ON THE DISTRIBUTION OF THE LENGTH OF THE SHORTEST PATH IN A GENERALISED BARAK-ERDŐS GRAPH

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We consider a random graph $\mathcal{G}_n$ that is an oriented version of Erdős-Renyi graph with set of vertices $\mathcal{V}_n = \{0, 1, 2, \ldots, n\}$ and random set of edges $\mathcal{E}_n \subset \mathcal{V}_n^2$. We assume that all the edges are directed from the smaller vertices to the larger. For $i < j$ let

$$p_{i,j}(n) := \mathbb{P}\left((i,j) \in \mathcal{E}_n\right).$$

Assume additionally that the events $\{(i,j) \in \mathcal{E}_n\}$ are mutually independent.

The graph $\mathcal{G}_n$ is called Barak-Erdős graph if $p_{i,j}(n)$ does not depend on the total number of vertices n and on vertices i and j. We study so-called generalised Barak – Erdős graph assuming $p_{i,j}(n)$ be a function of i, j and n.

Let L_n be the length of the shortest path between vertices 0 and n.

Assume that

$$p_{i,j}(n) = \frac{f(\frac{i}{n}, \frac{j}{n})}{n^\gamma},$$

where $f(x,y)$ – is a function Riemann-integrable on $[0,1]^2$ and $\gamma \in (0,1)$ is a positive constant.

The main result of our study is the following theorem.

Theorem. *Let $\gamma = 1 - 1/k$ for some $k \in \mathbb{N}$. Then*

$$\lim_{n\to\infty} \mathbb{P}(L_n = k+1) = 1 - \lim_{n\to\infty} \mathbb{P}(L_n = k) = \exp\left(-c_k(f)\right),$$

where

$$c_k(f) = \int_{0<u_1<\ldots<u_{k-1}<1} \prod_{j=0}^{k-1} f(u_j, u_{j+1}) du_1 \cdots du_{k-1} \in [0, \infty],$$

$u_0 = 0$ *and* $u_k = 1$.

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