

Primitive recursive reverse mathematics

Marta Fiori Carones

Sobolev Institute of Mathematics, Russia

(Joint work with Nikolay Bazhenov, Jiayi Liu, and Alexander Melnikov) Reverse mathematics allows to formally answer questions like whether a certain theorem, from ordinary mathematics, implies another one. Since one works over the base theory RCA_0 , reverse mathematics allows to prove that theorem A implies theorem B , or not, provided that computable operations can be carried out. We want to consider the same type of questions, but restricting our tools to primitive recursive operations. To this end we need to consider a weaker base theory, which encapsulates the idea of primitive recursiveness. PRA^2 , introduced by Kohlenbach, serves our purposes. In fact, it is a second order extension of the first order theory PRA in the two sorted language with variables for natural numbers and for functions. Axioms of PRA^2 extend those of PRA with defining equations for all primitive recursive functionals of type 2 (i.e. functions with functions as argument), allowing thus to speak about e.g. analysis, infinite combinatorics and algebra. Taking PRA^2 as base theory allows thus to study whether a statement (possibly computably true) is also primitively recursively true, and if not, which is its strength. During the talk some examples are presented.