Symmetries in physical dilations of open quantum systems

Marco Cattaneo

New Trends in Mathematical Physics

November 8th, 2022

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

IFISC | Universitat de les Illes Balears | CSIC

Warning: incomplete results!

Warning: incomplete results!

Feel free to interact during the talk!

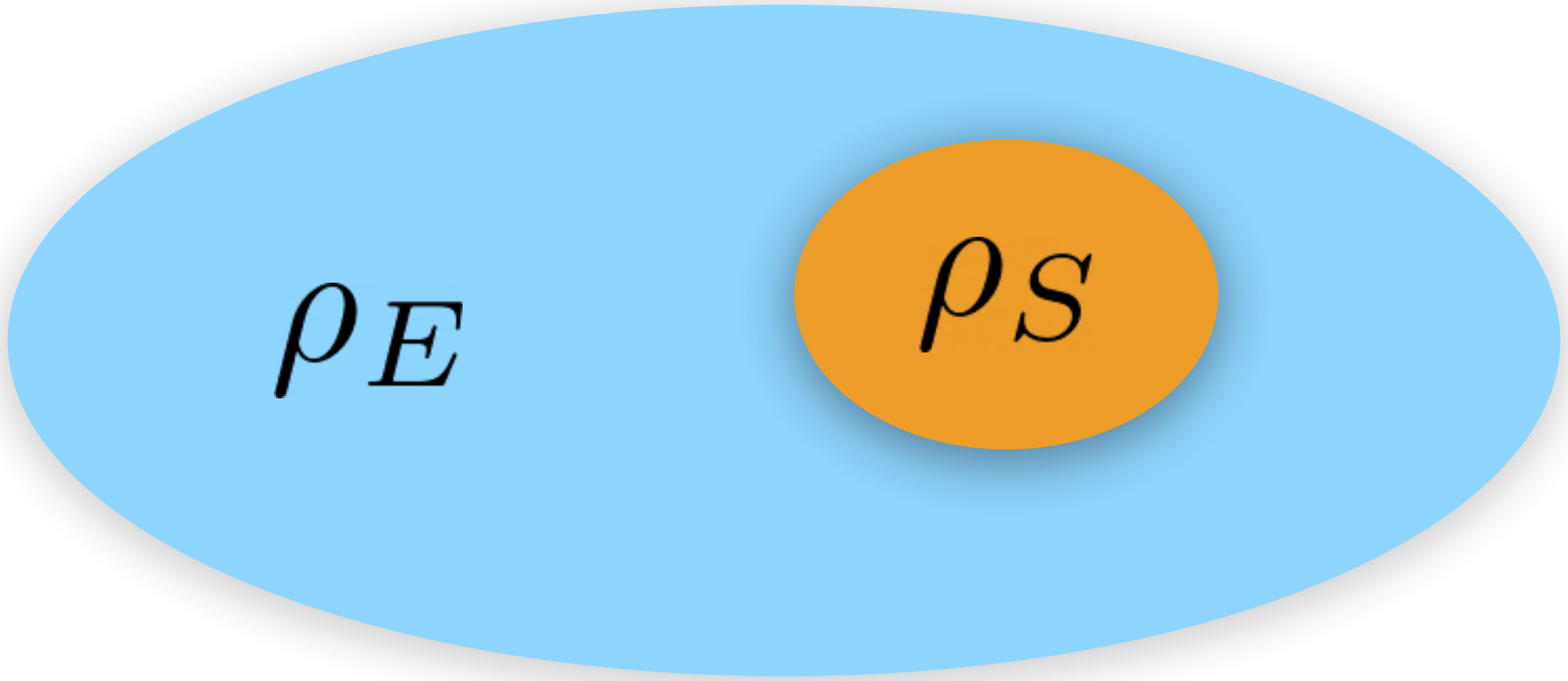
Warning: incomplete results!

Feel free to interact during the talk!

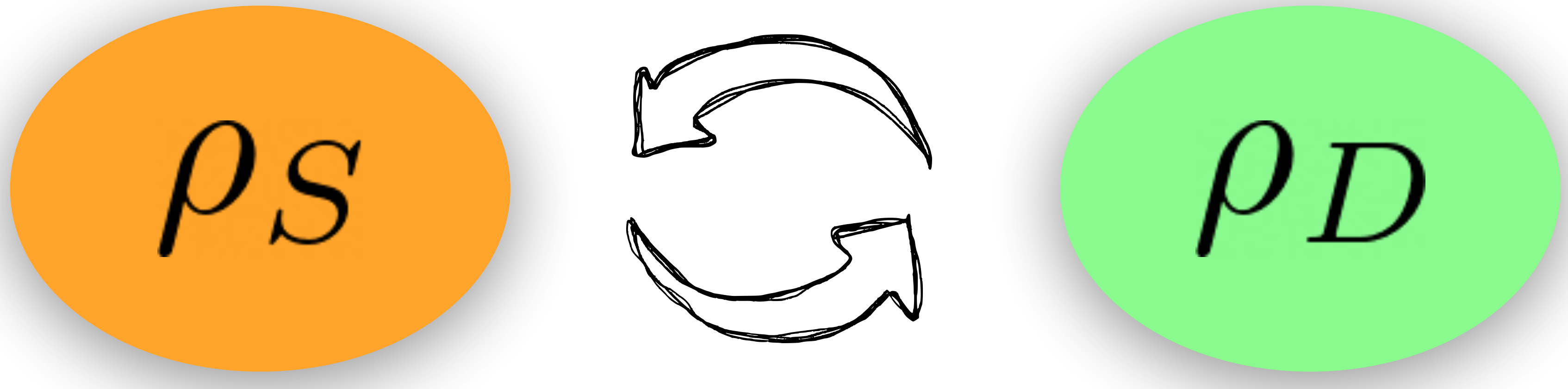
Feel free to contact me if you have questions/ideas about how to complete the results!

Main idea of my talk

An open quantum system dynamics



Its physical dilation

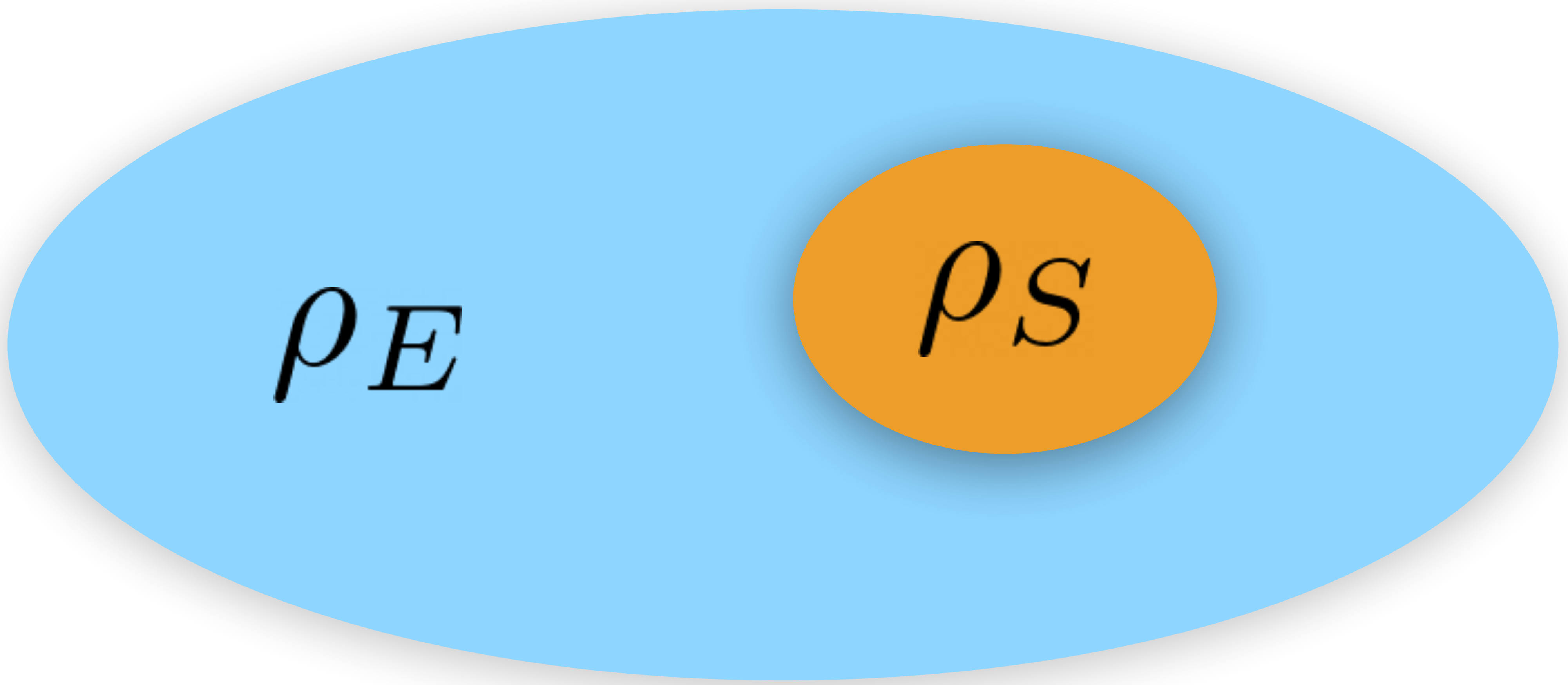


$$\phi_S(t) \rightleftarrows U_{SD}(t)$$

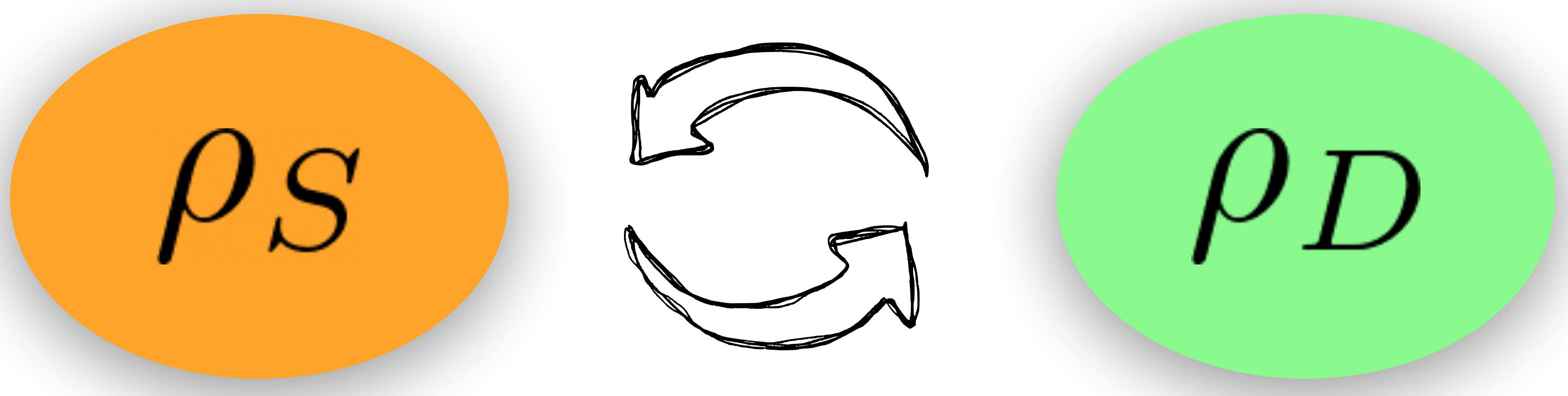
$$\phi_S(t)[\rho_S] = \mathrm{Tr}_D[U_{SD}(t)\rho_S \otimes \rho_D U^\dagger_{SD}(t)]$$

Main idea of my talk

An open quantum system dynamics



Its physical dilation

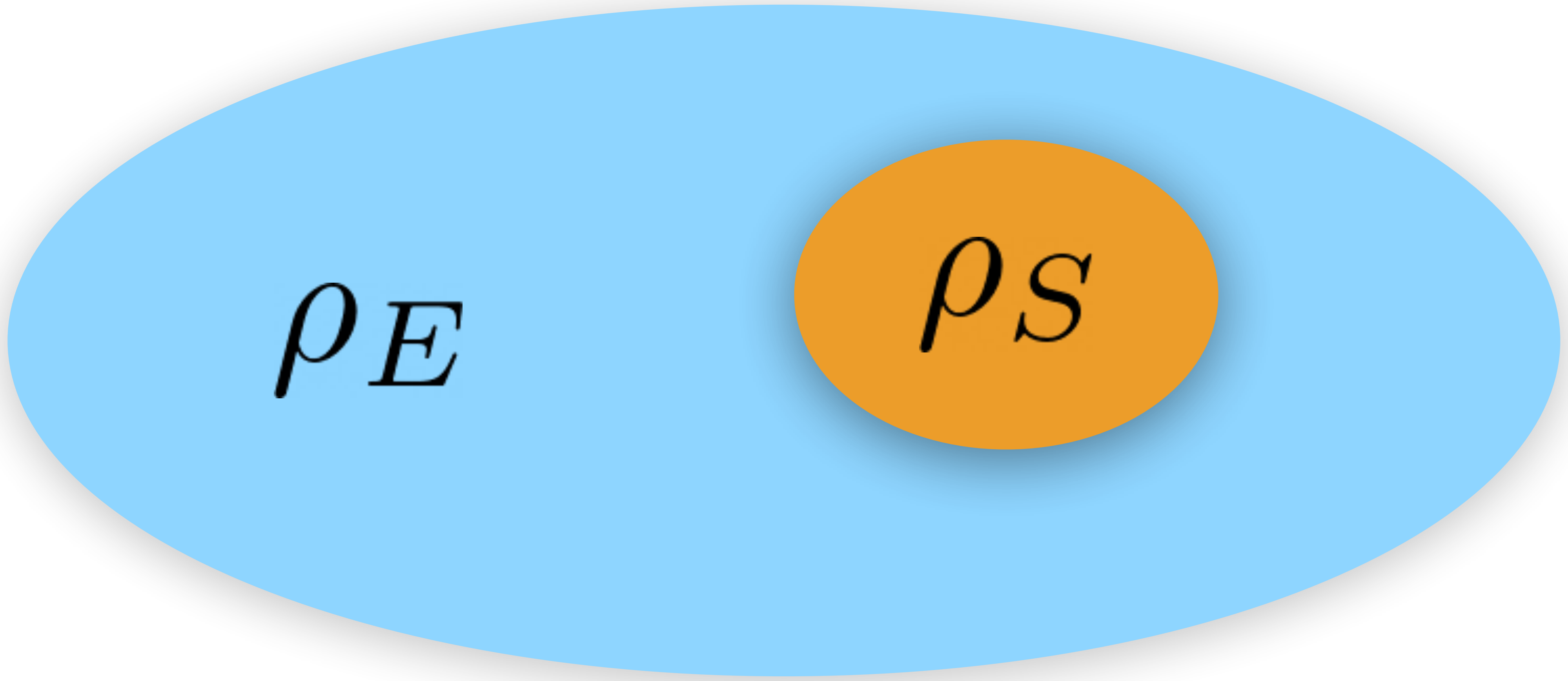


$$\phi_S(t) \rightleftarrows U_{SD}(t)$$

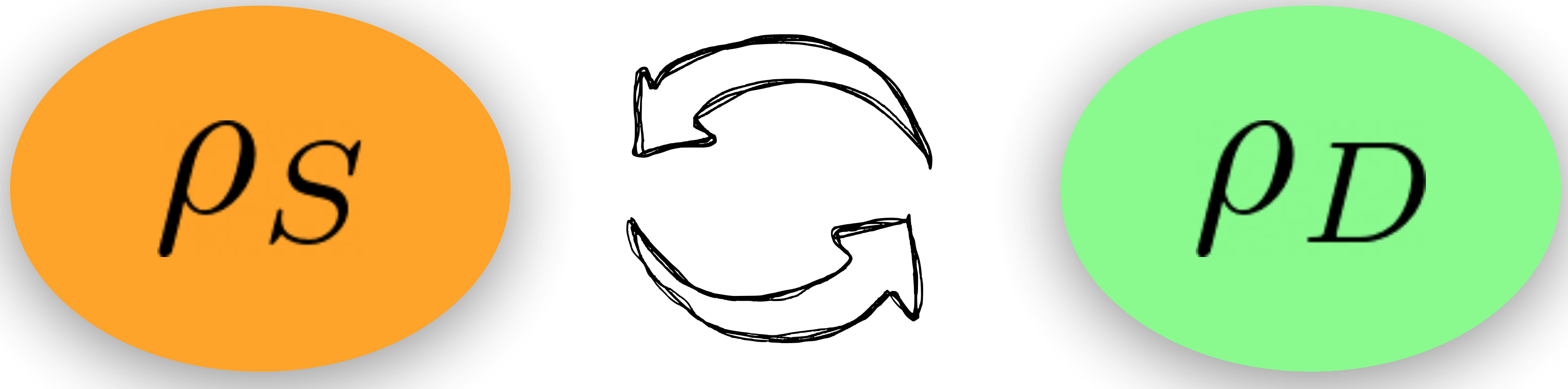
Symmetry: $\mathcal{U}_S(g)$

Main question of my talk

An open quantum system dynamics



Its physical dilation



$$\phi_S(t) \rightleftarrows U_{SD}(t)$$

Symmetry: $\mathcal{U}_S(g)$ $\rightleftarrows$ Symmetry: $\pi_{SD}(g)$

Conserved quantity: J_{SD}

Outline:

- Symmetries in open quantum systems
- Stinespring’s dilation and symmetries therein
- Results
- Open questions

Symmetries in open quantum systems

Quantum state: ρ_S

Quantum dynamical map: ϕ_S

Representation of a group as an operator on the system Hilbert space: $U_S(g)$

Superoperator representation: $\mathcal{U}_S(g)[O_S] = U_S(g) O_S U_S^\dagger(g)$

$\forall g$

I will keep it implicit in the remaining of the talk!

Symmetries in open quantum systems

Quantum state: ρ_S

Quantum dynamical map: ϕ_S

Representation of a group as an operator on the system Hilbert space: $U_S(g)$

Superoperator representation: $\mathcal{U}_S(g)[O_S] = U_S(g) O_S U_S^\dagger(g)$

Symmetry of the open system dynamics:

$$\mathcal{U}_S(g)[\phi_S]\mathcal{U}_S^\dagger(g) = \phi_S$$

Or equivalently ***covariance*** of the open system dynamics:

$$U_S(g)\phi_S[\rho_S]U_S^\dagger(g) = \phi_S[U_S(g)\rho_S U_S^\dagger(g)]$$

Symmetries in open quantum systems

Symmetry of the open system dynamics:

$$\mathcal{U}_S(g)[\phi_S]\mathcal{U}_S^\dagger(g) = \phi_S$$

<u>Noether's theorem is not valid anymore</u>

$$U_S(g) = \exp(igJ_S)$$

Then a symmetry does not imply a conserve quantity, and vice versa:

$$\mathcal{U}_S(g)[\phi_S]\mathcal{U}_S^\dagger(g) = \phi_S \quad \not\longleftrightarrow \quad \phi_S^\dagger[J_S] = J_S$$

Stinespring's dilation and symmetries therein

Quantum dynamical map: ϕ_S Hilbert space $\mathcal{H}_S$

Stinespring's dilation theorem:

It always exists

$\mathcal{H}_D$ Hilbert space $V : \mathcal{H}_S \to \mathcal{H}_{SD}$ Isometry

such that for any operator on the Hilbert space of the system

$$\phi_S^\dagger[O] = V^\dagger O \otimes \mathbb{I}_D V$$

Stinespring's dilation and symmetries therein

Quantum dynamical map: ϕ_S

Hilbert space $\mathcal{H}_S$

Physical dilation:

It always exists

$\mathcal{H}_D$ Hilbert space

$|\psi_D\rangle$ State of the dilation

U_{SD} Unitary operator

such that for any state in the Hilbert space of the system

$$\phi_S[\rho_S] = \mathrm{Tr}_D[U_{SD}\rho_S \otimes |\psi_D\rangle\langle\psi_D| U_{SD}^\dagger]$$

Mixed states of the dilation can be brought into this form via purification

Stinespring's dilation and symmetries therein

Stinespring's dilation theorem:

$$\phi_S^\dagger[O] = V^\dagger O \otimes \mathbb{I}_D V$$

Theorem (Scutaru 1979 and then Keyl, Werner 1999):

$$U_S(g)\phi_S[\rho_S]U_S^\dagger(g) = \phi_S[U_S(g)\rho_S U_S^\dagger(g)]$$

For minimal dilations!

It always exists $\tilde{U}_D(g)$ $\quad \pi_{SD}(g) = U_S(g) \otimes \tilde{U}_D(g)$

such that

$$VU_S(g) = \pi_{SD}(g)V$$

Main question of my talk

Quantum dynamical map labeled by time with a symmetry (or covariant property):

$$U_S(g)\phi_S(t)[\rho_S]U_S^\dagger(g) = \phi_S(t)[U_S(g)\rho_S U_S^\dagger(g)]$$

Let's consider a generic, continuous in time physical dilation of the quantum map driven by an interaction Hamiltonian:

$$\phi_S(t)[\rho_S] = \mathrm{Tr}_D[U_I(t)\rho_S \otimes \rho_D U_I^\dagger(t)]$$

$$U_I = \exp(-iH_I t)$$

What are the constraints on this Hamiltonian set by the presence of the symmetry on the quantum map?

Results

Using Scutaru 1979 and Keyl, Werner 1999:

For minimal dilations!

$$U_S(g) \otimes \tilde{U}_D(g,t) U_I(t) |\varphi_S\rangle \otimes |\psi_D\rangle = U_I(t) U_S(g) |\varphi_S\rangle \otimes |\psi_D\rangle$$

Results

Using Scutaru 1979 and Keyl, Werner 1999:

For minimal dilations!

$$U_S(g) \otimes \tilde{U}_D(g,t) U_I(t) |\varphi_S\rangle \otimes |\psi_D\rangle = U_I(t) U_S(g) |\varphi_S\rangle \otimes |\psi_D\rangle$$

Stationary covariance conjecture:

$$\tilde{U}_D(g,t) = \tilde{U}_D(g)$$

The same for all time, *to be proven!!!*

Results

Using Scutaru 1979 and Keyl, Werner 1999 and the stationary covariance conjecture:

$$U_S(g) \otimes \tilde{U}_D(g) U_I(t) |\varphi_S\rangle \otimes |\psi_D\rangle = U_I(t) U_S(g) |\varphi_S\rangle \otimes |\psi_D\rangle$$

Then:

$$\boxed{\tilde{U}_D(g)|\psi_D\rangle = |\psi_D\rangle \quad \forall g}$$

Results

Using Scutaru 1979 and Keyl, Werner 1999 and the stationary covariance conjecture:

$$U_S(g) \otimes \tilde{U}_D(g) U_I(t) |\varphi_S\rangle \otimes |\psi_D\rangle = U_I(t) U_S(g) |\varphi_S\rangle \otimes |\psi_D\rangle$$

Then:

$$\boxed{\tilde{U}_D(g)|\psi_D\rangle = |\psi_D\rangle \quad \forall g}$$

Is this always possible for non-Abelian groups?

Results

Using Scutaru 1979 and Keyl, Werner 1999 and the stationary covariance conjecture:

$$U_S(g) \otimes \tilde{U}_D(g) U_I(t) |\varphi_S\rangle \otimes |\psi_D\rangle = U_I(t) U_S(g) |\varphi_S\rangle \otimes |\psi_D\rangle$$

Then:

$$\boxed{\pi^{\dagger}_{SD}(g) H_I \pi_{SD}(g)|_{\mathcal{K}_{\|}} = H_I|_{\mathcal{K}_{\|}} \quad \forall g}$$

$$\pi_{SD}(g) = U_S(g) \otimes \tilde{U}_D(g)$$

Results

$$\pi^\dagger_{SD}(g)H_I\pi_{SD}(g)|\mathcal{K}_{||} = H_I|\mathcal{K}_{||}$$

Krylov subspace:

Basis of the system Hilbert space

$$\mathcal{K}_{||} = \sum_j \mathcal{K}(H_I, |\varphi_{S,j}\rangle \otimes |\psi_D\rangle)$$

Maximal power, after which the vectors are not linearly independent anymore

$$\mathcal{K}(H, |v\rangle) = \mathrm{Span}\{|v\rangle, H|v\rangle, H^2|v\rangle, \ldots, H^r|v\rangle\}$$

For an Hermitian operator, the system Hilbert space can be decomposed as a direct sum of Krylov subspace starting from different initial states that form an orthogonal basis

Results

$$\pi^{\dagger}_{SD}(g) H_I \pi_{SD}(g)|_{\mathcal{K}_{||}} = H_I|_{\mathcal{K}_{||}}$$

$$\pi_{SD}(g) = U_S(g) \otimes \tilde{U}_D(g)$$

$$\pi_{SD}(g) = \exp(ig(J_S \otimes \mathbb{I}_D + \mathbb{I}_S \otimes J_D))$$

Conserved quantity in the sum of Krylov subspaces $\mathcal{K}_{||}$

Further results:

- Non minimal dilations
- Mixed states of the environment
- Non-Hamiltonian evolutions
- Examples with Abelian and non-Abelian groups

Open questions

- Prove the stationary covariance conjecture
- Find constraints on the type of groups for which we can find a suitable state of the environment
- Is there a minimal dimension of the subspace $\mathcal{K}_{||}$?
- Application to collision models (discrete time but in the limit of very small timestep)
- What if there is a strong symmetry on the quantum map level (a conserved quantity in the open system dynamics)?

Thanks for nice discussions on the topic to:

Roberta Zambrini

Sabrina Maniscalco

Gian Luca Giorgi

Sergey Filippov

THANK YOU!