Generic Muchnik reducibility

Joseph Miller
University of Wisconsin–Madison, USA

Abstract: If A and B are countable structures, then A is Muchnik reducible to B if every ω-copy of B computes an ω-copy of A. This can be interpreted as saying that B is intrinsically at least as complicated as A. Schweber suggested a natural extension of Muchnik reducibility to arbitrary structures: if A and B are (possibly uncountable) structures, then A is generically Muchnik reducible to B if in some (equivalently, any) forcing extension that makes both A and B countable, A is Muchnik reducible to B.

I will survey most of what is known about the generic Muchnik degrees, culminating in work with Andrews, Schweber, and Soskova. We have proved the existence of a structure with degree strictly between Cantor space and Baire space. It remains open whether an expansion of Cantor space can be strictly in between, but we have proved that no closed expansion or unary expansion can work. Similar results hold for the interval between Baire space and the Borel complete degree (i.e., the degree that bounds all Borel structures). The proofs mix descriptive set theory (including some use of determinacy) with injury and forcing constructions native to computable model theory.