# ON ASYMPTOTICS OF THE PROBABILITY FOR A RANDOM PROCESS TO STAY ABOVE A MOVING BOUNDARY

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Let $X_1, X_2, \dots$ be independent random variables. We always assume that the random walk $S_n := X_1+\dots+X_n$, $n = 1, 2, \dots$, belongs to the domain of attraction of the normal distribution: i.e. there exists an increasing to infinity sequence $\{b_n\}$ such that $S_n/b_n$ converges in distribution towards the standard normal law as $n \to \infty$.

Let $T := \inf\{k \geq 1 : S_k \leq g_k\}$ be the first crossing time over the moving boundary $\{g_n = o(b_n)\}$ by the random walk $\{S_n\}$. We consider in the talk the asymptotic behavior of the upper tail $\mathbf{P}(T > n)$.

The known classical case is when random walks have zero means, finite variances and $B_n^2 := \mathbf{E}[S_n^2] \to \infty$. If the Lindeberg condition is satisfied then

$$\mathbf{P}(T > n) \sim \sqrt{\frac{2}{\pi}}\frac{U_n}{B_n} \quad \text{with} \quad U_n := \mathbf{E}[S_n - g_n; T_g > n]. \tag{1}$$

(See *Ann. Probab.*, 2018, pp. 3313-3350.)

In the present talk we focus on the further results in this direction.

In particular, we are not going to assume that all summands have finite variances or even finite expectations. Denote by $X_n^{[u_n]}$ the truncation of the random variable $X_n$ on the levels $\pm u_n$, where $u_n/b_n \to 0$ sufficiently slow. In this case

$$\mathbf{P}(T > n) \sim \sqrt{\frac{2}{\pi}}\frac{U_n(u_n)}{b_n} + J_n(u_n, b_n), \tag{2}$$

where $U_n(u_n)$ is defined similar to $U_n$ in (1), but for the random walk $X_1^{[u_n]} + \dots + X_n^{[u_n]}$ instead of $S_n$. Note that the value $J_n(u_n, b_n)$ from (2) is found in explicit way as a function of distributions of positive jumps of random variables $X_1 - u_n, \dots, X_n - u_n$.

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