Kleene Star in Substructural Logics

Stepan Kuznetsov
Steklov Mathematical Institute, Russia

Abstract: A logical system is called substructural if it lacks all or some of the structural rules: contraction, weakening, exchange. The basic substructural logic is multiplicative-additive Lambek calculus (MALC), where none of these rules are allowed. We also consider the commutative modification of MALC which allows exchange. These substructural logics may be extended by structural modalities called subexponentials, which allow applying structural rules in a controlled way. Such modalities may lead to undecidability. Another source of undecidability is Kleene iteration, or Kleene star. The most interesting things with this operation start happening when it is axiomatised using the omega-rule. In this case, the combination of Kleene star with subexponentials raises the complexity up to Π^1_1, while without subexponentials the system is only Π^0_1-complete. We also show some natural fragments of intermediate complexity, for one of which we conjecture that its complexity is properly hyperarithmetical. The talk is partially based on joint work with Stanislav Speranski.