DOUBLE BOOTSTRAP METHOD FOR TAIL INDEX ESTIMATION BY EXPECTILES

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Tail index estimation is an important topic in extreme value theory and has wide applications in financial and actuarial mathematics. Tail index in distribution defines the existence of high-order moments and allows estimating the decreasing rate of survival function. On the applied side of topic, such estimation gives opportunity to insure risky or extremal situations, which are modelled by power-law distributions.

There are plenty of approaches to estimate tail index. Most common is a group of estimators, which use properties of order statistics. Most well-known of them is a Hill estimator (Hill, [1975]). But the problem of such methods is their dependence from the number of selected order statistics k(n). For estimating the optimal value of k(n), a double bootstrap method was presented(Draisma, [1999]). Consistency and asymptotic normality holds, the representation of function k(n) is found.

Other methods for tail index estimation use so-called expectiles (Phillips, [2022]; Daouia, [2020]). Expectiles are solutions of the best weighted mean-squared predictor optimization problem and have similar properties to quantiles of distribution, but have even more: for example, expectile function is continuous, monotonically increasing function on [0,1]. Such methods like Hill-type estimators face the problem of searching for the optimal part of data from a sample.

The aim of this work is to explore the double bootstrap method and study the optimal level of subsample for tail index estimation by expectiles and statistical properties of such method. Also the aim is to implement a given method on Python programming language and compare it with existing double bootstrap method for Hill-type estimators.