Computability in C[0,1]

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Abstract: I will talk about several recent results about the space of continuous functions on [0, 1] and, more generally, on a compact domain. I will discuss several results many of which I view as highly unexpected, including:

- 1. A characterization of continuous regular (i.e., automatic) functions on [0, 1].
- 2. An effective characterization of C[0,1] among other separable Banach spaces; this answers a question of McNicholl.
- 3. The effective Banach–Stone duality between K and C(K; R) for profinite K, and its corollaries.
- 4. The computable and punctual universality of C[0,1] among all Polish spaces; arguably, this settles a problem posed by Sierpinski in the 1940-s.