Computability-theoretic reduction games in reverse mathematics

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Abstract: Reverse mathematics gives us a way to compare the relative strength of theorems by establishing implications and nonimplications over a weak subsystem of second-order arithmetic, typically RCA₀, which corresponds to computable mathematics, but with restricted induction. In many, but not all, cases, nonimplications over RCA₀ are proved using ω -models, i.e., models of RCA₀ with standard first-order part. Implication over RCA₀ and over ω -models are not fine enough for some purposes, however, so other notions of computability-theoretic reduction between theorems have been studied. Reduction games give us a framework for these notions, and also allow us to transfer them from the ω -model setting to arbitrary models of RCA₀, or other weak base theories such as RCA₀* or RCA₀ with additional induction axioms. We will discuss work on these games in work of Hirschfeldt and Jockusch; Dzhafarov, Hirschfeldt, and Reitzes; and Reitzes.