Probing QCD critical point and induced gravitational wave by black hole physics

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w/. Rong-Gen Cai, Li Li, Yuan-Xu Wang, 2201.02004,
SH, Li Li, Zhibin Li, Shao-Jiang Wang, 2210.14094

Outline

I. Motivations & hQCD

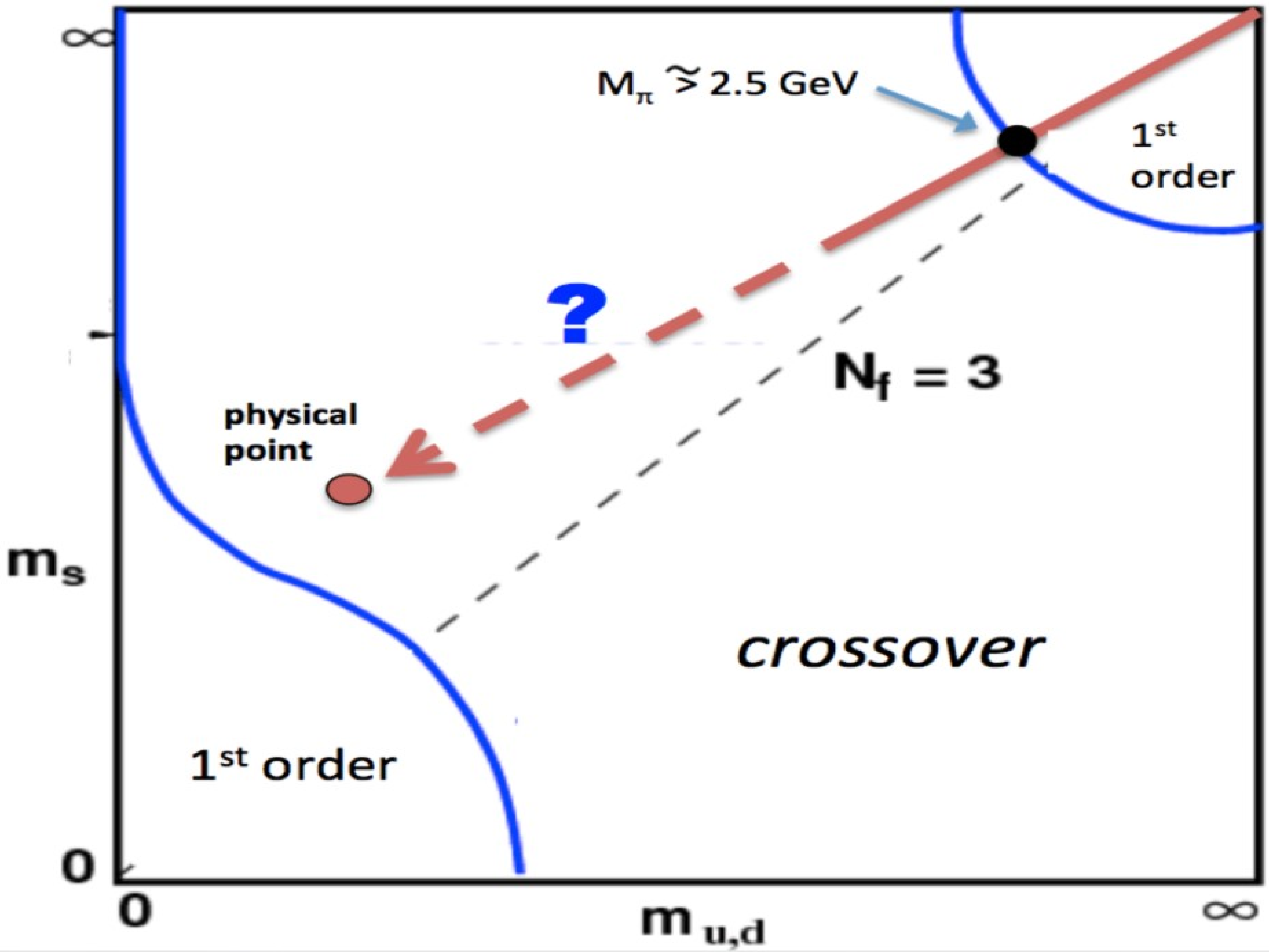
II. Holographic QCD model (hQCD)

III. Confront with QCD Phase diagram

IV. Holographic pure gluon model

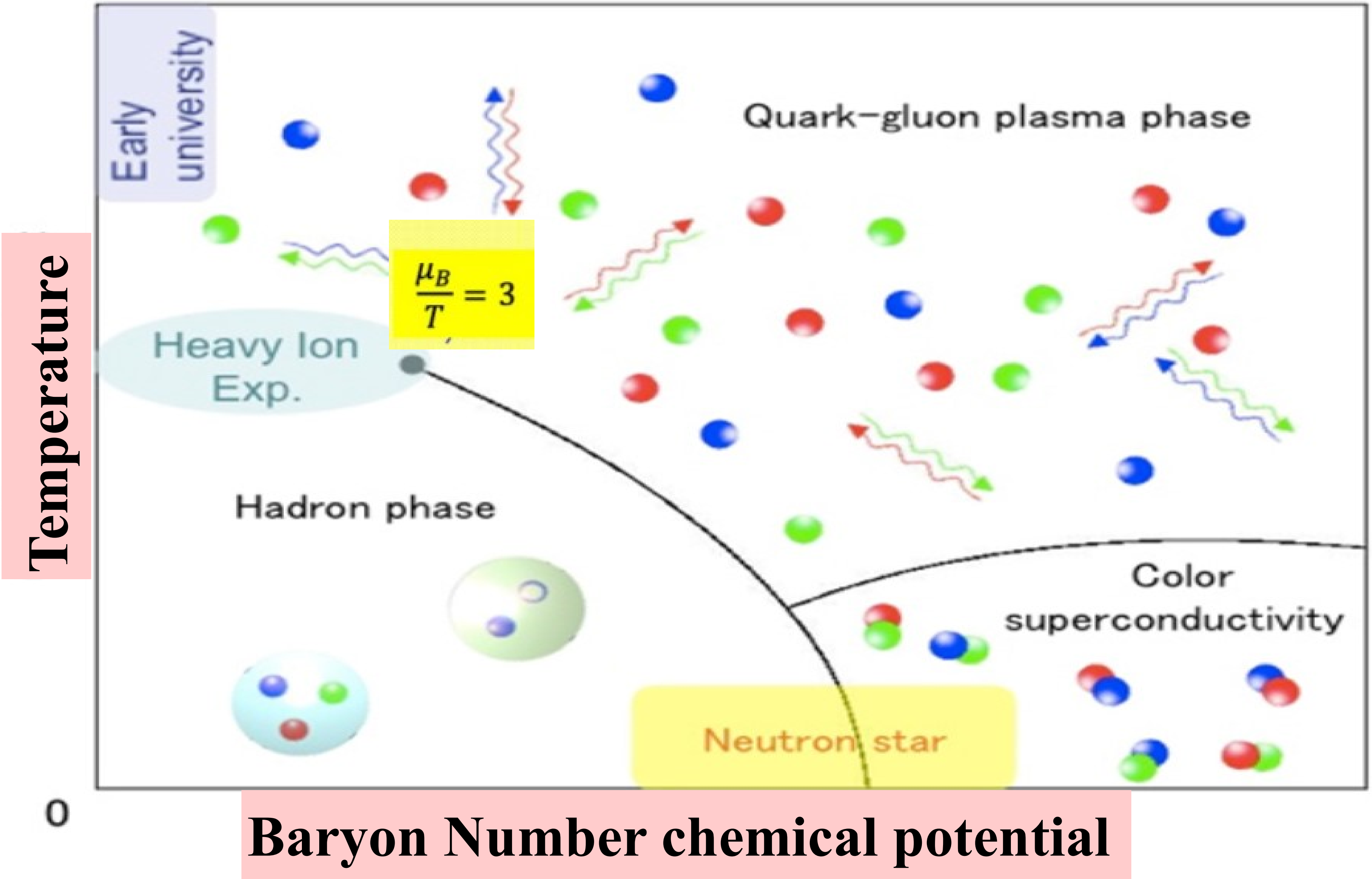
V. Summary

Motivations

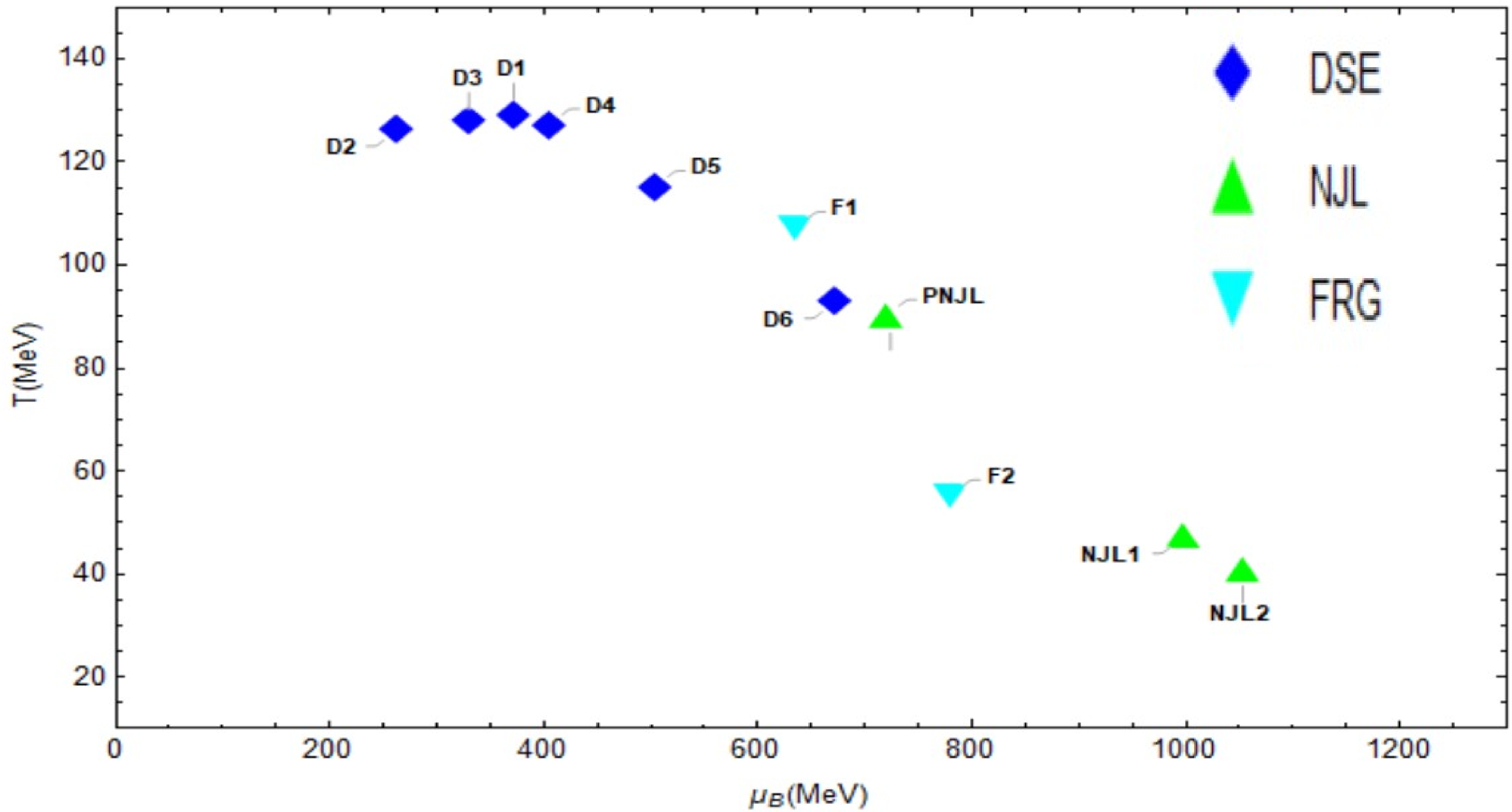


∞
$M_\pi \gtrsim 2.5$ GeV
1^{st} order
?
$N_f = 3$
physical point
m_s
crossover
1^{st} order
0
0
$m_{u,d}$
∞

A schematic view of QCD Phase diagram



Status of searching CEP



Schwinger–Dyson equation (DSE), 2109.09935 [hep-ph], 1607.01675 [hep-ph], 1011.2876 [nucl-th], 1403.3797 [hep-ph], 1405.4762 [hep-ph]], 2002.07500 [hep-ph]].
Nambu–Jona-Lasinio models (NJL, PNJL), arXiv:1801.09215 [hep-ph]], Nucl. Phys. A 504 (1989), 668-684
Functional renormalization group (FRG). 1909.02991 [hep-ph]], 1709.05654 [hep-ph]].

Motivations

I. Low energy QCD is strong coupled system

P. Braun-Munzinger and J. Wambach, Rev. Mod. Phys. 81 (2009), 1031-1050 [arXiv:0801.4256 [hep-ph]].

S. Gupta, X. Luo, B. Mohanty, H. G. Ritter and N. Xu, Science 332 (2011), 1525-1528 [arXiv:1105.3934[hep-ph]].

II. Finite density QCD v.s. sign Problem

O. Philipsen, Prog. Part. Nucl. Phys. 70 (2013), 55-107 [arXiv:1207.5999 [hep-lat]].

III. AdS/QCD offers a practical approach

O. DeWolfe, S. S. Gubser and C. Rosen, Phys. Rev. D 84 (2011), 126014 [arXiv:1108.2029 [hep-th]].

R. G. Cai, S. He and D. Li, JHEP 03 (2012), 033 [arXiv:1201.0820 [hep-th]].

U. Gursoy, M. Jarvinen and G. Nijs, Phys. Rev. Lett. 120 (2018) no.24,242002 [arXiv:1707.00872 [hep-th]].

J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo,C. Ratti and R. Rougemont,Phys. Rev. D 104 (2021) no.3, 034002 [arXiv:2102.12042[nucl-th]]. ….

Intro AdS/QCD

I. AdS5/CFT4

II. AdS4/CFT3 (ABJM)

III. AdS3/CFT2

IV. nAdS2/nCFT1, nAdS2/SYK4

V. Non-AdS/CFT (Celestial Holograph)

VI. …

AdS/QCD

AdS/CFT conjecture

$$AdS_5 \times S^5 \longleftrightarrow N = 4 \text{ SYM theory}$$

If it is true for any gauge theory (???)

String theory, quantum gravity	$\longleftrightarrow$	**Non-Abelian gauge theory**

Then what is the dual string theory of QCD?
(It is nature to ask the question here)

? $\longleftrightarrow$ **QCD**

Question: Is it possible to find a string theory dual to QCD?

Top-Down models: D3-D7, D4-D8D8bar ...

hQCD model for 2+1-flavor QCD

HQCD model for 2+1 flavor system

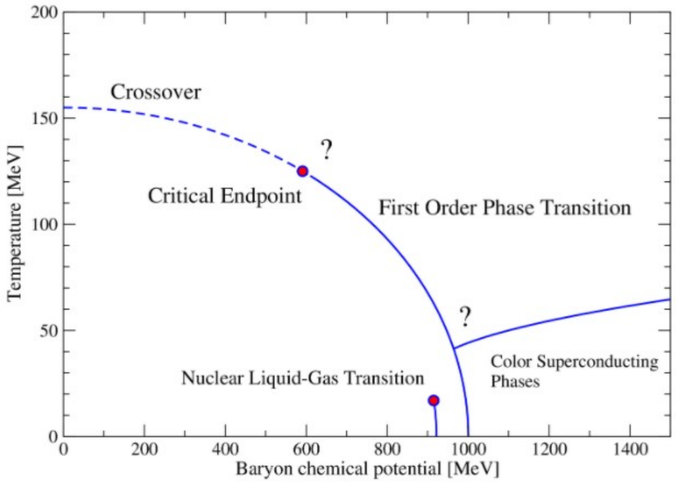
Einstein-Maxwell-Dilaton system

Motivation

To cover the degree of freedom in QCD phase Diagram.
Quarks (chemical potential) & gluons(dilaton potential)

Gravity Action

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\Big[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\Big],$$



ϕ Break conformal symmetry

$F^{\mu\nu}$ Introduce baryon chemical potential

HQCD model

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

Ansatz

$$ds^2 = -f(r)e^{-\eta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 d\boldsymbol{x}_3^2$$

$$\phi = \phi(r), \quad A_t = A_t(r)$$

EOM

$$\nabla_\mu \nabla^\mu \phi - \frac{\partial_\phi Z}{4} F_{\mu\nu}F^{\mu\nu} - \partial_\phi V = 0$$

$$\nabla^\nu (Z F_{\nu\mu}) = 0$$

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{Z}{2}F_{\mu\rho}F_\nu{}^\rho + \frac{1}{2}\left(-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z}{4}F_{\mu\nu}F^{\mu\nu} - V\right)g_{\mu\nu}$$

Model Parameters

$$V(\phi) = -12\cosh[c_1\phi] + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6\,,$$

$$Z(\phi) = \frac{1}{1+c_3}\operatorname{sech}[c_4\phi^3] + \frac{c_3}{1+c_3}e^{-c_5\phi}\,,$$

c_1, c_2, c_3, c_4, c_5 Effective Newton constant κ_N^2 Scalar source ϕ_s +Renormalization **b**

Boundary Stress Tensor

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

On-Shell Action

$$-\Omega V = T(S + S_\partial)_{on-shell} \,,$$

$$S_\partial = \frac{1}{2\kappa_N^2}\int_{r\to\infty} dx^4\sqrt{-h}\Big[2K - 6 - \frac{1}{2}\phi^2 - \frac{6c_1^4-1}{12}\phi^4\ln[r] - b\,\phi^4 + \frac{1}{4}F_{\rho\lambda}F^{\rho\lambda}\ln[r]\Big]$$

c_1, c_2, c_3, c_4, c_5 Effective Newton constant κ_N^2 Scalar source ϕ_s+Renormalization **b**

Boundary stress tensor

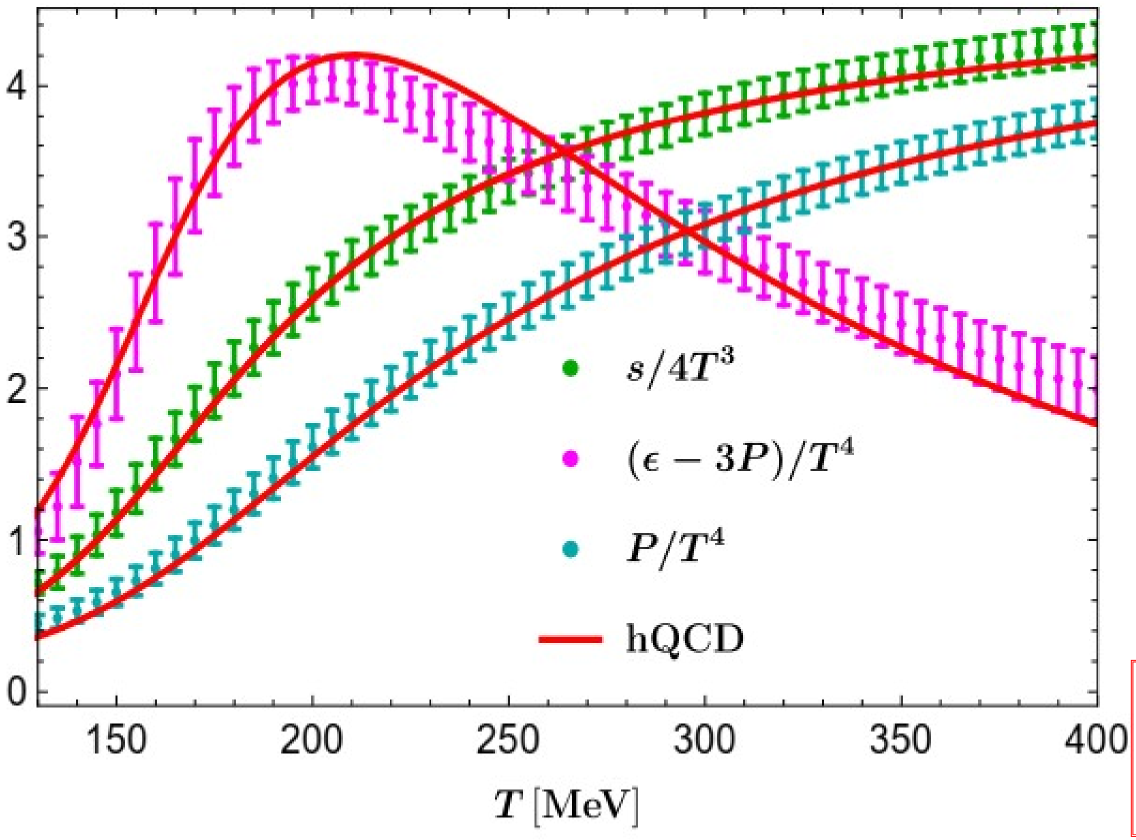
$$\begin{aligned}T_{\mu\nu} &= \lim_{r\to\infty}\frac{2}{\sqrt{-\det g}}\frac{\delta(S+S_\partial)_{on-shell}}{\delta g^{\mu\nu}}\,,\\ &= \frac{1}{2\kappa_N^2}\lim_{r\to\infty} r^2\Big[2(Kh_{\mu\nu} - K_{\mu\nu} - 3h_{\mu\nu}) - (\frac{1}{2}\phi^2 + \frac{6c_1^4-1}{12}\phi^4\ln[r] + \boxed{b\,\phi^4})h_{\mu\nu}\\ &\quad - (F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}h_{\mu\nu}F_{\rho\lambda}F^{\rho\lambda})\ln[r]\Big]\,,\end{aligned}$$

To fix model parameters by thermal dynamics

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

$c_1, c_2, \boxed{c_3, c_4, c_5}$ Effective Newton constant κ_N^2 Scalar source ϕ_s+**Renormalization b**

Equations of state at vanishing chemical potential, s, trace anomaly, pressure



$$T = \frac{1}{4\pi} f'(r_h) e^{-\eta(r_h)/2},$$

$$s = \frac{2\pi}{\kappa_N^2} r_h^3$$

$$\epsilon := T_{tt}$$

$$P := T_{xx}$$

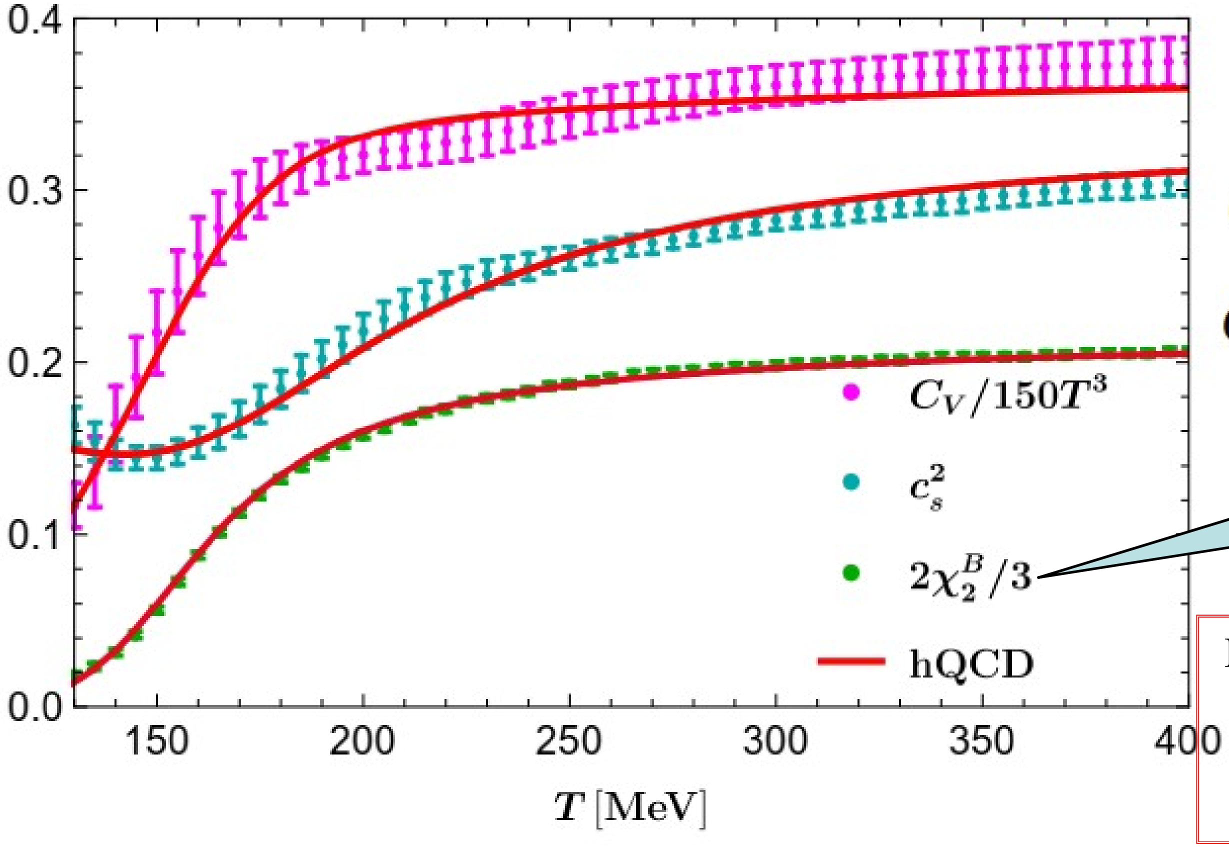
A. Bazavov *et al.* [HotQCD], Phys. Rev. D 90 (2014), 094503[arXiv:1407.6387 [hep-lat]].

To fix model parameters by thermal dynamics

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

$c_1, c_2, \boxed{c_3, c_4, c_5}$ Effective Newton constant κ_N^2 Scalar source ϕ_s+Renormalization **b**

Sound speed,specific heat, second-order baryon susceptibility **at vanishing chemical potential**



$$c_s = \sqrt{(dP/d\epsilon)_{\mu_B}}$$

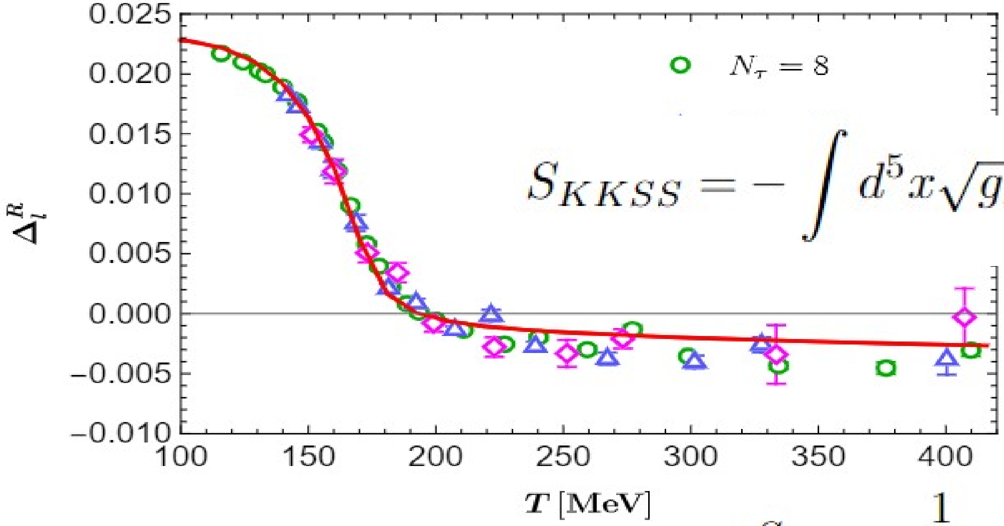
$$C_V = (d\epsilon/dT)_{\mu_B}$$

$$\boxed{\chi_2^B = (d\rho_B/d\mu_B)_T}$$

Bazavov *et al.* [HotQCD], Phys.Rev. D 90 (2014), 094503[arXiv:1407.6387 [hep-lat]].

Prediction of hQCD model

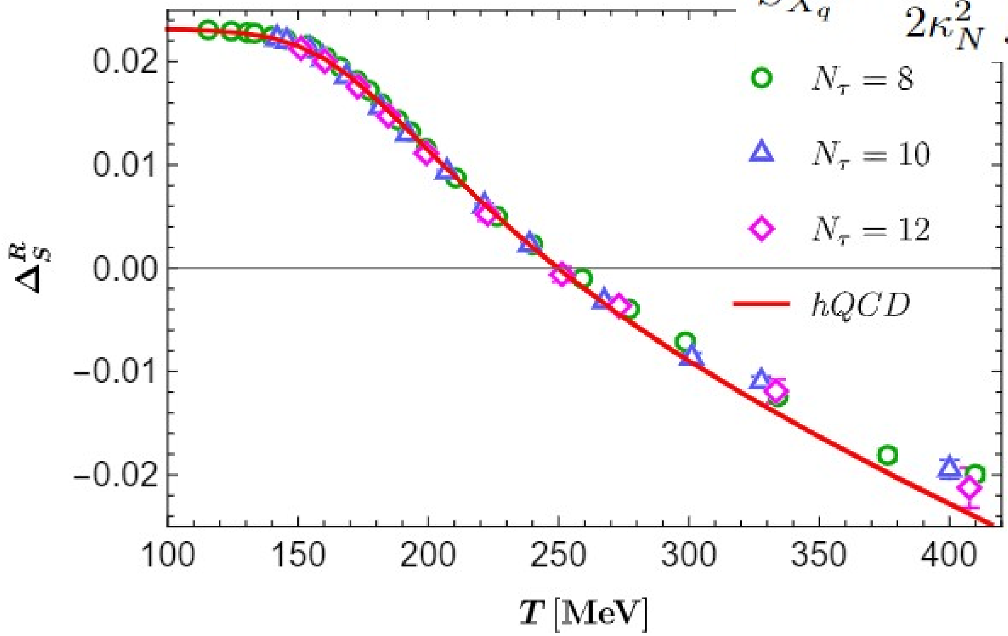
Condensation



$$S_{KKSS} = -\int d^5x\sqrt{g_s}e^{-\Phi}\,\mathrm{Tr}\left(|DX|^2 + V_X + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right)$$

(2+1)-flavors $\qquad m_u = m_d < m_s$

$$S_{X_q} = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\,Z_q(\phi)\left[-\frac{1}{2}\nabla_\mu X_q \nabla^\mu X_q - V(X_q)\right]$$

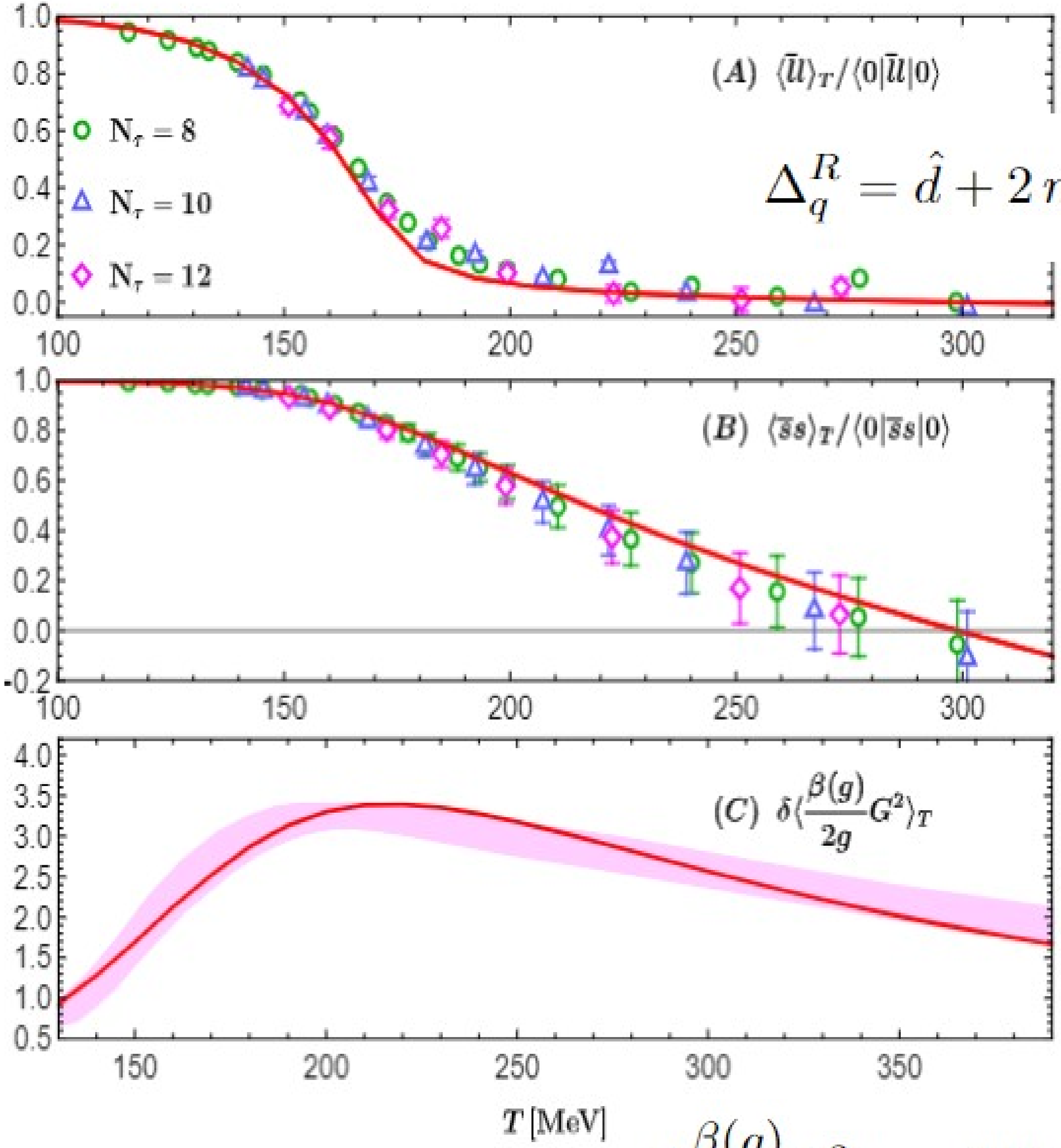


$$Z_q(\phi) = a_1\,e^{a_q\,\phi^2}\,,$$

$$V(X_q) = -\frac{3}{2}X_q^2 + a_0\,X_q^4\,,$$

***[1812.00385v2]: Gubler P, Satow D. Recent progress in QCD condensate evaluations and sum rules.**

Condensation



$$\Delta_q^R = \hat{d} + 2\, m_q r_1^4 \left[\langle\bar{\psi}\psi\rangle_{q,T} - \langle\bar{\psi}\psi\rangle_{q,0} \right], \quad q = l, s\,,$$

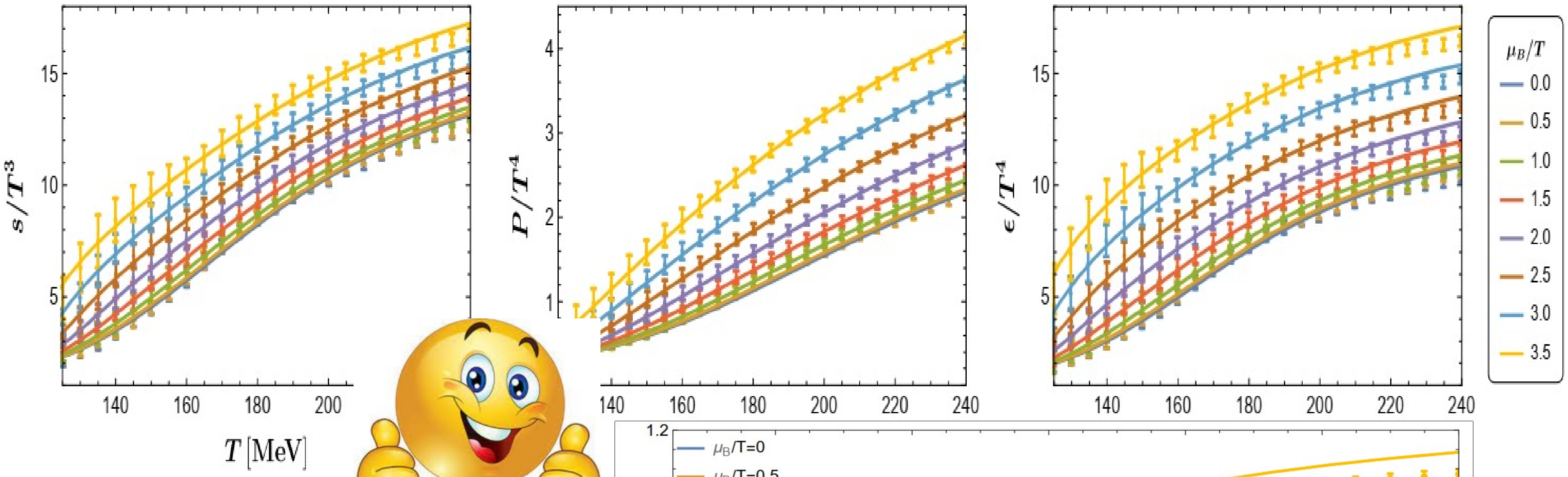
$$\frac{\langle\bar{l}l\rangle_T}{\langle 0|\bar{l}l|0\rangle} = 1 - \frac{\hat{d} - \Delta_l^R(T)}{\hat{d} - \Delta_l^R(\infty)}\,,$$

$$\frac{\langle\bar{s}s\rangle_T}{\langle 0|\bar{s}s|0\rangle} = 1 + \frac{\hat{d} - \Delta_s^R(T)}{2m_s r_1^4 \langle 0|\bar{s}s|0\rangle}\,.$$

$$\delta\langle\frac{\beta(g)}{2g}G^2\rangle_T = \theta(T) - \hat{m}_u \delta\langle\bar{u}u\rangle_T - \hat{m}_d \delta\langle\bar{d}d\rangle_T - \hat{m}_s \delta\langle\bar{s}s\rangle_T$$

Predictions of thermal dynamical quantities at finite chemical potential

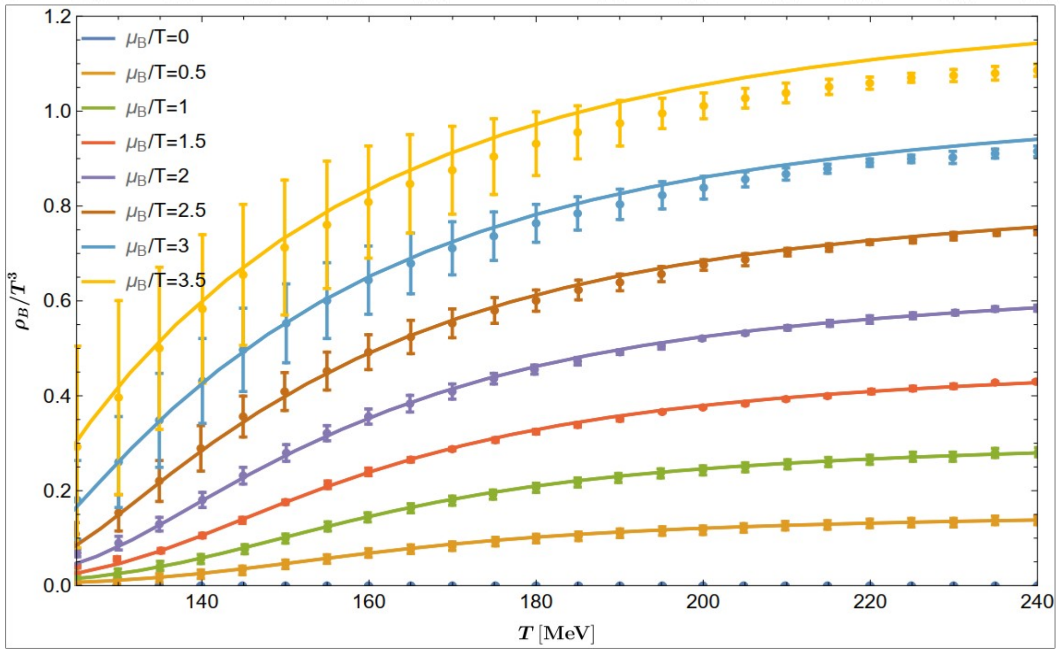
S. Bors´anyi, etc., Phys. Rev. Lett. 126 (2021) no.23, 232001 [arXiv:2102.06660 [hep-lat]]



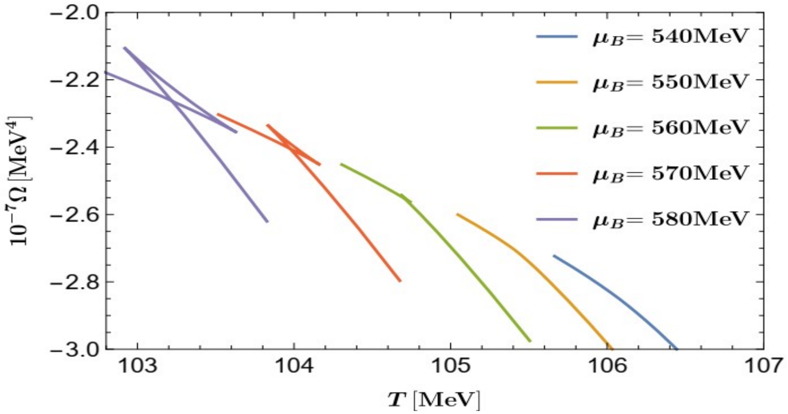
More challenging fitting

Rho_B

Rong-Gen Cai, Song He, Li Li, Yuan-Xu Wang, 2201.02004

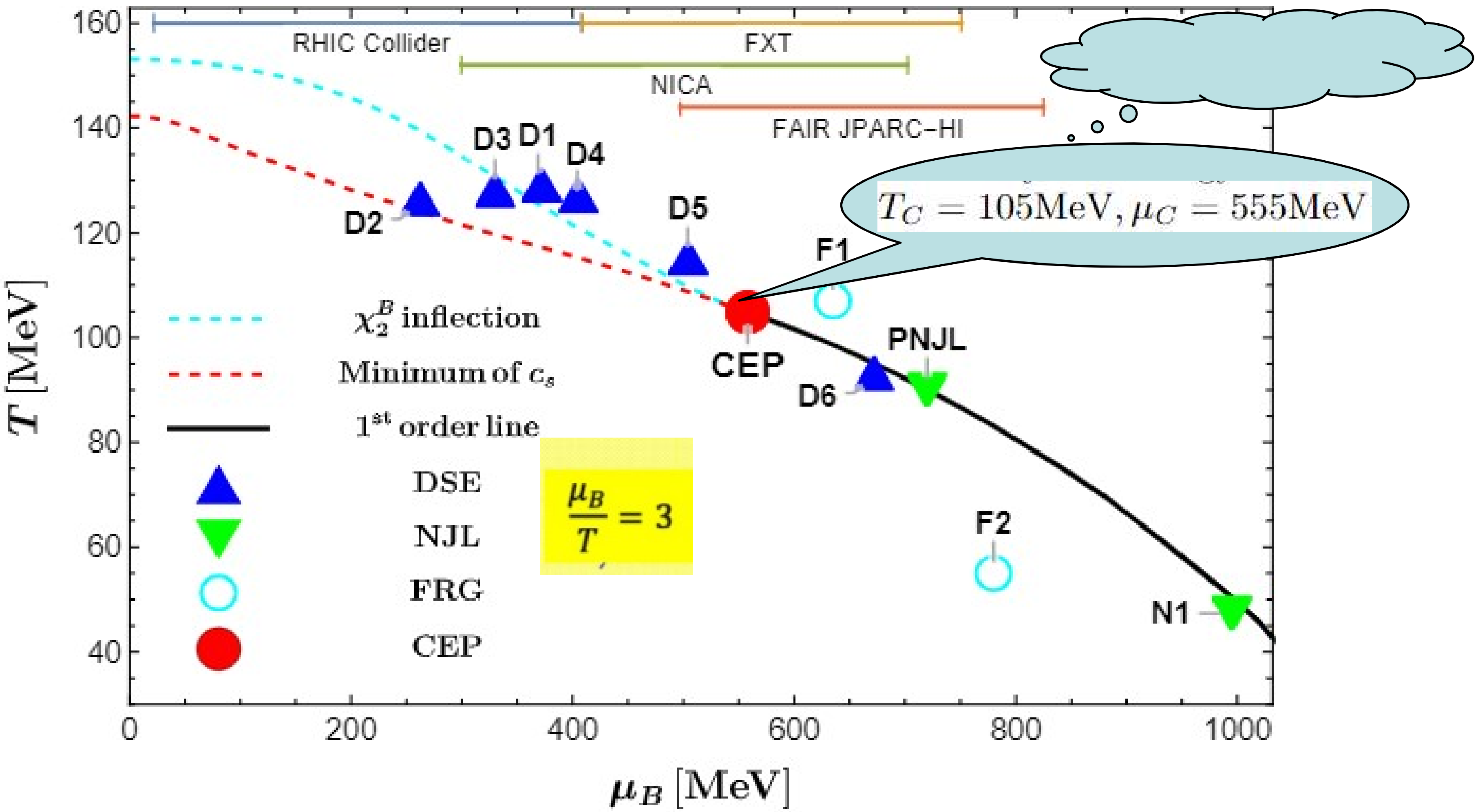


Predictions of QCD phase diagram



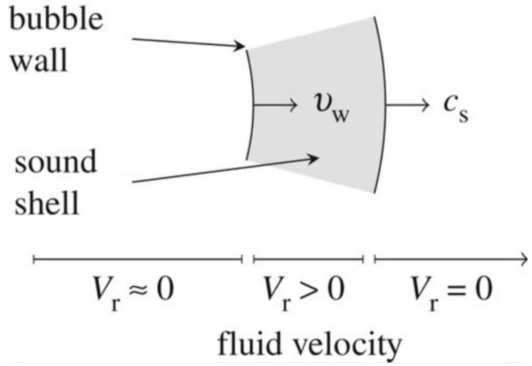
mu_B/T<=3 & mu_B< 300 MeV excluded by lattice simulation

A. Bazavov, etc.Phys. Rev. D 95 (2017) no.5, 054504 [arXiv:1701.04325 [hep-lat]].



Induced gravitational wave

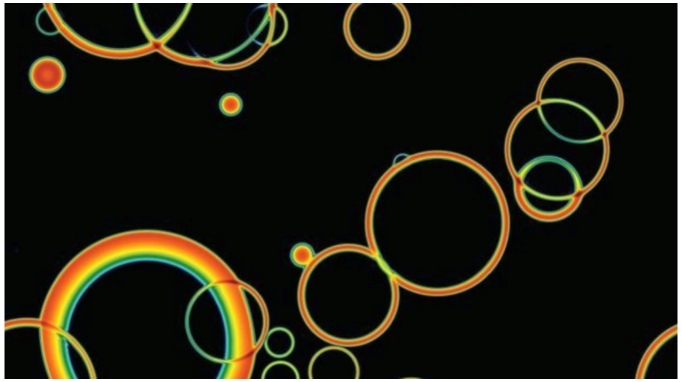
Strong first order phase transition will result in the production of GWs:



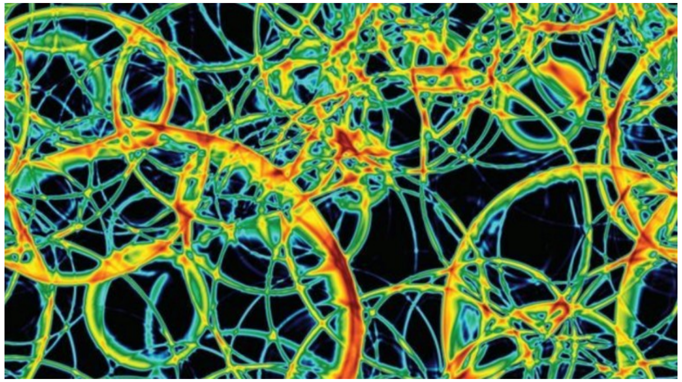
bubble collision + sound wave + MHD turbulence

GWs are dominated by **sound waves** with energy spectrum

$$h^2\Omega_{GW}(f) = 8.5 \times 10^{-6} \left(\frac{100}{g_n}\right)^{1/3} \left(\frac{\kappa\alpha}{1+\alpha}\right)^2 \times \left(\frac{H_n}{\beta}\right) v_w S_{SW}(f)\,.$$

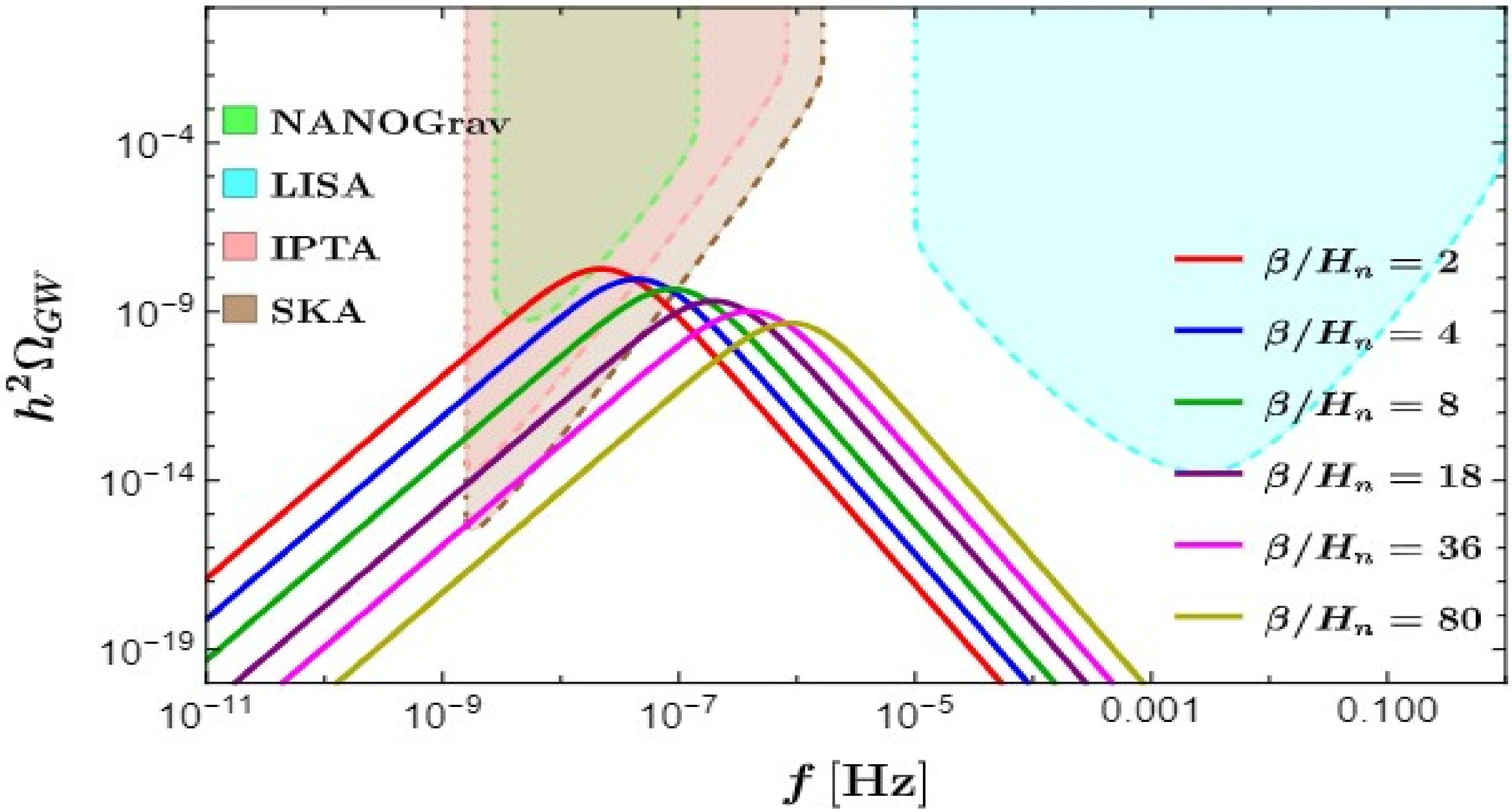


α: phase transition strength parameter
v_w: bubble wall terminal velocity
$\frac{\beta}{H_n}$: the inverse time duration of the phase transition
H_n: Hubble rate at the nucleation temperature T_n
g_n: the number of degrees of freedom
κ: the fraction of bulk kinetic energy relative to the available vacuum energy.



Weir David J. R. Soc. A.376: 20170126(2018)

Stochastic GW spectrum from the first order QCD phase transition



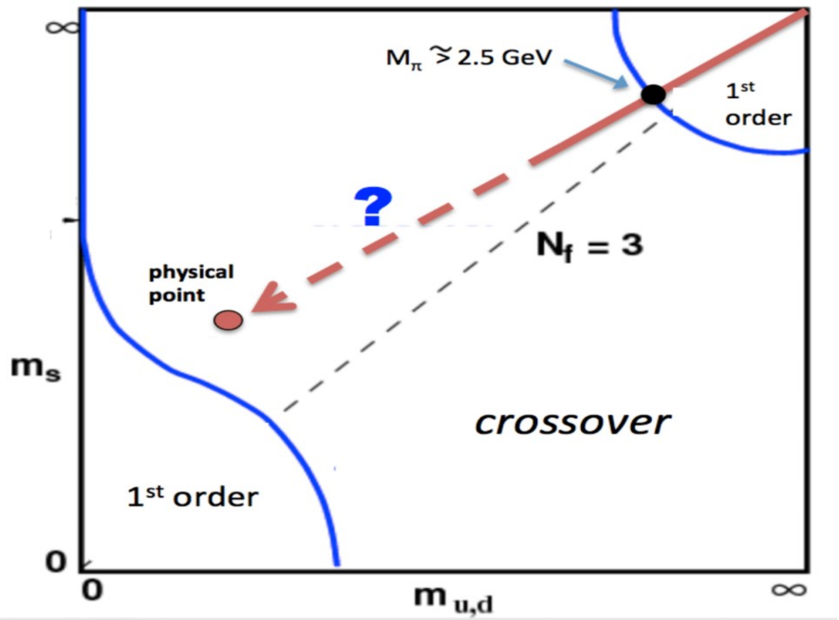
(μ, T, α) = (560 MeV, 104.71 MeV, 0.13) (red band) and (1000 MeV, 49.53 MeV, 0.33) (blue band). The upper curve in each band is for $\beta/Hn = 4$ and the lower curve is for $\beta/Hn = 80$.

The GW energy spectrum from our hQCD model is within the projected sensitivity of IPTA and SKA, thus can be potentially detected in the near future.

hQCD model for Pure gluon

HQCD model for Pure gluon syste in Einstein-Dilaton system

Motivation



SH, Li Li, Zhibin Li, Shao-Jiang Wang, in preparation

Gravity Action

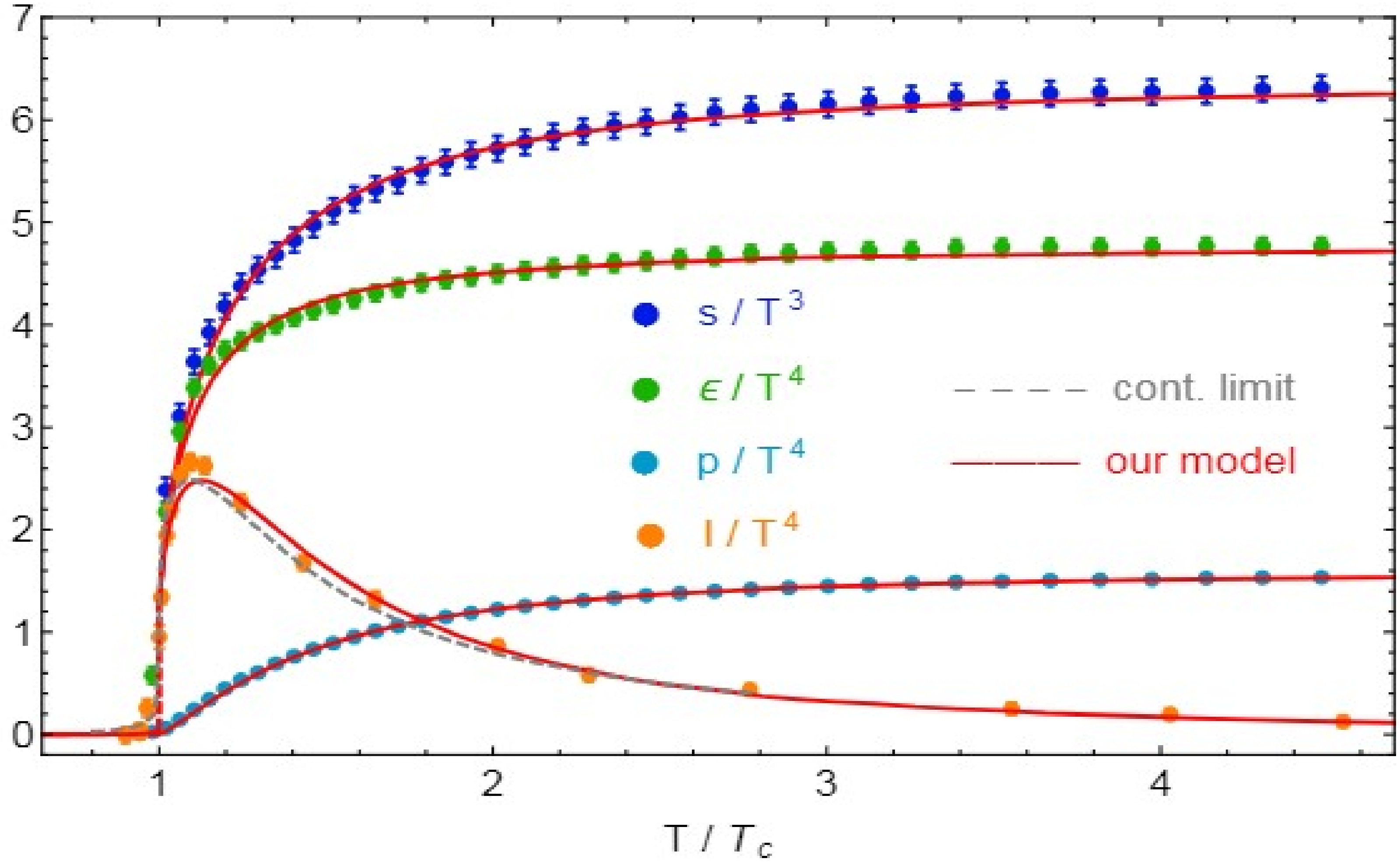
$$S=\frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[\mathcal{R}-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-V(\phi)\right]$$

$$ds^2=-f(r)e^{-\eta(r)}dt^2+\frac{dr^2}{f(r)}+r^2d\mathbf{x}_3^2,\quad \phi=\phi(r),$$

$$V(\phi)=\left(6\gamma^2-\frac{3}{2}\right)\phi^2-12\cosh(\gamma\phi),$$

Break conformal symmetry

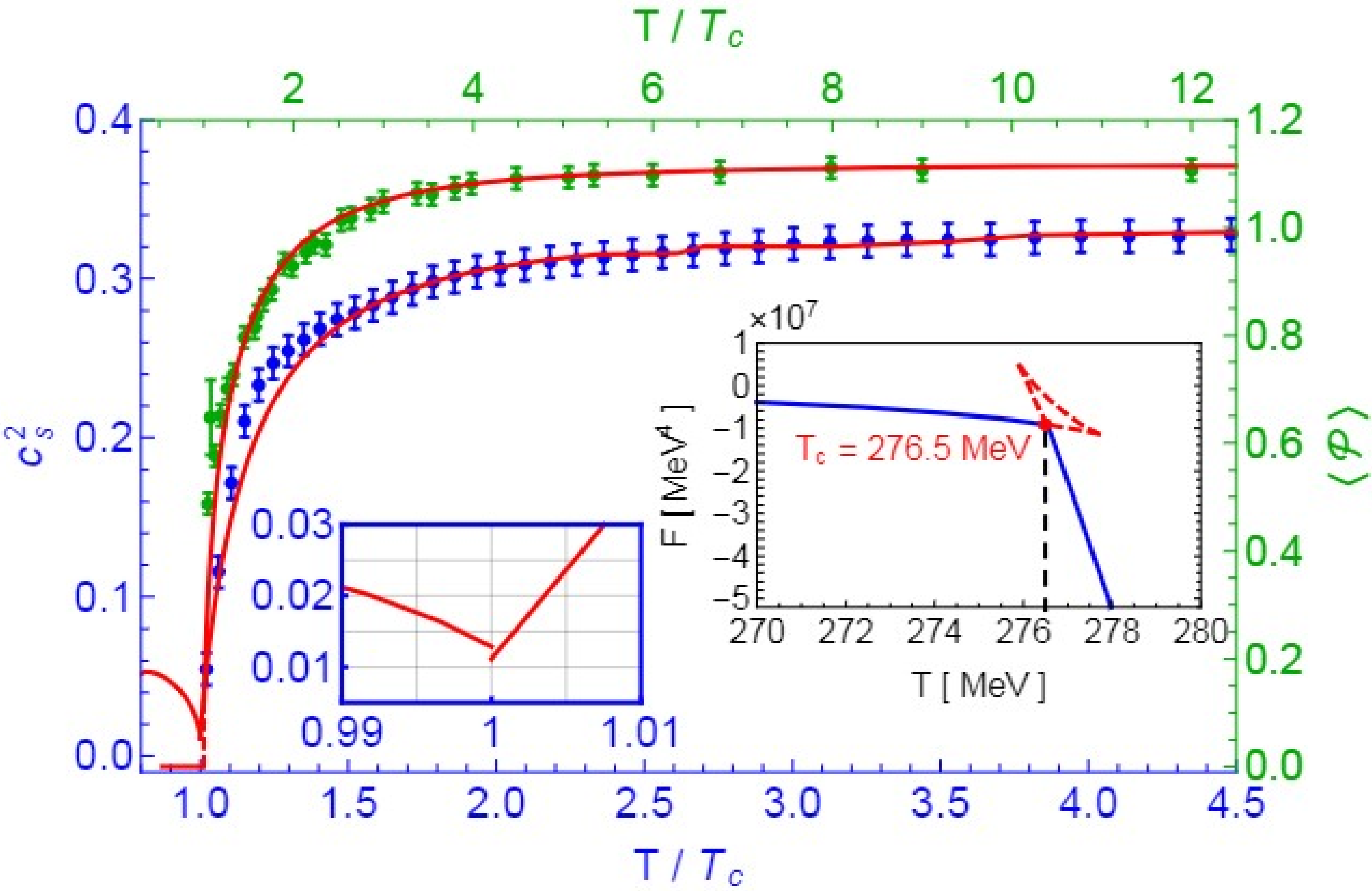
One parameter to fix all thermal dynamics lattice data



http://arxiv.org/abs/hep-lat/9602007

http://arxiv.org/abs/0711.2251

First order Phase transition vs Polyakov Loop



http://arxiv.org/abs/hep-lat/9602007

http://arxiv.org/abs/0711.2251

Gluon condensation- Probe

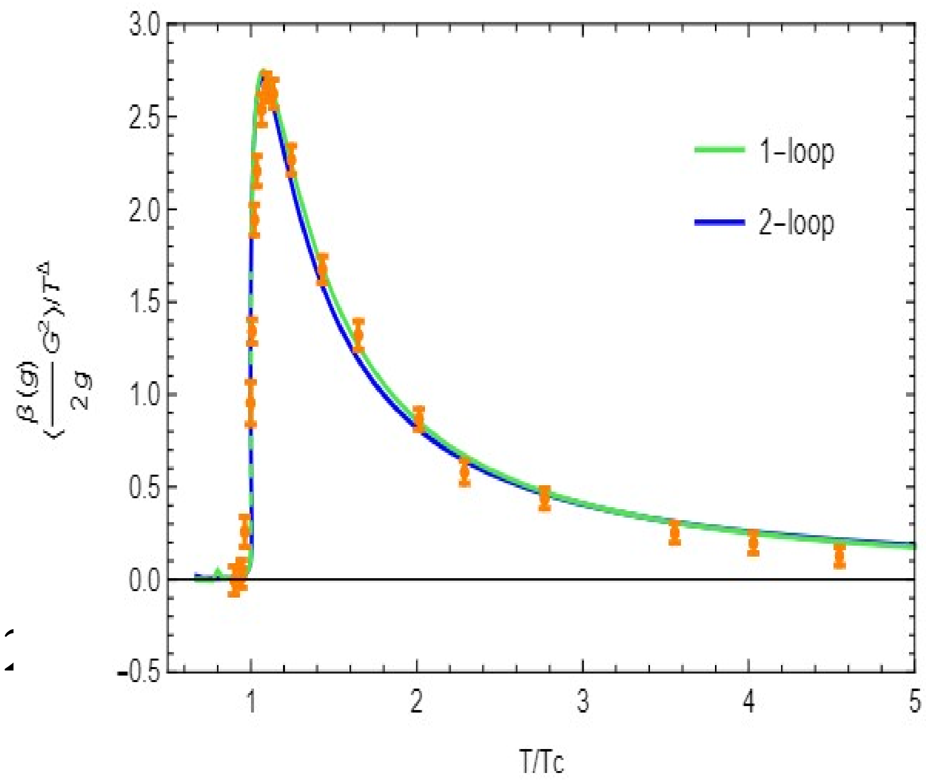
$$S = -\frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g^s}e^{-\sqrt{\frac{3}{8}}\phi}\left[\frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + \frac{1}{2}m_\chi^2\chi^2\right]. \quad (25)$$

$$\chi(r) = \chi_0 r^{\Delta-4} + \cdots + \chi_4 r^{-\Delta}$$

$$m_\chi^2 = \Delta(\Delta-4)$$

$$\chi_4 = \langle G^2 \rangle$$

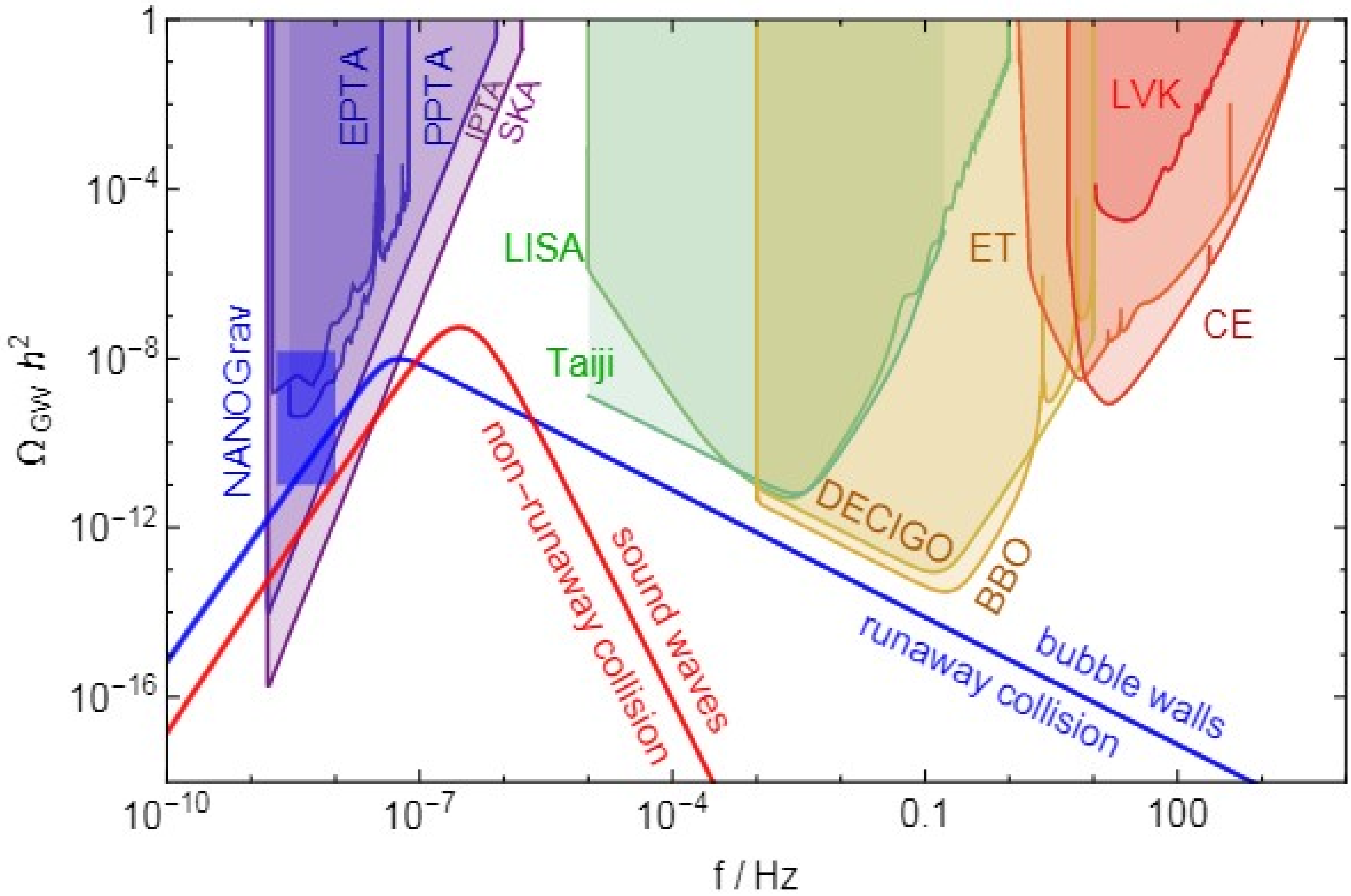
$$\left\langle \frac{\beta(g)}{2g} G^2 \right\rangle_T = \langle T^\mu_\mu \rangle = \epsilon - 3P$$



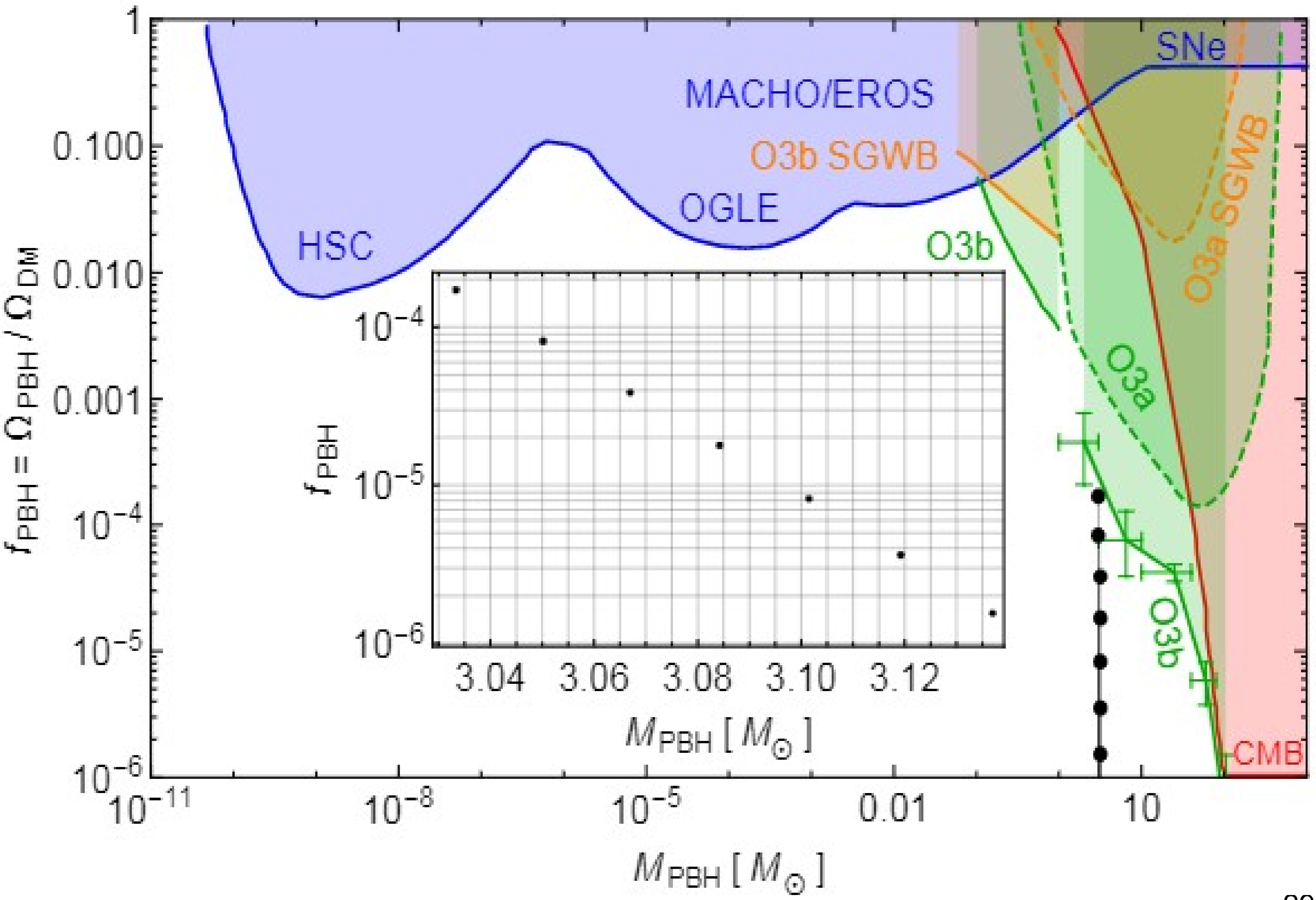
http://arxiv.org/abs/hep-lat/960

http://arxiv.org/abs/0711.2251

First order Phase transition vs GW



Gluon condensation vs Primidal Black hole



Summary

I. Propose a hQCD model on quantitative level to describe QCD phase diagram.

II. EOS confront with lattice simulations at zero/non-zero chemical potential.

III. Realize QCD CEP and quantitatively agrees with effective field results.

IV. Stochastic GW spectrum induced by QCD phase transition predicted.

V. Construct holographic pure gluon model.

Thanks for your attention！