}$ be indecomposable, $p^2 < \rho$, $n \to \infty$, and $s = s(n)$ changes so that $s^2/n \to 0$ and $n^2 R^2_{H_{\mathbb{P}}}/2 \to \lambda \in (0,\infty)$. Then the distribution of the random variable $\tilde{\xi}_2(n,s,H)$ converges to Poisson distribution with parameter λ.*

References

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