

Describing the Wadge hierarchy on arbitrary zero-dimensional Polish spaces

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Abstract: Let X be a topological space. The continuous reducibility relation among subsets of X is a quasi-order, and its quotient is called Wadge hierarchy. Under suitable set-theoretic assumptions, such hierarchy has been completely described when X is the Baire space: it is well-founded and has antichains of size at most 2, and moreover one can precisely determine at which levels we have such antichains. Using a simple argument, one can deduce that if we move to any other zero-dimensional Polish space, then the Wadge hierarchy is again induced by a well-quasi-order, but its exact behaviour has apparently been overlooked in the literature. We fill this gap by classifying (up to isomorphism) all possible shapes of the Wadge hierarchy on an arbitrary zero-dimensional Polish space X . This analysis naturally involves the Cantor-Bendixson rank of X , together with a new rank based on the so-called “compact derivative”. This is joint work with Carroy and Scamperti.