CRITICAL GALTON-WATSON BRANCHING PROCESSES WITH A COUNTABLE SET OF PARTICLE TYPES AND RANDOM GRAPHS

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We consider genealogical trees of Galton-Watson branching processes and study the critical case corresponding to a one-vertex random tree with an independent, identically distributed number of edges for all vertices. The average number of edges coming out of a lower level vertex is 1. One of the fundamental theorems for these processes is a Yaglom-type theorem, which states that processes that do not degenerate to a distant time n contain at a given time the number of particles equal to this time n times exponentially distributed random variable. It is convenient to describe these conditional processes in terms of reduced trees, which are obtained from genealogical trees by eliminating subtrees that do not reach level n. A more complex model of Galton-Watson branching processes with a countable set of particle types, in which the types of descendants are obtained by summing the parent type with independent identically distributed multidimensional random variables, can be represented as trees with weights of edges and vertices defined above in the one-dimensional case.

We describe the averages and variances of a number of characteristics of reduced weighted trees, including the total weight of all vertices at a fixed level. We prove a number of limit theorems for reduced trees.

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