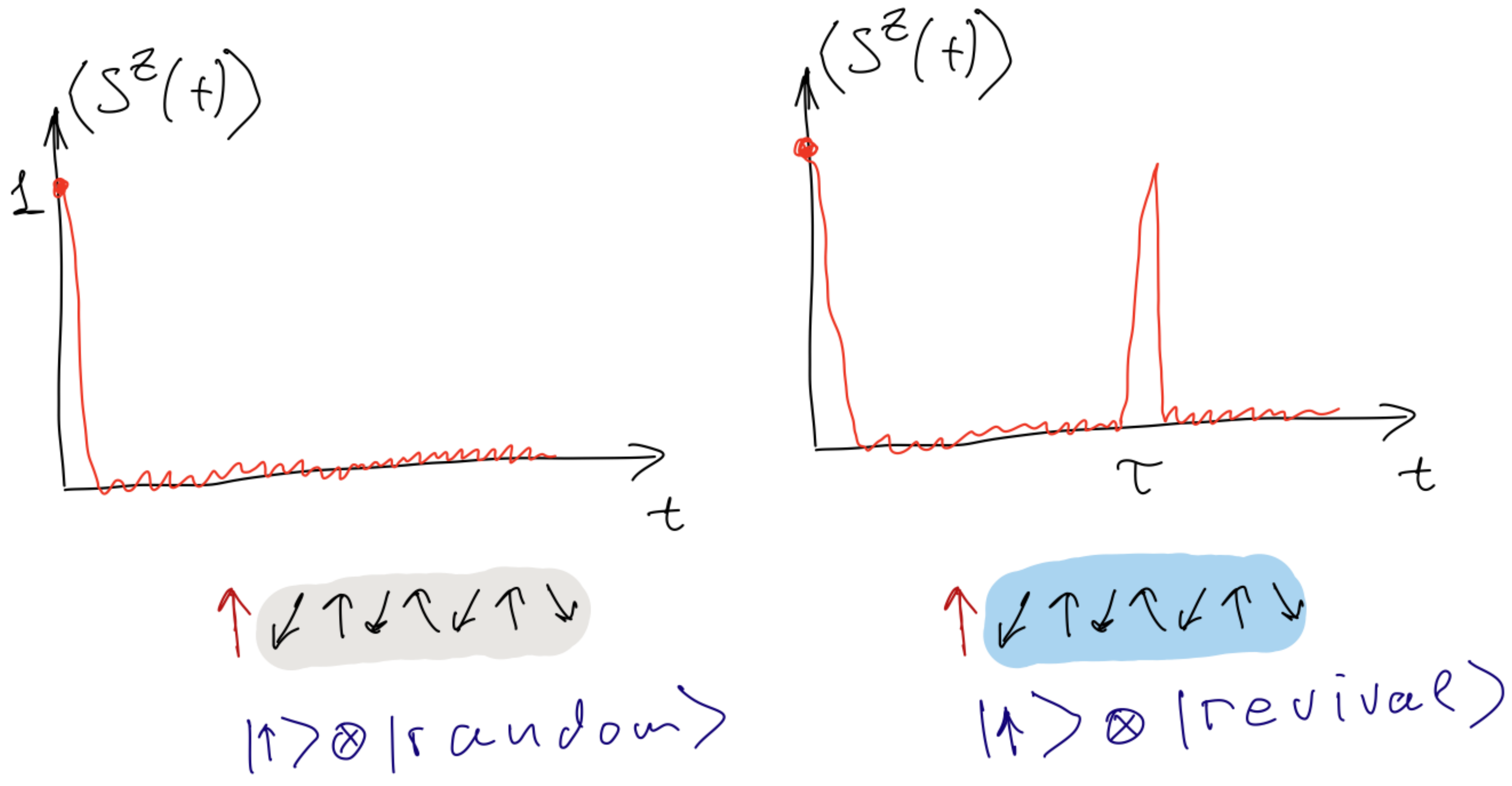
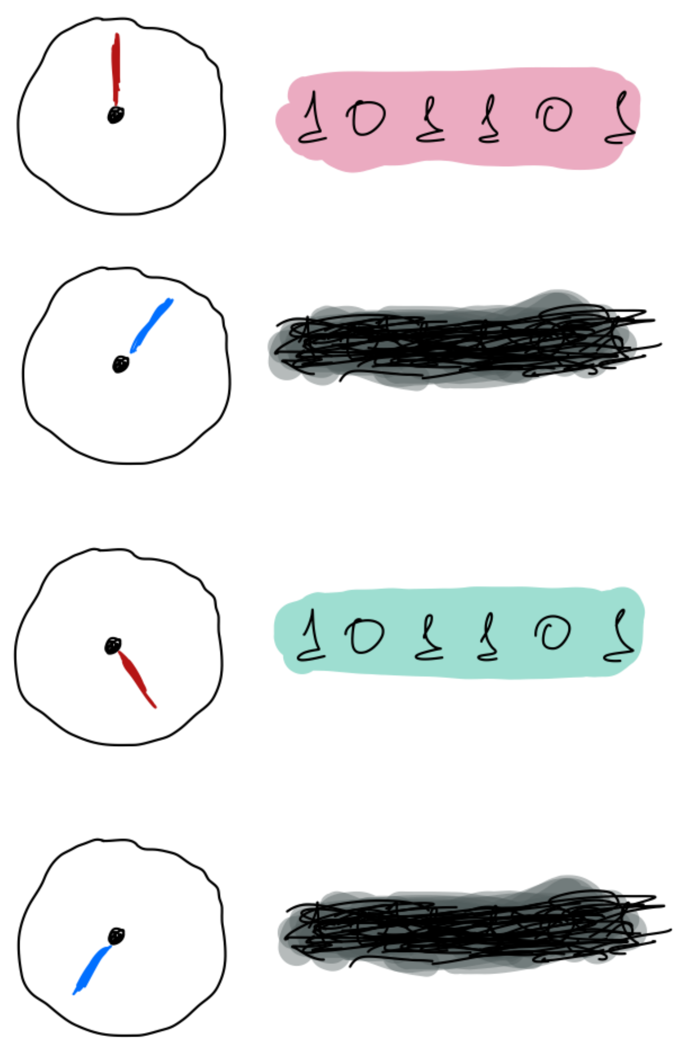
Almost complete local revivals and delayed disclosure of a secret



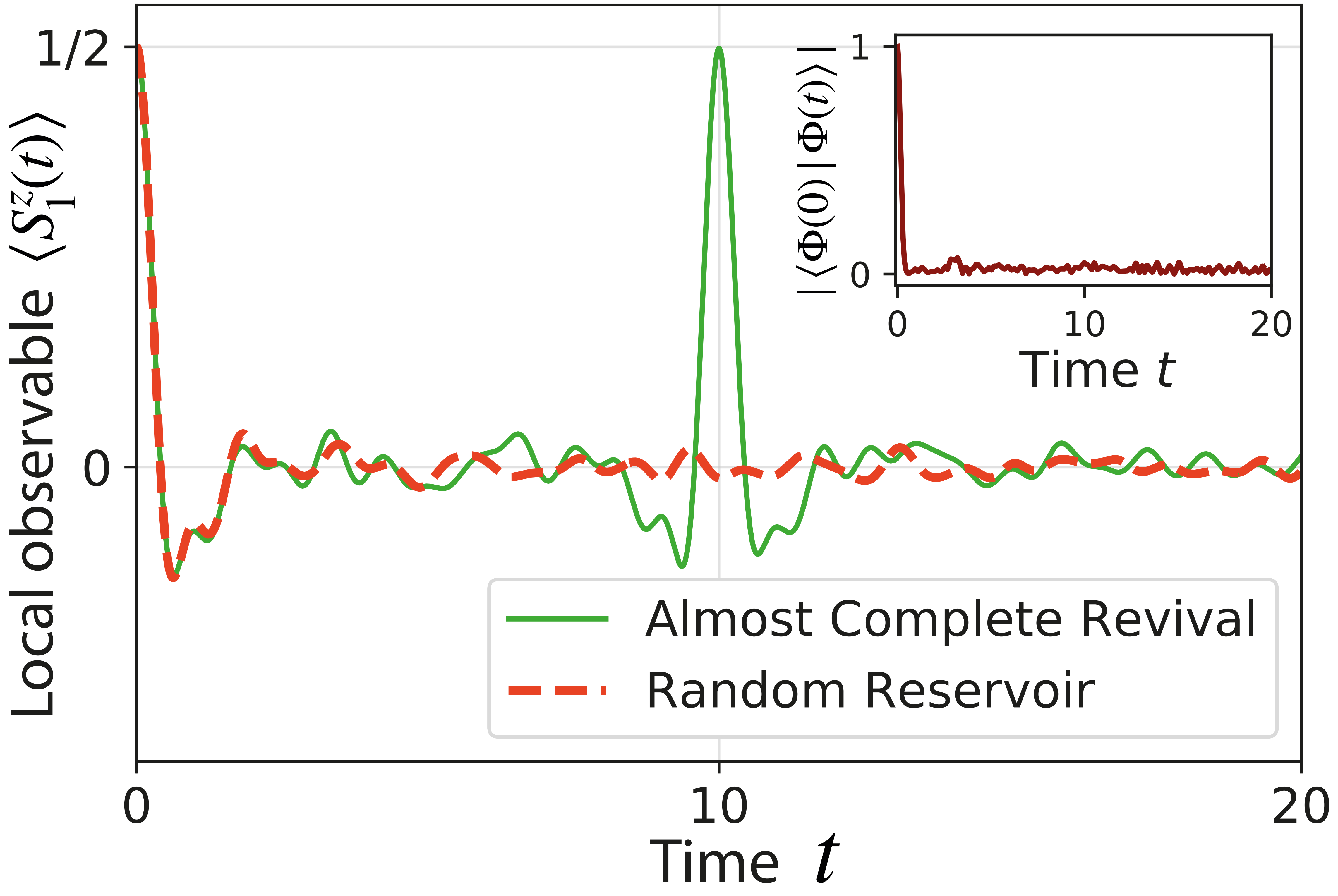


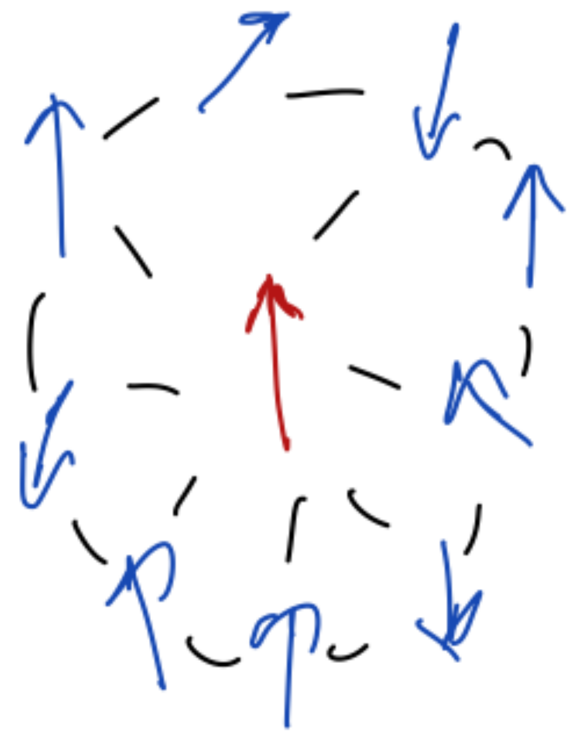
Ermakov, I., & Fine, B. V. (2021). Almost complete revivals in quantum many-body systems. Physical Review A, 104(5), L050202.
Ermakov, Igor. "Generalized Almost Complete Revivals in quantum spin chains." *arXiv preprint arXiv:2205.05584* (2022).

Igor Ermakov, 10 November 2022, MIAN

The work of I.E. was funded by Russian Federation represented by the Ministry of Science and Higher Education (Grant Number 075-15-2020-788).
Basis Foundation (Grant No. 18-1-5-19-1).

Key takeaway





Spin inside the reservoir

Random **VS** Finely tuned

There are special initial states such that selected local observable exhibits almost complete revival at predetermined moment of time

✅ Almost arbitrary spin Hamiltonian

✅ Accessible in small and experimentally relevant systems

✅ Exists in large chains (thermodynamic limit)

Motivation

Assumption: many-body system must reach the state of thermal equilibrium **(thermalize)**

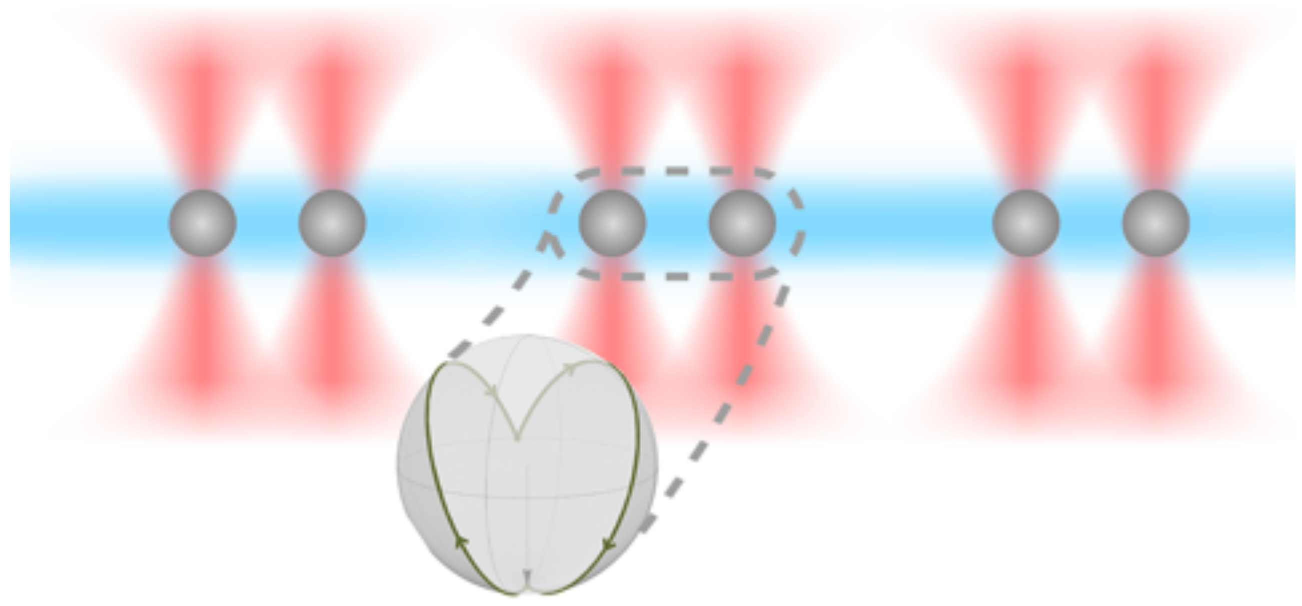
Reality: we observe many systems out of equilibrium

Questions: Timescales for thermalization?
What happens during thermalization?
Physical mechanisms preventing system from thermalization?

Why: Applicability limits for the above assumption
Non-equilibrium states are interesting

Living cell

Sun



Array of Rydberg atoms

Glace coffee

Equilibration in spin systems

In quantum many-body systems local observables quickly relax to its thermal equilibrium

Consider some playground Hamiltonian

$$H = \sum_{j=1}^{L} \left(J_x S_j^x S_{j+1}^x + J_y S_j^y S_{j+1}^y \right) + \sum_{j=1}^{L} \left(h_x S_j^x + h_y S_j^y \right)$$

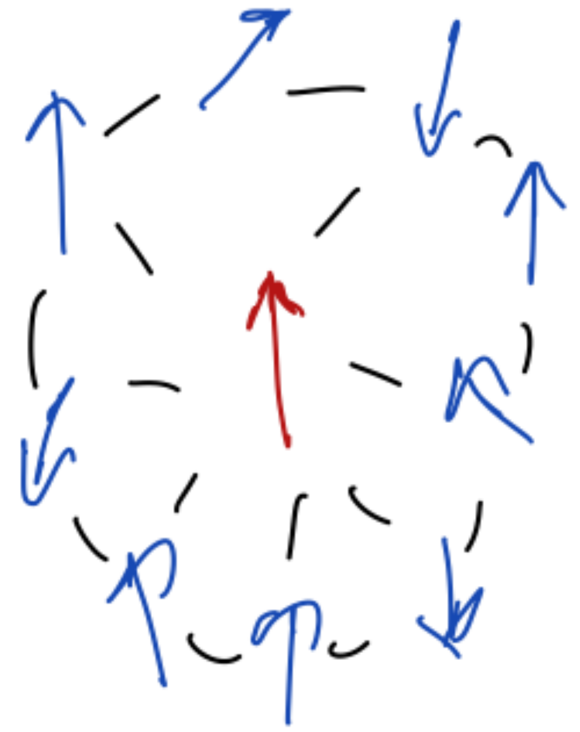
constants: $(J_x, J_y, h_x, h_y) = (-2.0, -4.0, 2.2, 2.2)$

This Hamiltonian is NOT special

No Integrability **No** MBL **No** Constrains

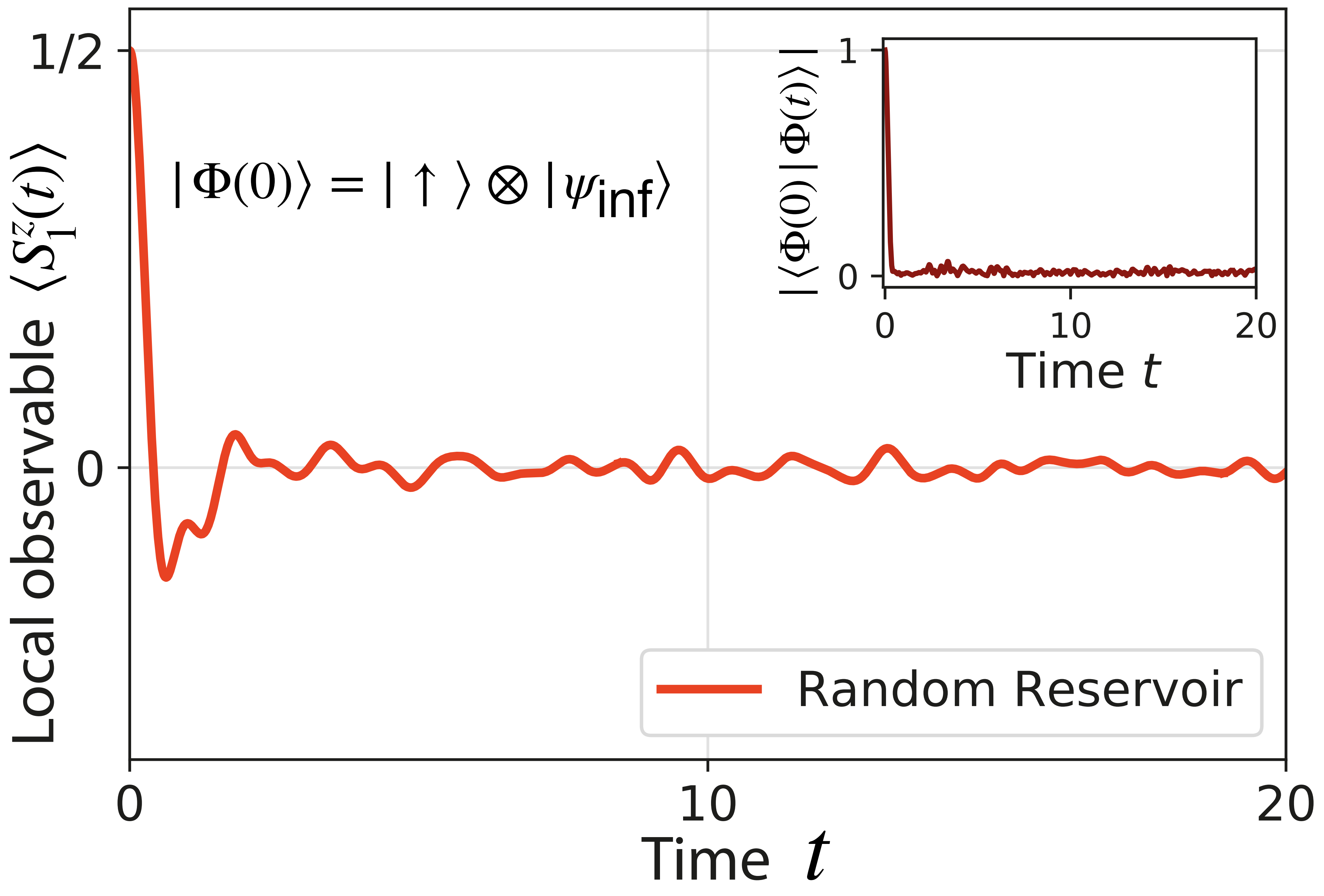
No Long-range interactions, etc

Satisfies ETH, r-value=0.58



Equilibrium value is in XY plane

Qubit is exited along Z axis



Number of spins: $L = 12$

Quantum recurrence theorem

If we do nothing, will this observable ever recover?

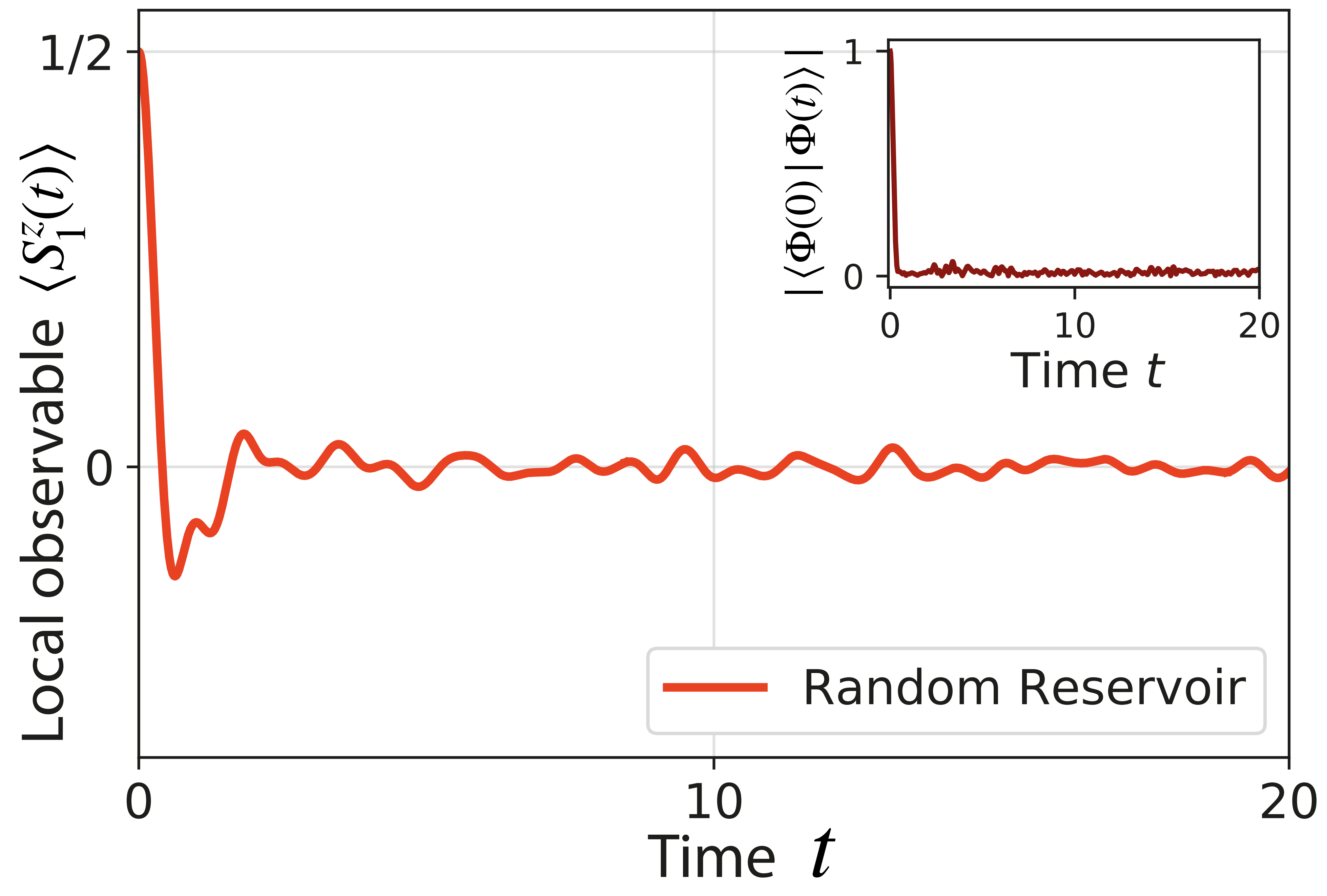
$|\Phi_{\text{in}}(0)\rangle = |1_1\rangle \otimes |\Psi_{\text{rnd}}\rangle$

In finite-size system **YES**, always.

Quantum Recurrence Theorem (1957) by P. Bocchieri and A. Loinger Phys. Rev. 107, 337

$$|\Psi(t) - \Psi(0)|^2 = 2\sum_{n=0}^{N} |c_n|^2 (1 - \cos(E_n t))$$

At some point in a (really) distant future $\psi(t)$ returns arbitrary close to $\psi(0)$



Number of spins: $L = 12$

However, for many-body system the recurrence time is exponentially large with respect to Hilbert space dimensionality N

Can we observe earlier recurrence?

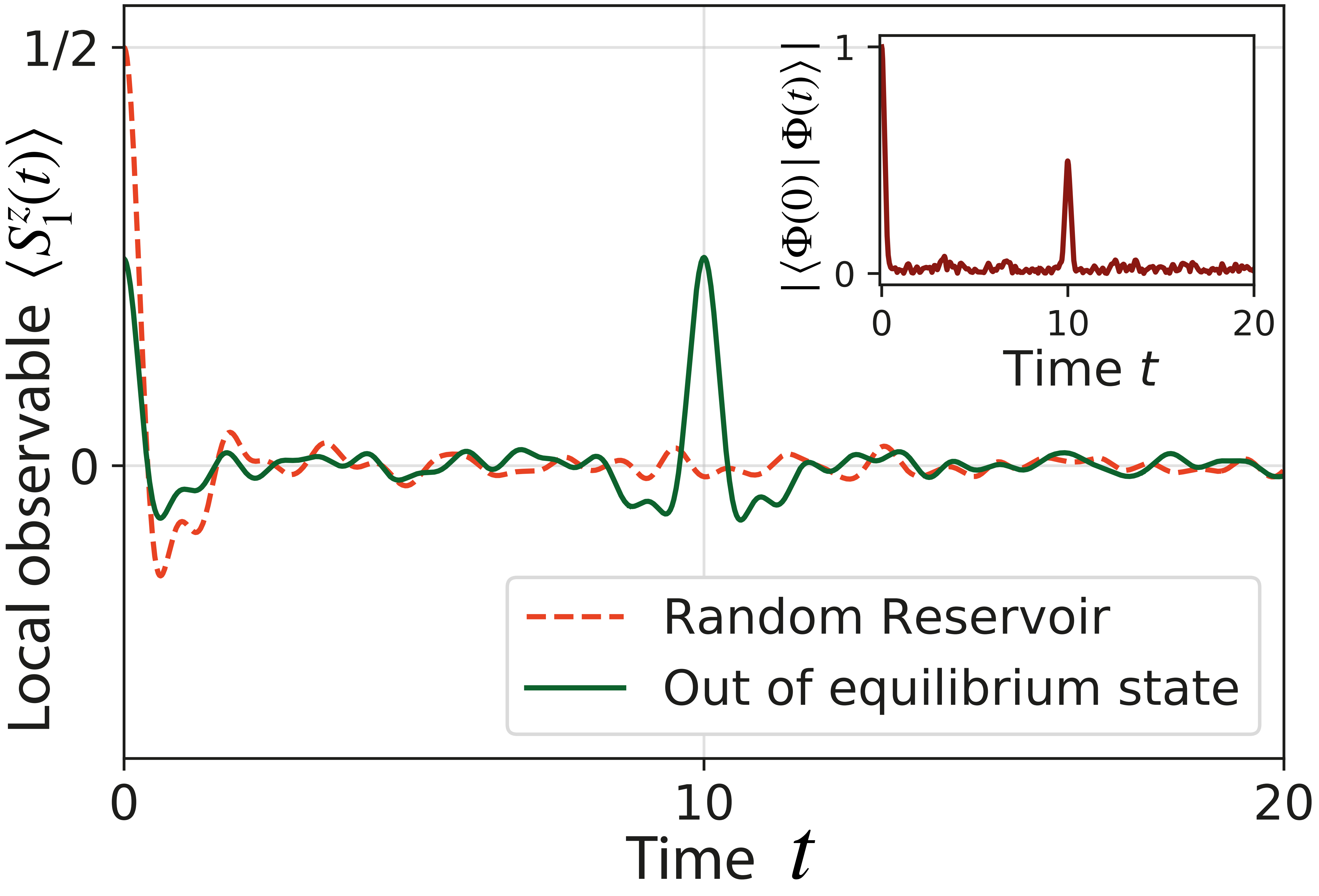
YES, we can. For example:

Dymarsky Anatoly. "Mechanism of macroscopic equilibration of isolated quantum systems." Physical Review B 99.22 (2019): 224302.

Number of spins: $L = 12$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\Psi_0(0)\rangle + |\Psi_0(-\tau)\rangle)$$

If $\langle\Psi_0|S_1^z|\Psi_0\rangle = 1/2$, then at any given moment of time τ we have $\langle S_1^z(0)\rangle = \langle S_1^z(\tau)\rangle = 1/4$



Almost compete revivals (ACR)

We can finely tune the whole reservoir such that the desired observable will exhibit almost complete revival at a predetermined moment of time

$$|\Phi_{\text{ACR}}(0)\rangle = |1_1\rangle \otimes |\Psi_{\text{res}}\rangle$$

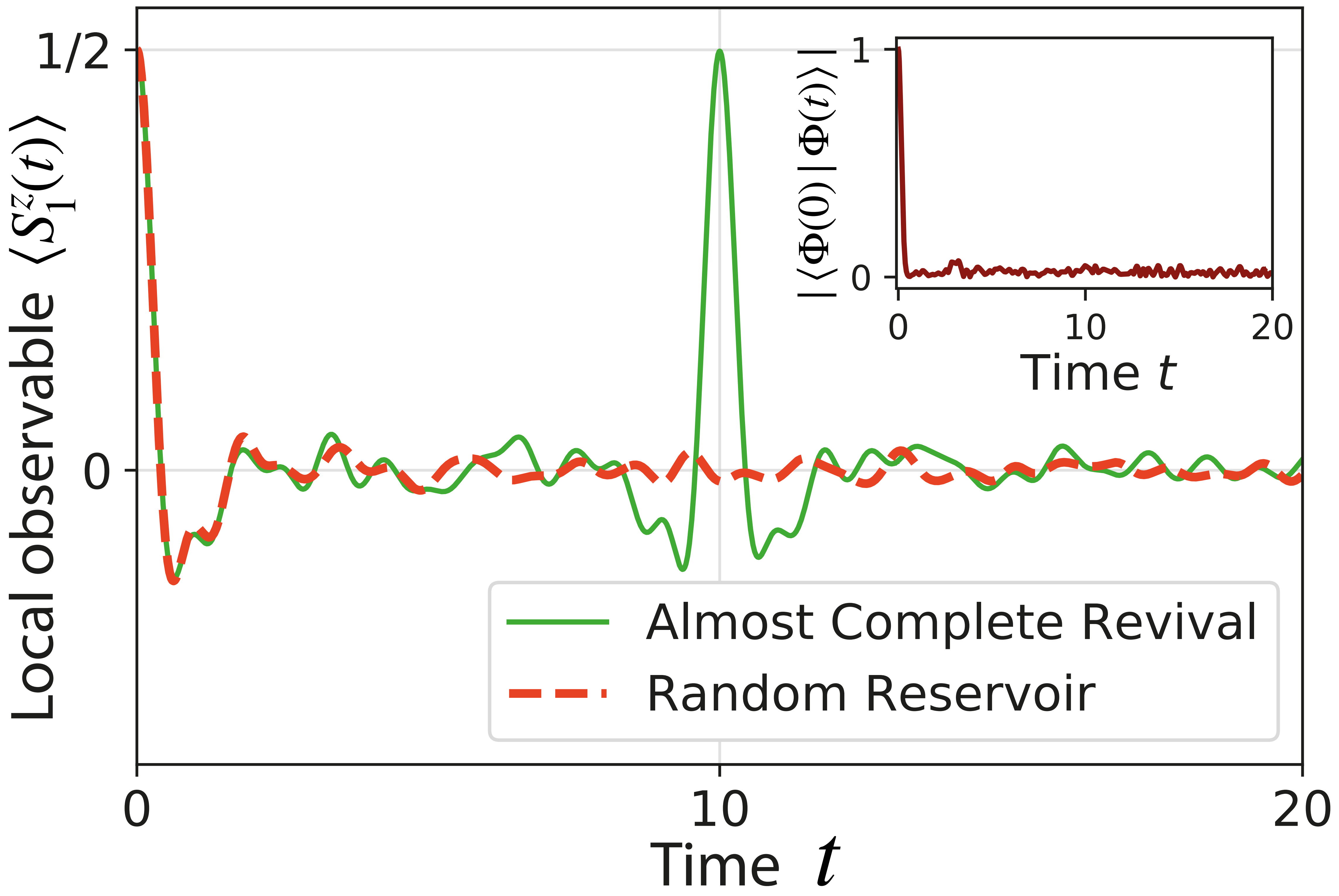
✅ Revival is almost complete

$$\langle S^z(\tau)\rangle = \langle S^z(0)\rangle - |\delta|^2, \ \ |\delta| \to 0$$

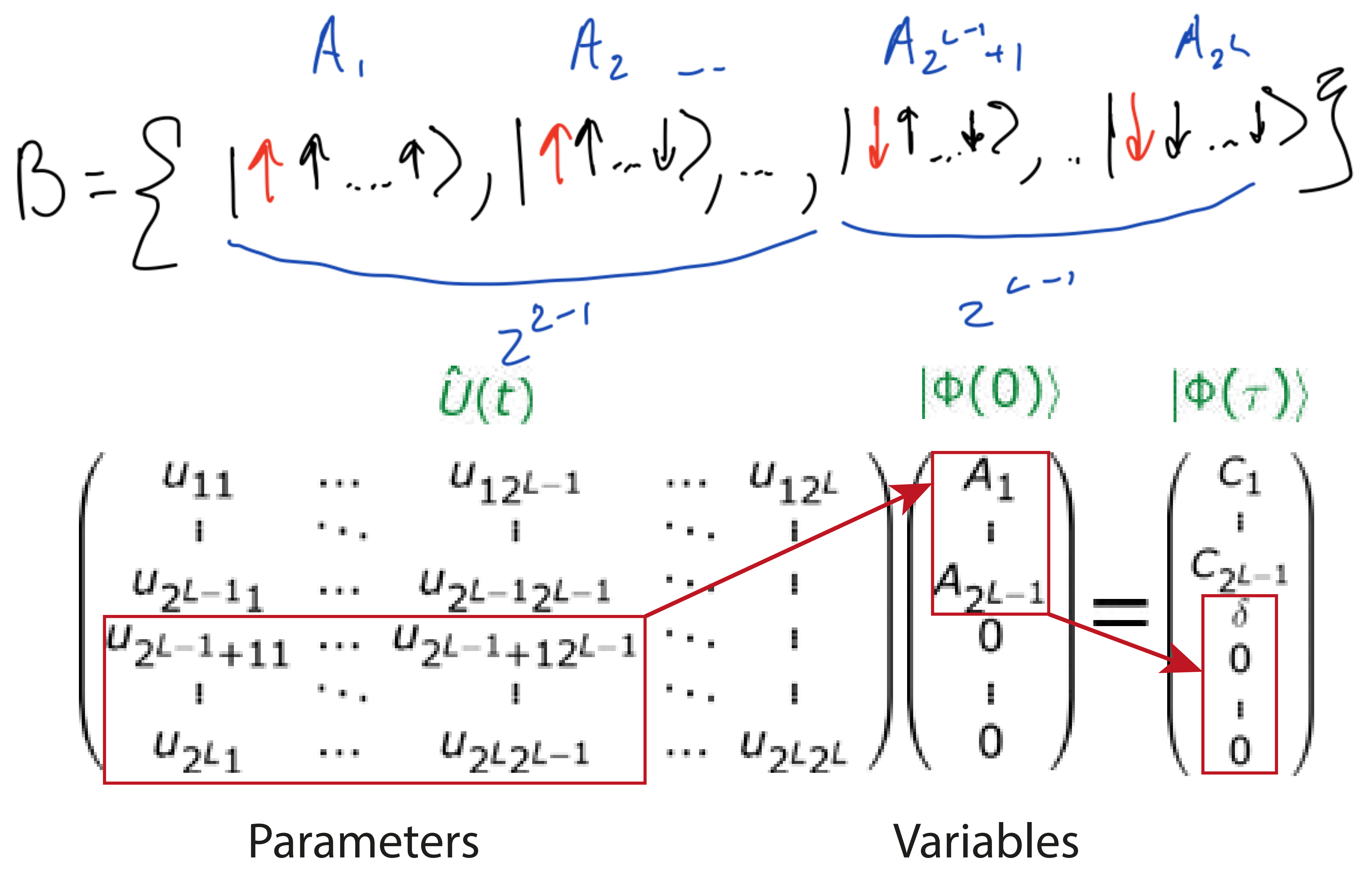
✅ NO recovery in fidelity (no time-reversal is implied)

⚠️ Quantum evolution has to be known to construct the ACR state

Number of spins: $L = 12$



How to build ACR



Basis

$$\mathscr{B}^+ = \{\,|1_1\ 1_2 \ldots 1_L\rangle,\ \ldots\ ,|1_1\ 0_2 \ldots 0_L\rangle\},$$
$$\mathscr{B}^- = \{\,|0_1\ 1_2 \ldots 1_L\rangle,\ \ldots\ ,|0_1\ 0_2 \ldots 0_L\rangle\}$$

Ansatz for wavefunction $N = 2^{L-1}$

$$|\Phi_{\mathrm{ACR}}(0)\rangle = \sum_{n=1}^{N} A_n |\varphi_n\rangle = |\uparrow\rangle \otimes |\psi_{\mathrm{res}}\rangle$$

Let us demand that:

$$|\Phi_{\mathrm{ACR}}(\tau)\rangle \equiv e^{-i\mathscr{H}\tau} |\Phi_{\mathrm{ACR}}(0)\rangle = \sum_{n=1}^{N} C_n |\varphi_n\rangle + \delta\, |\varphi_{N+1}\rangle,$$

Equations to satisfy (*), here $u \equiv e^{-i\mathscr{H}\tau}$

$$\begin{cases} u_{N+1,1}A_1 + \cdots + u_{N+1,N}A_N = \delta \\ u_{N+2,1}A_1 + \cdots + u_{N+2,N}A_N = 0 \\ \cdots \\ u_{2N,1}A_1 + \cdots + u_{2N,N}A_N = 0 \end{cases}$$

Solve for A_i and substitute

Observable at the revival time τ:

$$\langle S_1^z(\tau)\rangle = \frac{1}{2}\left(\sum_{i=1}^{N} |C_i|^2 - |\delta|^2\right)$$
$$= 1/2 - \mathscr{O}(1/N)$$

If u is close to a random rotation:

$$\delta \sim 1/\sqrt{N}$$
$$|C_n| \sim 1/\sqrt{N}$$

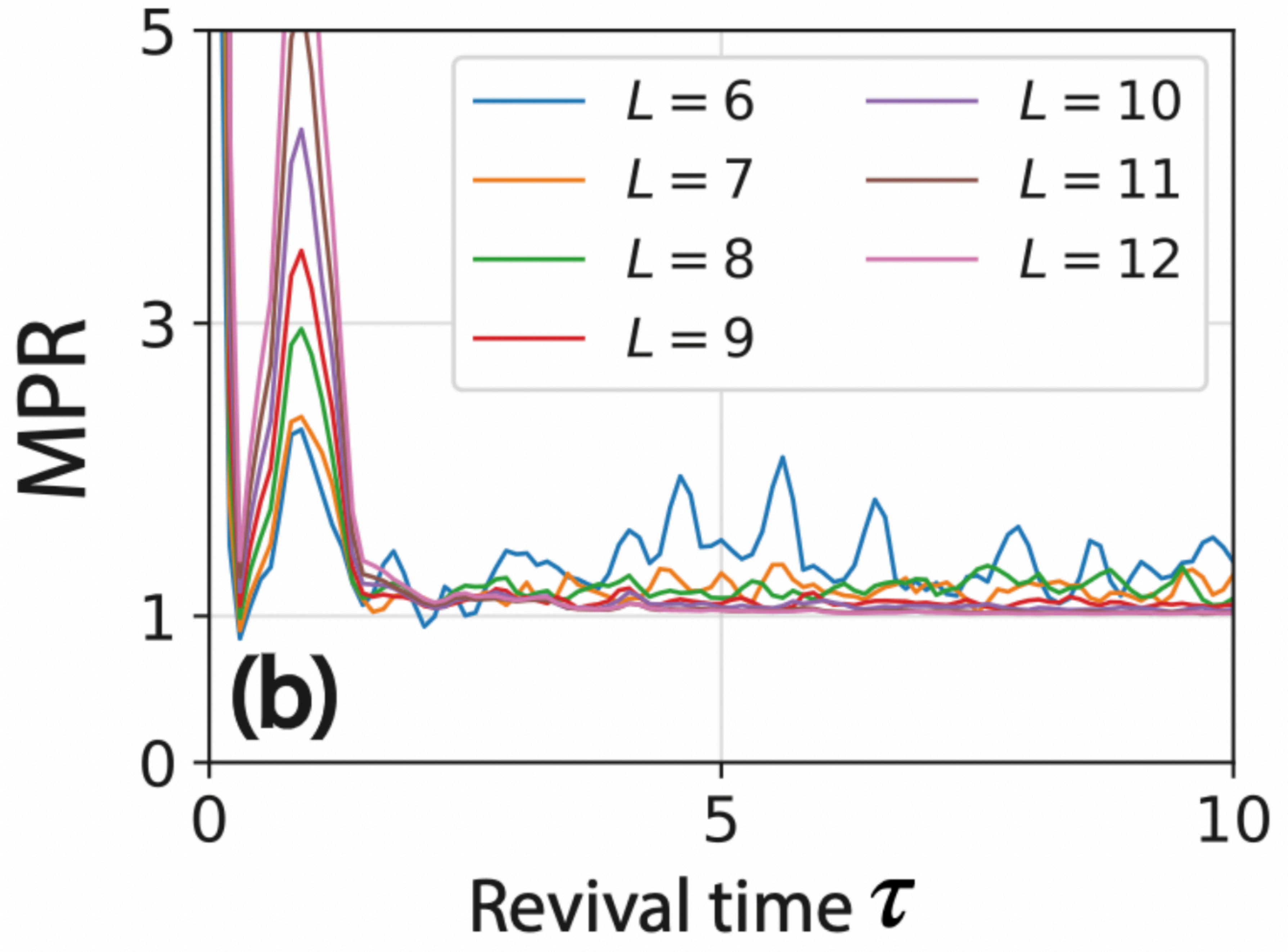
Why δ is small?

Evidenced by multiple numerical tests

If evolution matrix is close to a random rotation in the Hilbert space, then δ should be small

$$\begin{cases} u_{N+1,1}A_1 + \cdots + u_{N+1,N}A_N = \delta \\ u_{N+2,1}A_1 + \cdots + u_{N+2,N}A_N = 0 \\ \cdots \\ u_{2N,1}A_1 + \cdots + u_{2N,N}A_N = 0 \end{cases}$$

$$\text{MPR} = 1 - \frac{1}{2\sum |u_{ij}|^4}$$



$u_0 =$ typical u_{ij}	Assume that:	Then:
$A_0 =$ typical A_i	$A_0 \sim 1/\sqrt{N}$	$\delta \sim 1/\sqrt{N}$
$A_0 u_0 \sqrt{N} = \delta$	$u_0 \sim 1/\sqrt{N}$	$\lvert C_n \rvert \sim 1/\sqrt{N}$

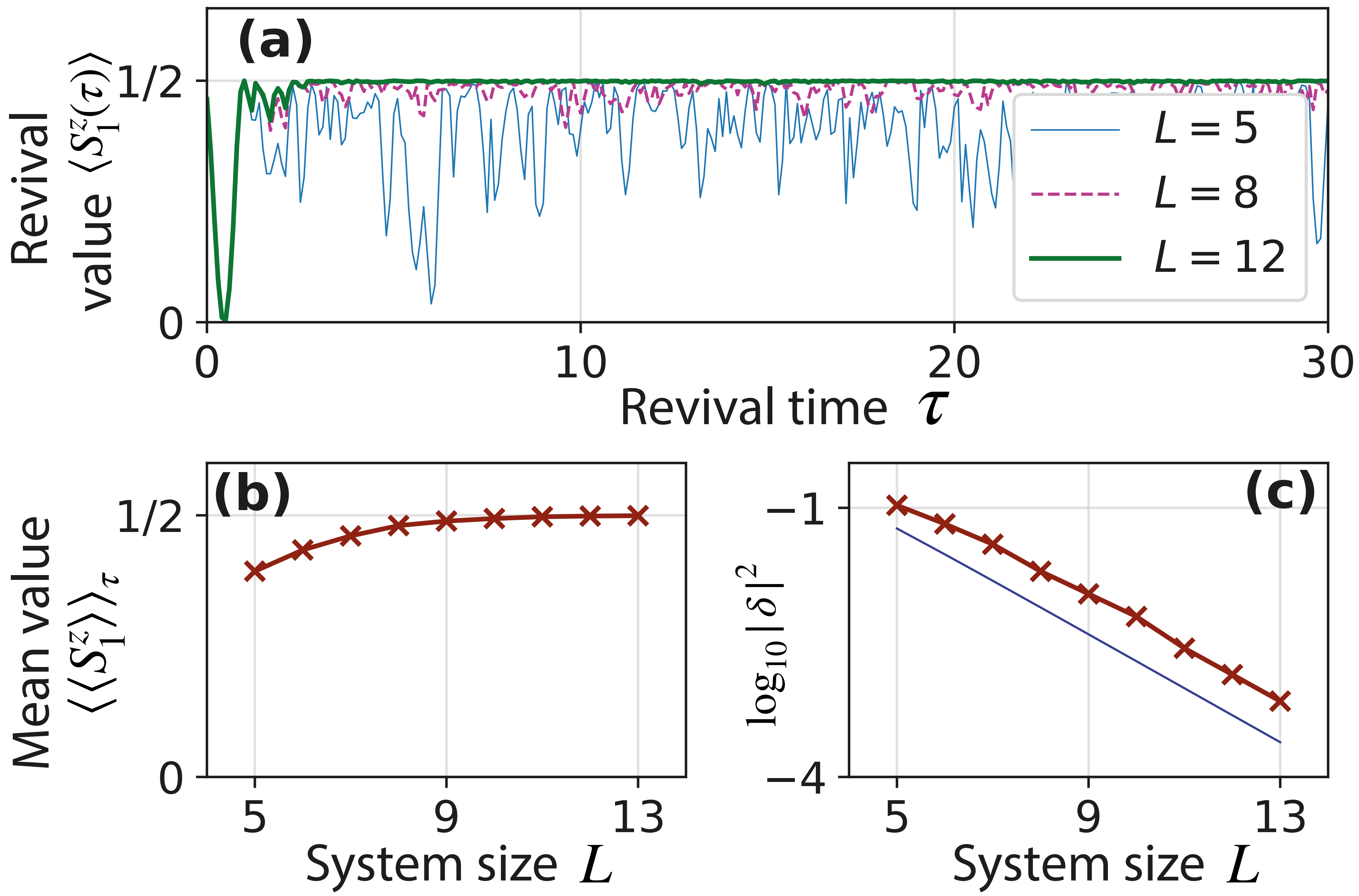
Dependence of the system size

✅ Thermodynamic limit. As we increase system size L the error $|\delta|$ vanishes exponentially

✅ Experimentally accessible sizes. Even for 5-15 spins revival is clearly pronounced above the average fluctuations

Mean revival value

$$\langle\langle S_1^z \rangle\rangle_\tau = \frac{1}{\tau_1 - \tau_0} \int_{\tau_0}^{\tau_1} \langle S_1^z(\tau)\rangle d\tau$$



Statistical properties

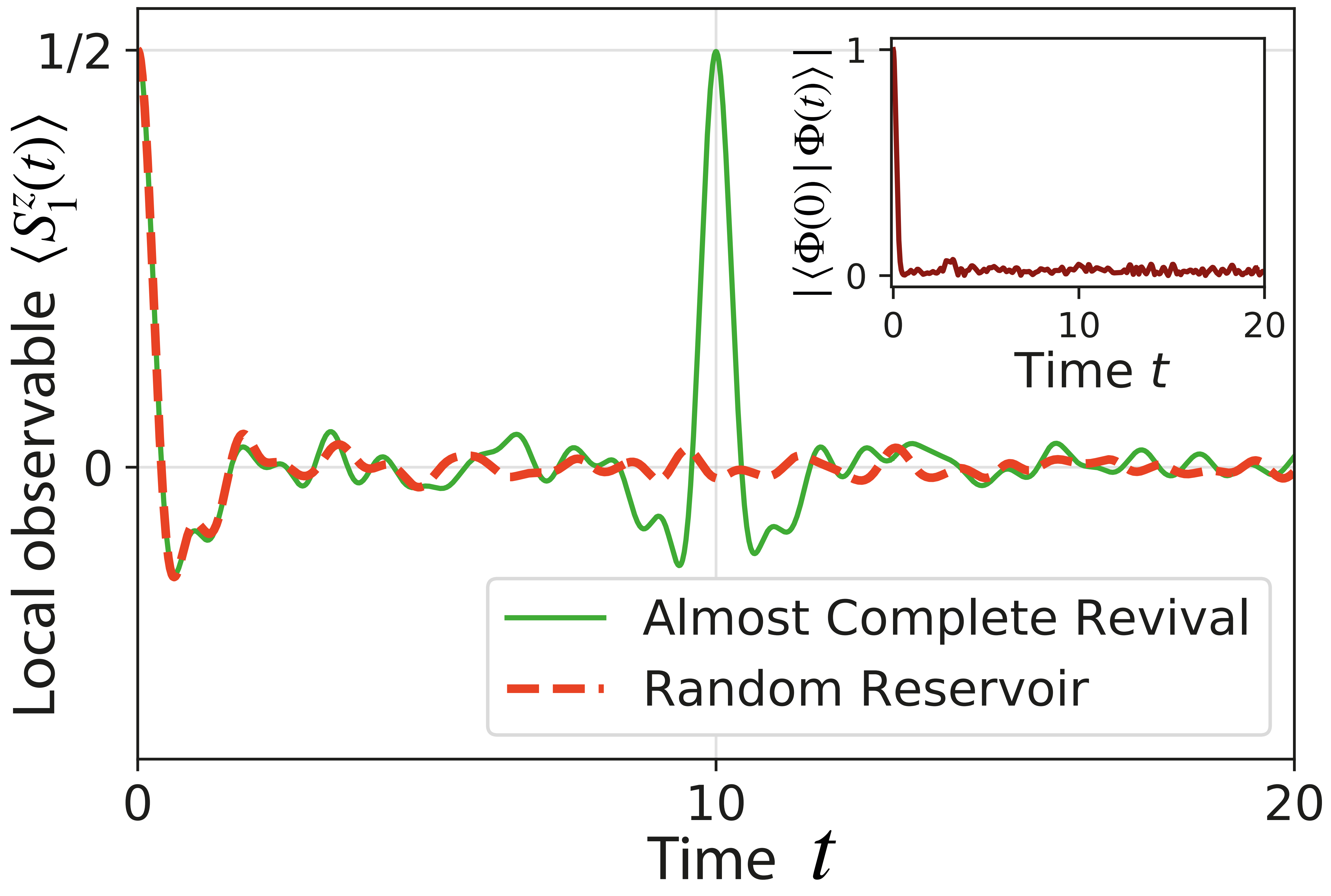
🔘 Practically indistinguishable from a random thermal state (except for two moments in time)

🔘 Early times — manifestation of quantum typicality

🔘 Character of revival is of reversed time relaxation

Dykman, M. I., & Schwartz, I. B. (2012). Large rare fluctuations in systems with delayed dissipation. Physical Review E, 86(3), 031145. Chicago

Number of spins: $L = 12$



Revivals to arbitrary points the Bloch Sphere

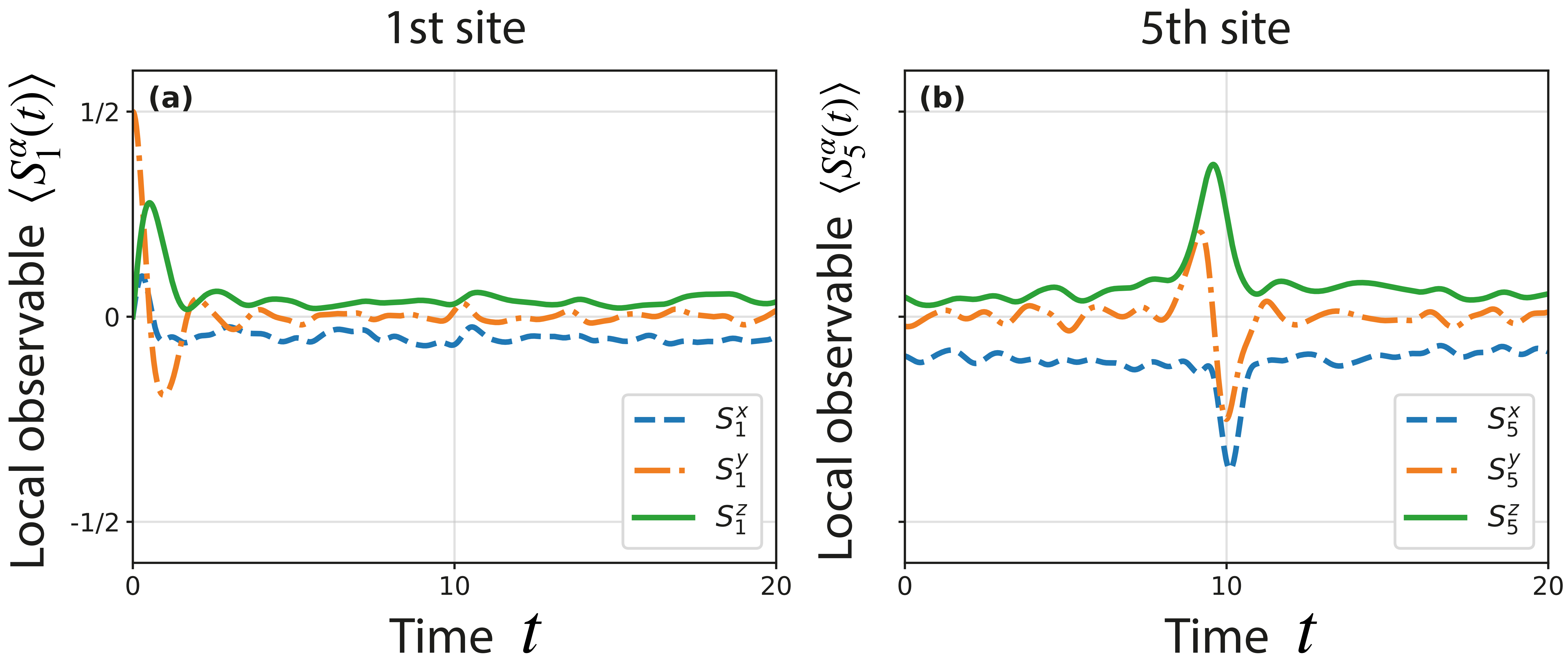
$$H_1 = \sum_{j=1}^{L} \left(g\sigma_j^x + h\sigma_j^z + J\sigma_j^z\sigma_{j+1}^z \right)$$

$$(g, h, J) = (0.9045, 0.8090, 1)$$

Revivals from arbitrary site q
to an arbitrary site p

$$|l_q\rangle = \frac{|0_q\rangle + \alpha|1_q\rangle}{\sqrt{1+|\alpha|^2}}$$

From arbitrary point α
to an arbitrary point β



$$\hat{V}\mathscr{A} = (\delta, 0, \ldots, 0)^T$$

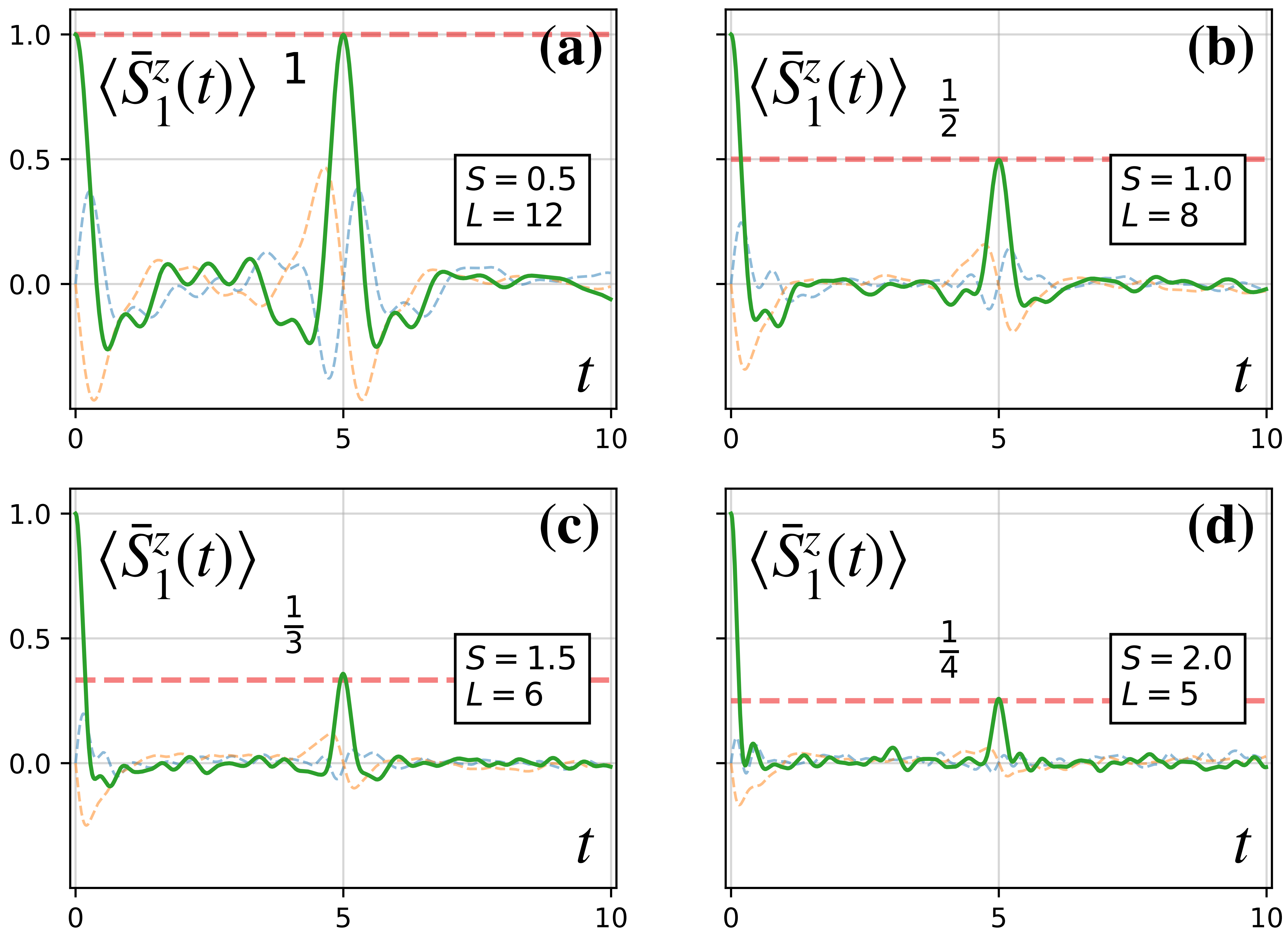
$$V_{ki} = u_{d[k],\bar{d}[i]} - \beta^{-1}u_{d[k]+2^{L-p},\bar{d}[i]} + \alpha u_{d[k],\bar{d}[i]+2^{L-q}} - \alpha\beta^{-1}u_{d[k]+2^{L-p},\bar{d}[k]+2^{L-q}}$$

$$d = \{\{s(k,n)\}_{k=1}^{2^{q-1}}\}_{n=1}^{2^{L-q}} \qquad s(k,n) = 2^{L-q+1}(k-1) + n$$

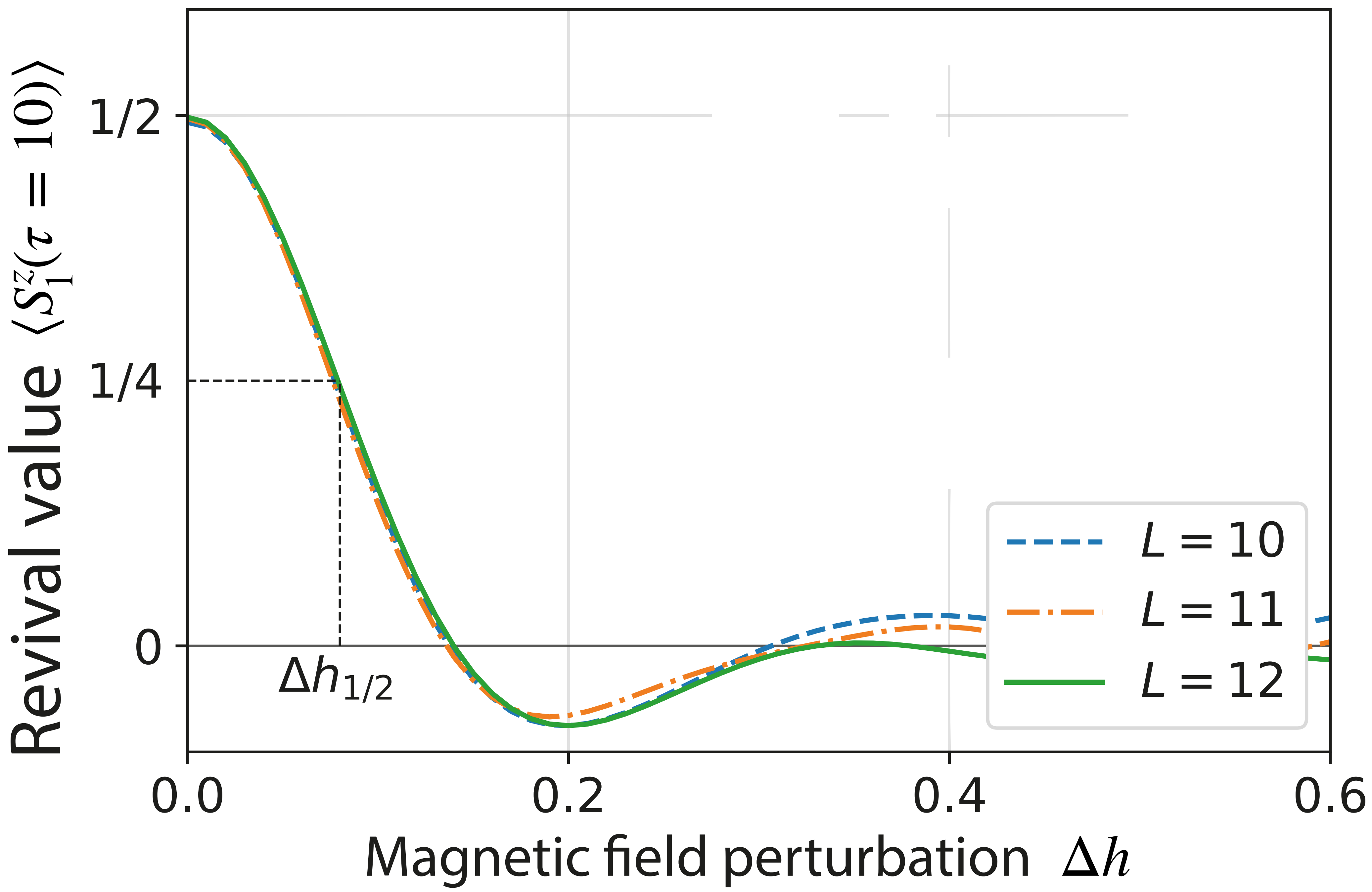
$$\bar{d} = \{\{\bar{s}(k,n)\}_{k=1}^{2^{p-1}}\}_{n=1}^{2^{L-p}} \qquad \bar{s}(k,n) = 2^{L-p+1}(k-1) + n$$

Higher spins

ACR are suppressed for higher quantum spins



Sensitivity to external perturbations



Let us perturb the external magnetic field as:

$$(h_x, h_y) = (2.2 - \Delta h,\ 2.2 - \Delta h)$$

Revival value decays linearly with Δh

✅ No exponential sensitivity to Hamiltonian parameters

✅ Can be utilized for entanglement assisted sensing

Delayed disclosure of a secret

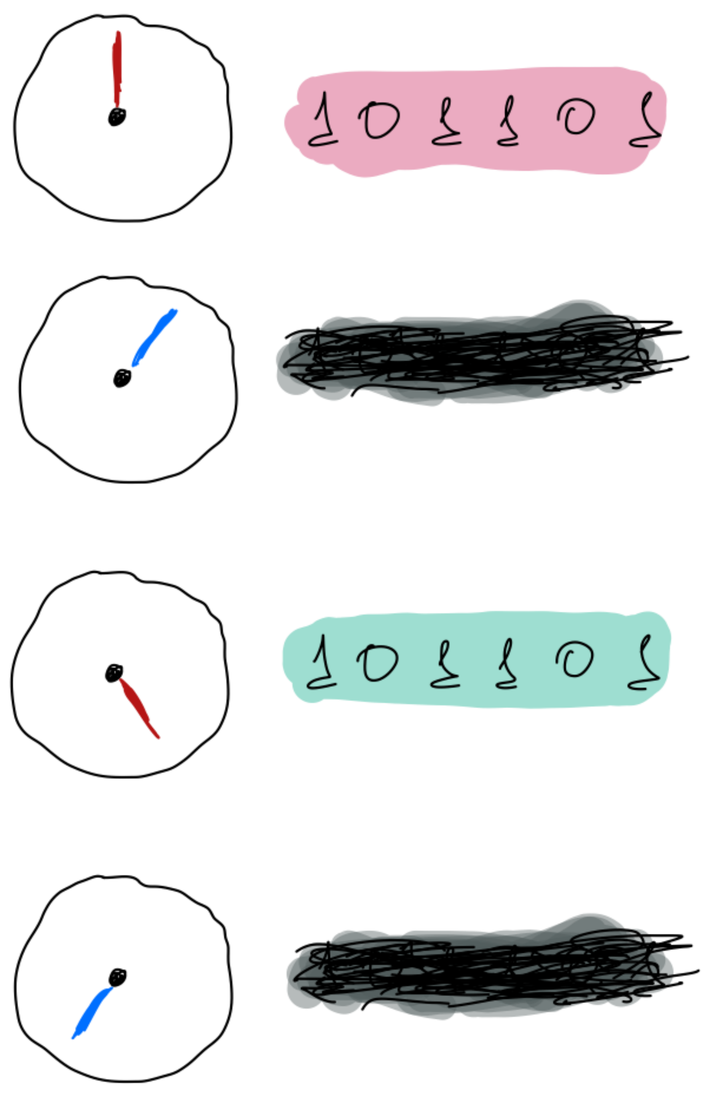
Can we record a piece of information such that it will only be available at certain moments of time?

$t = 0$

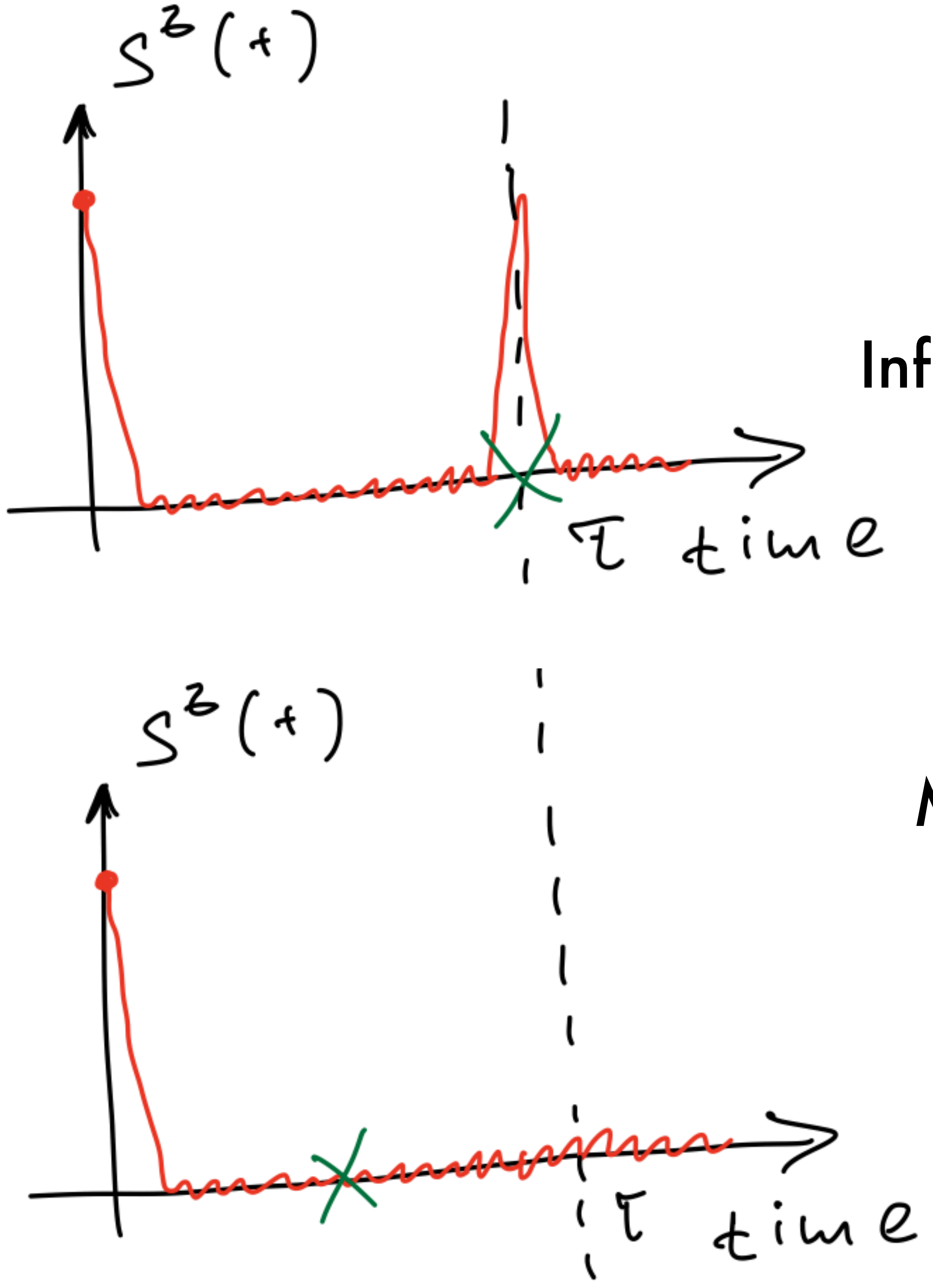
$0 < t < \tau$

$t = \tau$

$t > \tau$



Schematic representation of delayed disclosure of a secret



Measurement at τ

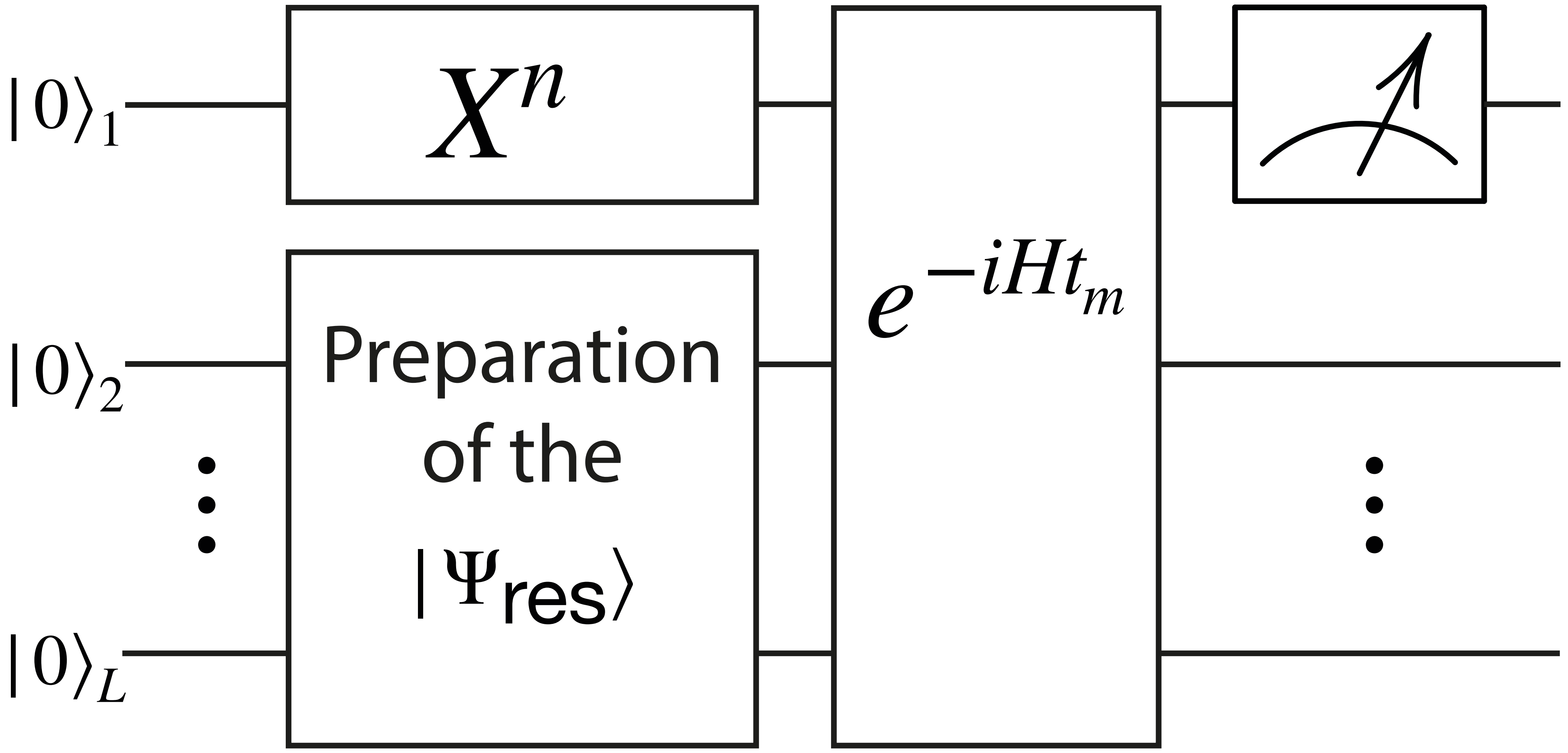
Information is recovered

Measurement earlier than τ

Information is lost

Quantum time capsule

$n = 0$ or 1 — bit of classical information



$$t_m = \tau$$

$\langle S_1^z \rangle = 1/2$ or $-1/2$

Bit "n" is disclosured

$$t_m \neq \tau$$

$\langle S_1^z \rangle \simeq \langle S_1^z \rangle_{\mathrm{eq}} = 0$

Bit "n" is lost

One can encrypt the information inside of the many-body quantum state such that it is fundamentally inaccessible before the revival time

Cannot be done classicaly

Sensitivity to external perturbations

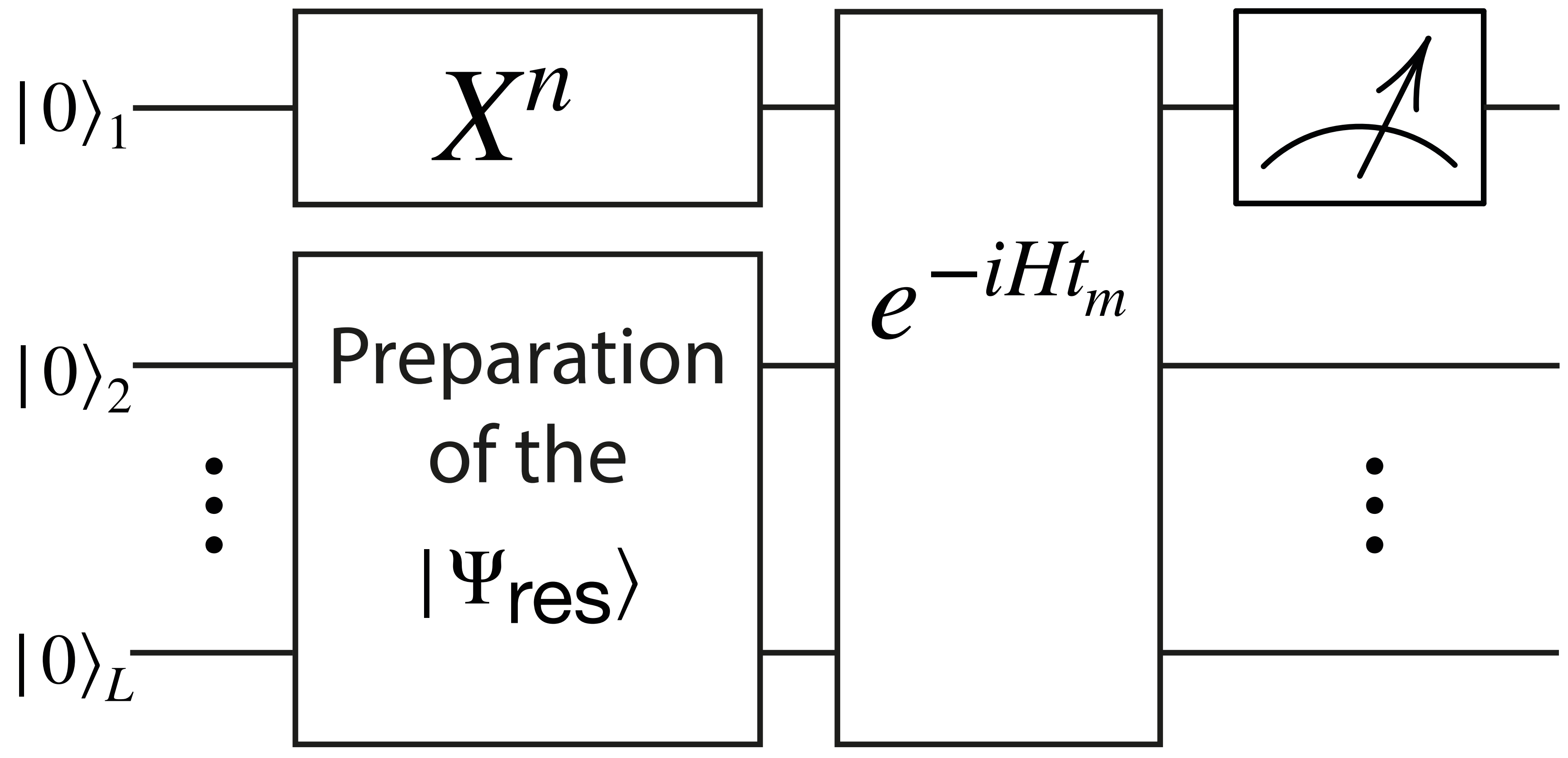
For quantum simulation we want to prepare some many-body state $|\Psi_{\text{target}}\rangle$

State prepared in experiment $|\Psi_{\text{exp}}\rangle$ is always different $|\langle\Psi_{\text{exp}}|\Psi_{\text{target}}\rangle| < 1$

Straightforward quantum tomography of many-body state requires exponential number of measurements

What if we use ACR states as a target state?

$$|\Psi_{\text{target}}\rangle = |\Psi_{\text{res}}\rangle$$



The successful observation of the revival means that **both** state preparation and quantum evolution were executed correctly

Thanks for attention!