MOMENTS OF THE FIRST DESCENDING EPOCH FOR A RANDOM WALK WITH NEGATIVE DRIFT

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We consider the first descending ladder epoch $\tau = \min\{n \geq 1 : S_n \leq 0\}$ of a random walk $S_n = \sum_1^n \xi_i, n \geq 1$ with i.i.d. summands having a negative drift $\mathbb{E}\xi = -a < 0$. Let $\xi^+ = \max(0, \xi_1)$. It is well-known that, for any $\alpha > 1$, the finiteness of $\mathbb{E}(\xi^+)^\alpha$ implies the finiteness of $\mathbb{E}\tau^\alpha$ and, for any $\lambda > 0$, the finiteness of $\mathbb{E}\exp(\lambda\xi^+)$ implies that of $\mathbb{E}\exp(c\tau)$ where $c > 0$ is, in general, another constant that depends on the distribution of ξ_1. We consider the intermediate case, assuming that $\mathbb{E}\exp(g(\xi^+)) < \infty$ for a positive increasing function g such that $\liminf_{x\to\infty} g(x)/\log x = \infty$ and $\limsup_{x\to\infty} g(x)/x = 0$, and that $\mathbb{E}\exp(\lambda\xi^+) = \infty$, for all $\lambda > 0$. Assuming a few further technical assumptions, we show that then $\mathbb{E}\exp((1-\varepsilon)g((1-\delta)a\tau)) < \infty$, for any $\varepsilon, \delta \in (0, 1)$.