A Mathematical Journey

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Main Sites of the Journey

1. Kazan.
2. Novosibirsk.
3. Germany.
4. Dagstuhl.
5. CA and CID.
6. France++.
7. Japan.
8. Mathematical Centers.

Kazan

The first lucky experience was the status of a student of Kazan University, along with many nice guys like Nikolay Lobachevsky, Lev Tolstoy, and Vladimir Lenin:)
The second one was to be among first students of Marat Arslanov, the founder of Kazan computability school. In the early times, we discussed degrees of unsolvability and things around Arslanov's completeness criterion.

My first published results, though, were about computable numberings. In this field, I
a) proved that any nontrivial semilattice of computable numberings is not a lattice;
b) constructed an example of a not effectively discrete family of total recursive functions having the unique (up to equivalence) computable numbering;
c) constructed an example of a not discrete family of recursively enumerable sets having the unique (up to equivalence) computable numbering.

Kazan–Novosibirsk

Any of these results was an answer to a well–known open problem on computable numberings.
These became the central results of my candidate dissertation (PhD thesis) "On computable numberings" defended at Novosibirsk University.
Later some of them found applications in Computable Model Theory — a central topic in the Siberian Logical School.

I have also proved several facts on degrees of unsolvability, e.g. classified the possible versions of *tt*-reducibilities (this was independently done by Bulitko) and completely described relationships between Ershov's hierarchy and the high–low hierarchy of degrees of unsolvability (this was independently done by C. Jockusch and R. Shore).

In the beginning 1980s, a detailed investigation of the possibility to extend analogs of the Rice–Shapiro theorem for other levels of the arithmetic hierarchy (as well as of some of its refinements), were made for a broad class of natural numberings including the standard numberings of c.e. sets and of computable partial functions.

This required to adjust the definition of Borel hierarchy to non-Hausdorff spaces which turned out standard in the later development of DST for quasi-Polish spaces. Recent results of M. Hoyrup on DST for non-countably-based spaces are reminiscent of the mentioned facts on extensional characterization of index sets.

Novosibirsk

As shown by Y.L. Ershov, the difference hierarchy over c.e. sets is closely related to the so called m-jump operator.

In 1979–1982 I defined and investigated several generalizations of this operator. It turned out that these generalized operators are closely related to the theory of complete numberings developed by A.I. Mal'cev and Y.L. Ershov, and to the study of structures of m-degrees of index sets of the numberings of c.e. sets and of computable partial functions.

These results led to simplification and generalization of results of L. Hay about index sets and were developed further by several mathematicians including J. Mohrherr.

Novosibirsk

In 1983, using the above-mentioned jump operations, I defined a refinement of the arithmetical hierarchy (called the fine hierarchy) and proved that it contains many known hierarchies and has some rather strong closure properties with respect to refinements. The m-complete sets in levels of FH are the "natural m-degrees" recently rediscovered by T. Kihara and A. Montalban in a different approach.

These results informally mean that the fine hierarchy is in some sence the finest possible. In 1989, after acquaintance with set–theoretic operations introduced by W. Wadge and A. Louveau, I gave a set–theoretical description of the fine hierarchy showing that it is, in some exact sence, the finitary effective version of the Wadge hierarchy of Borel sets.

This led to the possibility of defining this hierarchy in very different contexts, e.g. in logic and complexity theory.

Novosibirsk

The above-mentioned results formed my Doctoral dissertation (Habilitation Thesis) "Hierarchical classification of arithmetical sets and index sets" defended at Sobolev Institute of Mathematics at 1989.

In the mid 1980s I developed a relativized version of the theory of precomplete and complete numberings which led to priority–free proofs of several known results about the structure of tt-type degrees and about the m-degrees of index sets in such structures.

This approach led also to several new results in this field which are also proved very easily (using a version of the Kleene fixed-point theorem in place of the complicated priority constructions).

Also, a broad extension of Arslanov's Completenes Criterion to the relativsed precomplete numberings was achieved which was recently rediscovered by Barendregt and Terwijn.

Novosibirsk

In the beginning of 1990s, I obtained general results on positive numbered boolean algebras (partly joint with Sergey Odintsov). I characterised also many natural and important index sets in the numbering of positive boolean algebras (using some earlier results by S. Lempp on index sets of classes of hyperhypersimple sets). Some results were later rediscovered and developed by A. Montalban and R. Shore.

Later some of these results were extended by me to a very general context of recursively axiomatizable quasivarieties. This research had different consequences, e.g. to classification of many natural index sets in the lattice of c.e. sets, in the semilattice of c.e. m-degrees, in the Lindenbaum algebra of sentences. Some of those results were later developed by S. Lempp, M. Peretiatkin, and R. Solomon.

Germany

During the Humboldt Fellowship in 1992–1993, I defined and considered some analogs of my hierarchies in the context of complexity theory. I extended Kadin's theorem about the non-collapse of the Boolean hierarchy over NP to a much richer fragment of the fine hierarchy over the PTIME-hierarchy.

Later, together with Christian Glasser and Christian Reitwiessner, we returned to this stuff and have solved the question of Blass and Gurevich about the status of reduction (or shrinking) property for the levels of PTIME-hierarchy.

I have also investigated a close relation of hierarchies in complexity theory with the corresponding hierarchies of automata theory via the popular leaf-language approach developed by Wagner's school.

I have also applied the fine hierarchy to classification of regular ω-languages—a classical object of theoretical computer science. It turned out that the resulting classification coincides with the Wagner hierarchy of regular ω-languages. This result established a close relationship between DST and the theory of regular ω-languages. It leads to different proofs of deep and complicated results of K. Wagner, as well as to several new results on regular ω-languages.

Several new results on the dot–depth hierarchy and Straubing-Therien hierarchy of regular languages were obtained in collaboration with Klaus Wagner and his students Christian Glasser and Heinz Schmitz.

Germany

Collaboration with Klaus Weihrauch's school in computable analysis (Vasco Brattka, Peter Hertling, Matthias Schröder, Xizhong Zheng, Norbert Müller), Klaus Keimel's group in domain theory, and Dieter Spreen's group in computable topology led to several concrete results, and to the formation of an active Novosibirsk group working in these areas and their applications including Margarita Korovina, Oleg Kudinov, Svetlana Selivanova, Anton Zhukov, and myself. It is hard to mention here numerous results obtained by members of this group in computable analysis and computable topologies within several national and international projects.

In general, collaborations with the mentioned German colleagues and also others (including Klaus Ambos-Spies, Wolfgang Thomas, Ludwig Staiger, and Martin Ziegler) were very fruitful and (hopefully) useful for both sides.

Dagstuhl

The above-mentioned research was highly influenced and speed-up by the possibility to organize thematic workshops at Dagstuhl Conference Center led by Leibniz Society for Computer Science. It was a great opportunity to organize (with some of the above-mentioned colleagues) a series of workshops on the mentioned topics. Many thanks to the Dagstuhl team for their fantastic work on promoting research in computer science!

These workshops attracted several brilliant young mathematicians and inspired new collaborations and projects which contributed a lot to the mentioned fields. Also many senior scientists took part, bringing their ideas, memories, and vision. For instance, the personal contact with W.W. Wadge helped to open new directions in the Wadge theory and led to a lot of progress in descriptive set theory, automata on infinite objects, and computable topology.

CA and CID

Among many consequences of the mentioned collaborations was the appearance of international projects “Computable Analysis” and “Computing with Infinite Data” coordinated by Dieter Spreen. Both projects led to many results and new research directions for the Novosibirsk team of computable analysis.

For instance, I attracted Oleg Kudinov (and then Anton Zhukov and Lyudmila Yartseva) to the investigation of definability in the structures on words, trees and forests, which is interested on its own and has non-trivial corollaries for the extension of Wagner hierarchy to k-partitions (suggested by me earlier). Some of our results are now well cited in TCS papers.

Later i became aware that independently a similar project for finite algebraic structures was initiated by Ralph McKenzie and continued by several his followers.

CA and CID

Jointly with Svetlana Selivanova we initiated the study of PDEs-solution operators by methods of computable analysis and computer algebra, complementing some earlier results by Klaus Weihrauch and Ning Zhong. Our approach yields new classification of PDEs complementing the classical ones.

Based on my joint work with Pavel Alaev about feasible presentations of countable algebraic structures, we jointly with Svetlana found non-trivial connections between computability and complexity in discrete (countable) and continuous (non-countable) structures which mix methods of precise symbolic algorithms and approximate numeric ones.

Applications of complexity theory to PDE were also investigated by Martin Ziegler and his students recently jointly with Svetlana. Their new approaches to PDE-theory are fast evolving and attracted attention of some famous experts in classical numerics.

France

Among consequences of events at the at two previous sites of the journey was collaboration with researchers from other countries, in particular from France and Japan. Collaborations with Jean-Eric Pin, Jacques Duparc, Olivier Finkel and other representatives of the powerful French school in automata theory (hopefully) had some influence on the extensive investigation of Wadge degrees of languages of infinite words and trees recognizable by different kinds of automata. This is currently an active research field where also the work of Polish school of automata and descriptive set theory is very important.

Also, collaboration with Matthew Hoyrup and his group was important for the work in computable topology within the CID-project. This led to essential progress in understanding effective quasi-Polish spaces (along with independent work of de Brecht-Pauli-Schröder) and (jointly with Takayuki Kihara) degree spectra of topological spaces (along with independent work of Harrison-Trainor–Melnikov–Ng).

Japan

Another consequence was a close collaboration with several Japanese colleagues, especially with Mizuhito Ogawa, Matthew de Brecht and Takayuki Kihara. This collaboration led to new results on automata and well quasi-orders (jointly with Mizuhito), computable topology (jointly with Matthew and Takayuki) and Wadge theory (jointly with Takayuki).

This collaboration apparently influenced not only our work but also work of other members of the Japanese and Novosibirsk teams, in particular within the very successful separate joint RFBR-JSPS joint project and a series of workshops related to that project.

Mathematical Centers

A recent site of the long journey was the creation of several mathematical centers at leading Russian universities for promoting mathematical research and education. This big project seems a real success because it intensified many mathematical collaborations between researchers in Russia and abroad.

Collaborations in our field were quite intensive, many results were already obtained in Kazan and Novosibirsk Centers, supported by activities of Moscow and Sochy Centers. The collaboration is intensified by a lot of visits of young and senior mathematicians between the centers, and organization of several interesting conferences. One might only hope that the activities of mathematical centers will continue even in the current hard time, and new events follow.

Thanks

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