On the complexity of describing topological bases for QCB_0-spaces

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Abstract: I will present some previous joint work with Victor Selivanov and Matthias Schröder on the descriptive complexity of describing topological bases for QCB_0-spaces (T_0 quotients of countably based spaces). QCB_0-spaces are an important class of topological spaces that includes all countably based T_0-spaces, forms a countably (co-)complete cartesian closed category, and supports a natural computability theory. Although classical descriptive set theory naturally extends to all countably based spaces, the situation becomes more complicated when extending to non-countably based QCB_0-spaces. The basis complexity we discuss here is just one aspect of the descriptive set theory of QCB0-spaces. Its purpose is to provide a more careful organization of the topological complexity of QCB_0-spaces beyond the simple "countably based" versus "non-countably based" dichotomy. There have been important developments in this area (particularly by Mathieu Hoyrup and his colleagues) since we presented this joint work more than seven years ago, but it is a vast field that is still mostly unexplored. We hope this talk attracts more attention to this area of descriptive set theory, and also provides a small sample of Victor Selivanov's many contributions to extending the field of descriptive set theory to rich classes of mathematical structures.