Anisotropic Holographic Models Supported by Einstein-dilaton-four-Maxwell action

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Based on work in progress
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Motivation. Heavy-ion collisions

AdS/CFT correspondence
J.Maldacena, D.Witten, S.Gubser and others
AdS/CFT correspondence $\Longrightarrow$ $\frac{\eta}{s} = \frac{1}{4\pi}$
G.Policastro, D.Son, A.Starinets, PRL '01

Experiment: STAR at RHIC; LHC; NICA; FAIR

- Quark gluon plasma formation $\tau \sim 0.1$ fm/c
- Spatial anisotropy with $\tau_{iso} \sim 2$ fm/c
- Strongly correlated system $\Longrightarrow$ Non-perturbative methods

Our purpose: Study of the QCD phase diagram in (μ, T) plane within the fully anisotropic holographic models.

- Multiplicity $\mathcal{M} \propto s_{AdS}^{0.33}$ vs $\mathcal{M} \propto s_{LHC}^{0.155}$ $\mathcal{M} \propto s^{\frac{1}{\nu+2}}$, $\nu = 4.5$ I.Aref'eva, A.Golubtsova, JHEP '14
- Direct/inverse magnetic catalysis

Anisotropic holographic models

Holographic approach to the study of QGP is actively developed

- Isotropic models for light and heavy quarks
  M.-W. Li, Y. Yang, P.-H. Yuan, PRD '17
  Y. Yang and P.-H. Yuan JHEP '15
  O. Andreev and V.I. Zakharov, PRD '06
- Anisotropic models for light and heavy quarks
  I. Aref'eva, K.Rannu, P.S., JHEP '21
  I. Aref'eva, K.Rannu, P.S. PLB '19
  I. Aref'eva, K.Rannu, JHEP '18
- Magnetic catalysis (direct/inverse)
  U. Gürsoy, M. Järvinen, G. Nijs and J. F. Pedraza, JHEP '21
  I. Aref'eva, K.Rannu, P.S., JHEP '21
  H. Bohra, D. Dudal, A. Hajilou and S. Mahapatra PRD'21
  A. Ballon-Bayona, J. P. Shock and D. Zoakos, JHEP '20
  S. He, Y. Yang and P. H. Yuan 2004.01965

The action and metric

$$S = \int d^5x\,\sqrt{-g}\Big[R - \frac{f_0(\phi)}{4}\,F_{(0)}^2 - \frac{f_1(\phi)}{4}\,F_{(1)}^2 - \frac{f_2(\phi)}{4}\,F_{(2)}^2 - \frac{f_3(\phi)}{4}\,F_{(3)}^2$$
$$-\frac{1}{2}\,\partial_M\phi\partial^M\phi - V(\phi)\Big],\ \ \textit{where}\ \ F_{(i),MN} = \partial_M A_{(i),N} - \partial_N A_{(i),M}$$

$$ds^2 = -\,g_{00}(z)dt^2 + g_{11}(z)dx_1^2 + g_{22}(z)dx_2^2 + g_{33}(z)dx_3^2 + g_{44}(z)dz^2 \quad (*)$$

Specific anzats in previous works:

$$ds^2 = \frac{L^2}{z^2}\,b(z)\left[-\,g(z)\,dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)}\right]$$

Aref'eva, Golubtsova (2014), Giataganas (2013) *Gürsoy, Järvinen et al. (2019)*
$b(z) = e^{2\mathcal{A}(z)}$, $\mathcal{A}(z) = -\,cz^2/4$, $\mathcal{A}(z) = -\,a\ln(bz^2+1)$ for heavy quarks (b, t) and light quarks (d, u)

Anzats for the matter fields (for the scalar field and for the Maxwell fields)

$$\begin{aligned} \phi &= \phi(z) \;\; \textit{for the scalar field} \\ A_{(0),0} &= A_0(z), A_{(0),1} = \ldots A_{(0),4} = 0 \qquad \text{''}\textit{electric anzats}\text{''} \end{aligned}$$

$$F_{(1)} = dx^2 \wedge dx^3 \quad F_{(2)} = dx^3 \wedge dx^1 \quad F_{(3)} = dx^1 \wedge dx^2$$

"magnetic anzats"

Ricci tensor

The Ricci tensor is diagonal for the metric (*) because the metric is diagonal and depends only on z coordinate $g_{MN} = g_{MM}(z)$. This is a particular case of a general formula for the Ricci tensor for diagonal metric

Dingle (1933)

$$R_{00} = \frac{g''_{00}}{2g_{44}} - \frac{g'^2_{00}}{4g_{00}g_{44}} + \frac{g'_{00}g'_{11}}{4g_{11}g_{44}} + \frac{g'_{00}g'_{22}}{4g_{22}g_{44}} + \frac{g'_{00}g'_{33}}{4g_{33}g_{44}} - \frac{g'_{00}g'_{44}}{4g^2_{44}},$$

$$R_{jj} = -\frac{g''_{jj}}{2g_{44}} + \frac{g'^2_{jj}}{4g_{jj}g_{44}} - \frac{g'_{00}g'_{jj}}{4g_{00}g_{44}} - \sum_{i=1, i\neq j}^{3} \frac{g'_{jj}g'_{ii}}{4g_{ii}g_{44}} + \frac{g'_{jj}g'_{44}}{4g^2_{44}}, \ j = 1, 2, 3$$

$$\begin{aligned} R_{44} = & -\frac{g''_{00}}{2g_{00}} - \frac{g''_{11}}{2g_{11}} - \frac{g''_{22}}{2g_{22}} - \frac{g''_{33}}{2g_{33}} + \frac{g'^2_{00}}{4g^2_{00}} + \frac{g'^2_{11}}{4g^2_{11}} + \frac{g'^2_{22}}{4g^2_{22}} + \frac{g'^2_{33}}{4g^2_{33}} + \\ & + \frac{g'_{00}g'_{44}}{4g_{00}g_{44}} + \frac{g'_{11}g'_{44}}{4g_{11}g_{44}} + \frac{g'_{22}g'_{44}}{4g_{22}g_{44}} + \frac{g'_{33}g'_{44}}{4g_{33}g_{44}}, \end{aligned}$$

Einstein equations

Einstein EOMs can be written as:

$$D_M D^M \phi + V'(\phi) + \sum_{a=1,2,B} \frac{f'_a(\phi)}{4} \, F^2_{(a)} = 0,$$

$$\partial_M(\sqrt{-g}\, f_a \, F^{MN}_{(a)}) = 0 \qquad\qquad a = 0, 1, 2, 3,$$

$$R_{MN} - \frac{1}{2}\, g_{MN} R = \, T^{(\phi)}_{MN} + \sum_a T^{(a)}_{MN}, \quad M, N = 0, 1, 2, 3, 4,$$

$$T^{(\phi)}_{MN} = \frac{1}{2}(\partial_M \phi \partial_N \phi - \frac{1}{2} g_{MN} \partial_K \phi \partial^K \phi) - \frac{1}{2} g_{MN} V(\phi),$$

$$T^{(a)}_{MN} = \frac{1}{2} f_a \left(F_{(a)\, ML} F_{(a)\, N}{}^{L} - \frac{1}{4}\, g_{MN} F_{(a)}{}^{2} \right).$$

Matter fiels

Tensors for all matter fiels are also diagonal.

$$T_{MN} = T_{MN}^{(F_0)} + \sum_{i=1}^{3} T_{MN}^{(F_{(i)})} + T_{MN}^{(\phi)}$$

EOM for magnetic fields are automatically satisfied:

$$\partial_M(-\sqrt{g}\, f\, F^{MN}) = 0$$

since all functions depend on z and $F^{4,M} = 0$, however eq. for electric field A_0 is nontrivial. Therefore all together we have equations:

ANISOTROPIC CASE

$$\#of\ \ equations: \underbrace{5}_{Einstein} + \underbrace{1}_{\phi} + \underbrace{1}_{A_0} = 7$$

ISOTROPIC CASE

$$\#of\ \ equations: \underbrace{3}_{Einstein} + \underbrace{1}_{\phi} + \underbrace{1}_{A_0} = 5$$

$$5(g_{MN}) + 1(A_0) + 1(\phi) + 1(V) + 4(f_0(\phi), f_i(\phi)) = 12 \quad func.$$

$$3(g_{MN}) + 1(A_0) + 1(\phi) + 1(V) + 1(f_0(\phi)) = 7 \quad func.$$

5 Einstein equations

$$
\begin{aligned}
\mathbf{00}: \quad & \sqrt{-g}\,\Bigg[\frac{g_{00}}{2g_{44}}\Bigg\{\frac{g_{11}''}{g_{11}}+\frac{g_{22}''}{g_{22}}+\frac{g_{33}''}{g_{33}}-\frac{g_{11}'}{2g_{11}}\left(\frac{g_{11}'}{g_{11}}-\frac{g_{22}'}{g_{22}}+\frac{g_{44}'}{g_{44}}\right)- \\
& -\frac{g_{22}'}{2g_{22}}\left(\frac{g_{22}'}{g_{22}}-\frac{g_{33}'}{g_{33}}+\frac{g_{44}'}{g_{44}}\right)-\frac{g_{33}'}{2g_{33}}\left(-\frac{g_{11}'}{g_{11}}+\frac{g_{33}'}{g_{33}}+\frac{g_{44}'}{g_{44}}\right)\Bigg\}+ \\
& + g_{00}\left(\frac{f_0A_0'^2}{4g_{00}g_{44}}+\frac{f_1}{4g_{22}g_{33}}+\frac{f_2}{4g_{11}g_{33}}+\frac{f_3}{4g_{11}g_{22}}+\frac{\phi'^2}{4g_{44}}+\frac{V}{2}\right)\Bigg]=0, \\
\mathbf{11}: \quad & \sqrt{-g}\,\Bigg[\frac{g_{11}}{2g_{44}}\Bigg\{\frac{g_{00}''}{g_{00}}+\frac{g_{22}''}{g_{22}}+\frac{g_{33}''}{g_{33}}-\frac{g_{00}'}{2g_{00}}\left(\frac{g_{00}'}{g_{00}}-\frac{g_{22}'}{g_{22}}+\frac{g_{44}'}{g_{44}}\right)- \\
& -\frac{g_{22}'}{2g_{22}}\left(\frac{g_{22}'}{g_{22}}-\frac{g_{33}'}{g_{33}}+\frac{g_{44}'}{g_{44}}\right)-\frac{g_{33}'}{2g_{33}}\left(-\frac{g_{00}'}{g_{00}}+\frac{g_{33}'}{g_{33}}+\frac{g_{44}'}{g_{44}}\right)\Bigg\}+ \\
& + g_{11}\left(-\frac{f_0A_0'^2}{4g_{00}g_{44}}+\frac{f_1}{4g_{22}g_{33}}-\frac{f_2}{4g_{11}g_{33}}-\frac{f_3}{4g_{11}g_{22}}+\frac{\phi'^2}{4g_{44}}+\frac{V}{2}\right)\Bigg]=0, \\
\mathbf{22}: \quad & \sqrt{-g}\,\Bigg[\frac{g_{22}}{2g_{44}}\Bigg\{\frac{g_{00}''}{g_{00}}+\frac{g_{11}''}{g_{11}}+\frac{g_{33}''}{g_{33}}-\frac{g_{00}'}{2g_{00}}\left(\frac{g_{00}'}{g_{00}}-\frac{g_{11}'}{g_{11}}+\frac{g_{44}'}{g_{44}}\right)- \\
& -\frac{g_{11}'}{2g_{11}}\left(\frac{g_{11}'}{g_{11}}-\frac{g_{33}'}{g_{33}}+\frac{g_{44}'}{g_{44}}\right)-\frac{g_{33}'}{2g_{33}}\left(-\frac{g_{00}'}{g_{00}}+\frac{g_{33}'}{g_{33}}+\frac{g_{44}'}{g_{44}}\right)\Bigg\}+ \\
& + g_{22}\left(-\frac{f_0A_0'^2}{4g_{00}g_{44}}-\frac{f_1}{4g_{22}g_{33}}+\frac{f_2}{4g_{11}g_{33}}-\frac{f_3}{4g_{11}g_{22}}+\frac{\phi'^2}{4g_{44}}+\frac{V}{2}\right)\Bigg]=0,
\end{aligned}
$$

5 Einstein equations

$$
\begin{aligned}
\mathbf{33}: \quad & \sqrt{-g}\left[\frac{g_{33}}{2g_{44}}\left\{\frac{g_{00}''}{g_{00}}+\frac{g_{11}''}{g_{11}}+\frac{g_{22}''}{g_{22}}-\frac{g_{00}'}{2g_{00}}\left(\frac{g_{00}'}{g_{00}}-\frac{g_{11}'}{g_{11}}+\frac{g_{44}'}{g_{44}}\right)-\right.\right.\\
& -\left.\frac{g_{11}'}{2g_{11}}\left(\frac{g_{11}'}{g_{11}}-\frac{g_{22}'}{g_{22}}+\frac{g_{44}'}{g_{44}}\right)-\frac{g_{22}'}{2g_{22}}\left(-\frac{g_{00}'}{g_{00}}+\frac{g_{22}'}{g_{22}}+\frac{g_{44}'}{g_{44}}\right)\right\}+\\
& +\left.g_{33}\left(-\frac{f_0 A_0'^2}{4g_{00}g_{44}}-\frac{f_1}{4g_{22}g_{33}}-\frac{f_2}{4g_{11}g_{33}}+\frac{f_3}{4g_{11}g_{22}}+\frac{\phi'^2}{4g_{44}}+\frac{V}{2}\right)\right]=0,\\
\mathbf{44}: \quad & \sqrt{-g}\left[\frac{g_{00}'}{4g_{00}}\left(\frac{g_{00}'}{g_{00}}+\frac{g_{11}'}{g_{11}}\right)+\frac{g_{11}'}{4g_{11}}\left(\frac{g_{22}'}{g_{22}}+\frac{g_{33}'}{g_{33}}\right)+\frac{g_{33}'}{4g_{33}}\left(\frac{g_{00}'}{g_{00}}+\frac{g_{22}'}{g_{22}}\right)+\right.\\
& +\left.g_{33}\left(\frac{f_0 A_0'^2}{4g_{00}g_{44}}+\frac{f_1}{4g_{22}g_{33}}+\frac{f_2}{4g_{11}g_{33}}+\frac{f_3}{4g_{11}g_{22}}-\frac{\phi'^2}{4g_{44}}+\frac{V}{2}\right)\right]=0,
\end{aligned}
$$

where $' = \partial/\partial z$.

5 Einstein equations

Let us form linear combinations of equations in the following way:

$$\frac{2g_{44}}{\sqrt{-g}}\left(-\frac{\mathbf{00}}{g_{00}}+\frac{\mathbf{11}}{g_{11}}\right),$$
$$\frac{2g_{44}}{\sqrt{-g}}\left(\frac{\mathbf{00}}{g_{00}}-\frac{\mathbf{44}}{g_{44}}\right),$$
$$\frac{-2g_{44}}{\sqrt{-g}}\left(\frac{\mathbf{11}}{g_{11}}-\frac{\mathbf{22}}{g_{22}}\right),$$
$$\frac{-2g_{44}}{\sqrt{-g}}\left(\frac{\mathbf{11}}{g_{11}}-\frac{\mathbf{33}}{g_{33}}\right),$$
$$\frac{2g_{44}}{\sqrt{-g}}\left(\frac{\mathbf{22}}{g_{22}}+\frac{\mathbf{44}}{g_{44}}\right).$$

All EOMs (5 Einstein equations, 1 for scalar field ϕ, 1 for electric potential A_0)

$$\phi'' + \frac{\phi'}{2}\left(\frac{g_{00}'}{g_{00}} + \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}}\right) +$$

$$+ \frac{(A_0')^2}{2g_{00}}\frac{\partial f_0}{\partial \phi} - \frac{g_{44}}{2g_{22}g_{33}}\frac{\partial f_1}{\partial \phi} - \frac{g_{44}}{2g_{11}g_{33}}\frac{\partial f_2}{\partial \phi} - \frac{g_{44}}{2g_{11}g_{22}}\frac{\partial f_3}{\partial \phi} - g_{44}\frac{\partial V}{\partial \phi} = 0,$$

$$A_0'' + A_0'\left(-\frac{g_{00}'}{2g_{00}} + \frac{g_{11}'}{2g_{11}} + \frac{g_{22}'}{2g_{22}} + \frac{g_{33}'}{2g_{33}} - \frac{g_{44}'}{2g_{44}} + \frac{f_0'}{f_0}\right) = 0,$$

$$\frac{g_{00}''}{g_{00}} - \frac{g_{11}''}{g_{11}} - \frac{g_{00}'^2}{2g_{00}^2} + \frac{g_{11}'^2}{2g_{11}^2} + \frac{1}{2}\left(\frac{g_{00}'}{g_{00}} - \frac{g_{11}'}{g_{11}}\right)\left(\frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}}\right) - \frac{f_0(A_0')^2}{g_{00}} - \frac{g_{44}\,f_2}{2g_{11}g_{33}} - \frac{g_{44}\,f_3}{2g_{11}g_{22}} = 0,$$

$$\frac{g_{11}''}{g_{11}} + \frac{g_{22}''}{g_{22}} + \frac{g_{33}''}{g_{33}} - \frac{1}{2}\left[\frac{g_{11}'^2}{g_{11}^2} + \frac{g_{22}'^2}{g_{22}^2} + \frac{g_{33}'^2}{g_{33}^2} + \left(\frac{g_{00}'}{g_{00}} + \frac{g_{44}'}{g_{44}}\right)\left(\frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}}\right)\right] + \phi'^2 = 0,$$

$$\frac{g_{11}''}{g_{11}} - \frac{g_{22}''}{g_{22}} - \frac{g_{11}'^2}{2g_{11}^2} + \frac{g_{22}'^2}{2g_{22}^2} + \frac{1}{2}\left(\frac{g_{11}'}{g_{11}} - \frac{g_{22}'}{g_{22}}\right)\left(\frac{g_{00}'}{g_{00}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}}\right) - \frac{g_{44}\,f_1}{g_{22}g_{33}} + \frac{g_{44}\,f_2}{g_{11}g_{33}} = 0,$$

$$\frac{g_{11}''}{g_{11}} - \frac{g_{33}''}{g_{33}} - \frac{g_{11}'^2}{2g_{11}^2} + \frac{g_{33}'^2}{2g_{33}^2} + \frac{1}{2}\left(\frac{g_{11}'}{g_{11}} - \frac{g_{33}'}{g_{33}}\right)\left(\frac{g_{00}'}{g_{00}} + \frac{g_{22}'}{g_{22}} - \frac{g_{44}'}{g_{44}}\right) - \frac{g_{44}\,f_1}{g_{22}g_{33}} + \frac{g_{44}\,f_3}{g_{11}g_{22}} = 0,$$

$$\frac{g_{00}''}{g_{00}} + \frac{g_{11}''}{g_{11}} + \frac{g_{33}''}{g_{33}} + \frac{g_{00}'}{2g_{00}}\left(-\frac{g_{00}'}{g_{00}} + \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}}\right) + \frac{g_{11}'}{2g_{11}}\left(\frac{g_{00}'}{g_{00}} - \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}}\right) +$$

$$+ \frac{g_{33}'}{2g_{33}}\left(\frac{g_{00}'}{g_{00}} + \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} - \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}}\right) + \frac{g_{44}\,f_2}{g_{11}g_{33}} + 2g_{44}V = 0.$$

All EOMs (5 Einstein equations, 1 for scalar field ϕ, 1 for electric potential A_0)

In the system on the previous slide, one of the equations is a consequence of the others. We can rewrite the remaining independent equations in the form:

$$A_0'' = -A_0' \left(-\frac{g_{00}'}{2g_{00}} + \frac{g_{11}'}{2g_{11}} + \frac{g_{22}'}{2g_{22}} + \frac{g_{33}'}{2g_{33}} - \frac{g_{44}'}{2g_{44}} + \frac{f_0'}{f_0} \right), (1)$$

$$g_{00}'' = g_{00} \left[\frac{g_{11}''}{g_{11}} + \frac{g_{00}'^2}{2g_{00}^2} - \frac{g_{11}'^2}{2g_{11}^2} - \frac{1}{2} \left(\frac{g_{00}'}{g_{00}} - \frac{g_{11}'}{g_{11}} \right) \left(\frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \frac{f_0 (A_0')^2}{g_{00}} + \frac{g_{44}\, f_2}{g_{11} g_{33}} + \frac{g_{44}\, f_3}{g_{11} g_{22}} \right], (2)$$

$$g_{11}'' = -g_{11} \left[\frac{g_{22}''}{g_{22}} + \frac{g_{33}''}{g_{33}} - \frac{1}{2} \left\{ \frac{g_{11}'^2}{g_{11}^2} + \frac{g_{22}'^2}{g_{22}^2} + \frac{g_{33}'^2}{g_{33}^2} + \left(\frac{g_{00}'}{g_{00}} + \frac{g_{44}'}{g_{44}} \right) \left(\frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} \right) \right\} + \phi'^2 \right], \quad (3)$$

$$f_2 = -\frac{g_{11} g_{33}}{g_{44}} \left[\frac{g_{11}''}{g_{11}} - \frac{g_{22}''}{g_{22}} - \frac{g_{11}'^2}{2g_{11}^2} + \frac{g_{22}'^2}{2g_{22}^2} + \frac{1}{2} \left(\frac{g_{11}'}{g_{11}} - \frac{g_{22}'}{g_{22}} \right) \left(\frac{g_{00}'}{g_{00}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) - \frac{g_{44}\, f_1}{g_{22} g_{33}} \right], \quad (4)$$

$$f_3 = -\frac{g_{11} g_{22}}{g_{44}} \left[\frac{g_{11}''}{g_{11}} - \frac{g_{33}''}{g_{33}} - \frac{g_{11}'^2}{2g_{11}^2} + \frac{g_{33}'^2}{2g_{33}^2} + \frac{1}{2} \left(\frac{g_{11}'}{g_{11}} - \frac{g_{33}'}{g_{33}} \right) \left(\frac{g_{00}'}{g_{00}} + \frac{g_{22}'}{g_{22}} - \frac{g_{44}'}{g_{44}} \right) - \frac{g_{44}\, f_1}{g_{22} g_{33}} \right], \quad (5)$$

$$\begin{aligned} V = -\frac{1}{2g_{44}} \Bigg[& \frac{g_{00}''}{g_{00}} + \frac{g_{11}''}{g_{11}} + \frac{g_{33}''}{g_{33}} + \frac{g_{00}'}{2g_{00}} \left(-\frac{g_{00}'}{g_{00}} + \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \\ & + \frac{g_{11}'}{2g_{11}} \left(\frac{g_{00}'}{g_{00}} - \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \frac{g_{33}'}{2g_{33}} \left(\frac{g_{00}'}{g_{00}} + \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} - \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \frac{g_{44}\, f_2}{g_{11} g_{33}} \Bigg] \quad (6) \end{aligned}$$

Bianchi identity and EOM connection

$$ds^2 = -\ g_{00}(z)dt^2 + g_{11}(z)dx_1^2 + g_{22}(z)dx_2^2 + g_{33}(z)dx_3^2 + g_{44}(z)dz^2(*) =$$

$$\mathfrak{B}^2\left[-\ g_0(z)dt^2 + \mathfrak{g}_1(z)dx_1^2 + \mathfrak{g}_2(z)dx_2^2 + \mathfrak{g}_3(z)dx_3^2 + \frac{dz^2}{g_0(z)}\right](**)$$

$$g_{00} = \mathfrak{B}^2 g_0, \quad g_{11} = \mathfrak{B}^2\mathfrak{g}_1, \quad g_{22} = \mathfrak{B}^2\mathfrak{g}_2, \quad g_{33} = \mathfrak{B}^2\mathfrak{g}_3, \quad g_{44} = \mathfrak{B}^2/g_0$$

Biianchi identity and EOM connection

If we consider stress-energy tensor for all the matter in parameterization (**):

$$T^{MN} = T^{MN}_{F_0} + T^{MN}_{F_1} + T^{MN}_{F_2} + T^{MN}_{F_3} + T^{MN}_{\phi},$$

$$\nabla_M T^{MN} = \delta^N_4 \nabla_M T^{M4},$$

$$\begin{aligned}
\nabla_M T^{M4} = \nabla_M T^{M4}_{F_0} + \nabla_M T^{M4}_{F_1} + \nabla_M T^{M4}_{F_2} + \nabla_M T^{M4}_{F_3} + \nabla_M T^{M4}_{\phi} = \\
= \frac{g^2\phi'}{2\mathfrak{B}^4}\Bigg[\phi'' + \phi'\left(\frac{3\mathfrak{B}'}{\mathfrak{B}} + \frac{g'}{g} + \frac{1}{2}\sum_{i=1}^{3}\frac{\mathfrak{g}_i'}{\mathfrak{g}_i}\right) - \\
- \frac{f_0 A_0'}{\mathfrak{B}^2 g\phi'}\left\{A_0'' + A_0'\left(\frac{\mathfrak{B}'}{\mathfrak{B}} + \frac{f_0'}{2f_0} + \frac{1}{2}\sum_{i=1}^{3}\frac{\mathfrak{g}_i'}{\mathfrak{g}_i}\right)\right\} - \\
- \frac{\partial f_1}{\partial\phi}\frac{1}{2\mathfrak{B}^2 g\,\mathfrak{g}_2\mathfrak{g}_3} - \frac{\partial f_2}{\partial\phi}\frac{1}{2\mathfrak{B}^2 g\,\mathfrak{g}_1\mathfrak{g}_3} - \frac{\partial f_3}{\partial\phi}\frac{1}{2\mathfrak{B}^2 g\,\mathfrak{g}_1\mathfrak{g}_2} - \frac{\mathfrak{B}^2}{g}\frac{\partial V}{\partial\phi}\Bigg].
\end{aligned}$$

This is a linear combination of the EOM for the vector potential and the EOM for the scalar field.

Bianchi identity and EOM connection

$$\frac{\nabla_M T^{MN}}{g} = \frac{1}{2\sqrt{-g}\,\mathfrak{B}^2}\left(A_0'\,\frac{\delta\mathcal{L}}{\delta A_0} + \phi'\,\frac{\delta\mathcal{L}}{\delta\phi}\right),$$

$$\frac{\delta\mathcal{L}}{\delta A_0} \;\sim\; A_0'' + A_0'\left(\frac{\mathfrak{B}'}{\mathfrak{B}} + \frac{f_0'}{f_0} + \frac{1}{2}\sum_{i=1}^{3}\frac{\mathfrak{g}_i'}{\mathfrak{g}_i}\right) = 0 \;(EOM\;\; for\;\; A_0),$$

$$\frac{\delta\mathcal{L}}{\delta\phi} \;\sim\; \phi'' + \phi'\left(\frac{3\mathfrak{B}'}{\mathfrak{B}} + \frac{g'}{g} + \frac{1}{2}\sum_{i=1}^{3}\frac{\mathfrak{g}_i'}{\mathfrak{g}_i}\right) +$$

$$+\;\frac{\partial f_0}{\partial\phi}\,\frac{A_0'^2}{2\mathfrak{B}^2 g} - \frac{\partial f_1}{\partial\phi}\,\frac{1}{2\mathfrak{B}^2 g\,\mathfrak{g}_2\mathfrak{g}_3} - \frac{\partial f_2}{\partial\phi}\,\frac{1}{2\mathfrak{B}^2 g\,\mathfrak{g}_1\mathfrak{g}_3} - \frac{\partial f_3}{\partial\phi}\,\frac{1}{2\mathfrak{B}^2 g\,\mathfrak{g}_1\mathfrak{g}_2} - \frac{\mathfrak{B}^2}{g}\,\frac{\partial V}{\partial\phi} = 0.$$

$(EOM\;\; for\;\; \phi)$

All EOMs (5 Einstein equations, 1 for scalar field ϕ, 1 for electric potential A_0)

One of the equations is a consequence of the others. We can rewrite the remaining independent equations in the form:

$$A_0'' = -A_0' \left(-\frac{g_{00}'}{2g_{00}} + \frac{g_{11}'}{2g_{11}} + \frac{g_{22}'}{2g_{22}} + \frac{g_{33}'}{2g_{33}} - \frac{g_{44}'}{2g_{44}} + \frac{f_0'}{f_0} \right), (1)$$

$$g_{00}'' = g_{00} \left[\frac{g_{11}''}{g_{11}} + \frac{g_{00}'^2}{2g_{00}^2} - \frac{g_{11}'^2}{2g_{11}^2} - \frac{1}{2} \left(\frac{g_{00}'}{g_{00}} - \frac{g_{11}'}{g_{11}} \right) \left(\frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \frac{f_0 (A_0')^2}{g_{00}} + \frac{g_{44}\, f_2}{g_{11} g_{33}} + \frac{g_{44}\, f_3}{g_{11} g_{22}} \right], (2)$$

$$g_{11}'' = -g_{11} \left[\frac{g_{22}''}{g_{22}} + \frac{g_{33}''}{g_{33}} - \frac{1}{2} \left\{ \frac{g_{11}'^2}{g_{11}^2} + \frac{g_{22}'^2}{g_{22}^2} + \frac{g_{33}'^2}{g_{33}^2} + \left(\frac{g_{00}'}{g_{00}} + \frac{g_{44}'}{g_{44}} \right) \left(\frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} \right) \right\} + \phi'^2 \right], \quad (3)$$

$$f_2 = -\frac{g_{11} g_{33}}{g_{44}} \left[\frac{g_{11}''}{g_{11}} - \frac{g_{22}''}{g_{22}} - \frac{g_{11}'^2}{2g_{11}^2} + \frac{g_{22}'^2}{2g_{22}^2} + \frac{1}{2} \left(\frac{g_{11}'}{g_{11}} - \frac{g_{22}'}{g_{22}} \right) \left(\frac{g_{00}'}{g_{00}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) - \frac{g_{44}\, f_1}{g_{22} g_{33}} \right], \quad (4)$$

$$f_3 = -\frac{g_{11} g_{22}}{g_{44}} \left[\frac{g_{11}''}{g_{11}} - \frac{g_{33}''}{g_{33}} - \frac{g_{11}'^2}{2g_{11}^2} + \frac{g_{33}'^2}{2g_{33}^2} + \frac{1}{2} \left(\frac{g_{11}'}{g_{11}} - \frac{g_{33}'}{g_{33}} \right) \left(\frac{g_{00}'}{g_{00}} + \frac{g_{22}'}{g_{22}} - \frac{g_{44}'}{g_{44}} \right) - \frac{g_{44}\, f_1}{g_{22} g_{33}} \right], \quad (5)$$

$$V = -\frac{1}{2g_{44}} \left[\frac{g_{00}''}{g_{00}} + \frac{g_{11}''}{g_{11}} + \frac{g_{33}''}{g_{33}} + \frac{g_{00}'}{2g_{00}} \left(-\frac{g_{00}'}{g_{00}} + \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \right.$$

$$\left. + \frac{g_{11}'}{2g_{11}} \left(\frac{g_{00}'}{g_{00}} - \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} + \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \frac{g_{33}'}{2g_{33}} \left(\frac{g_{00}'}{g_{00}} + \frac{g_{11}'}{g_{11}} + \frac{g_{22}'}{g_{22}} - \frac{g_{33}'}{g_{33}} - \frac{g_{44}'}{g_{44}} \right) + \frac{g_{44}\, f_2}{g_{11} g_{33}} \right] \quad (6)$$

The scheme of equations solution

The system of equations is solvable if consider the anzats (**) and set f_0, f_1, $\mathfrak{B}$, $\mathfrak{g}_1$, $\mathfrak{g}_2$, $\mathfrak{g}_3$.

$$ds^2 = -\ g_{00}(z)dt^2 + g_{11}(z)dx_1^2 + g_{22}(z)dx_2^2 + g_{33}(z)dx_3^2 + g_{44}(z)dz^2 (*) =$$

$$\mathfrak{B}^2 \left[-g_0(z)dt^2 + \mathfrak{g}_1(z)dx_1^2 + \mathfrak{g}_2(z)dx_2^2 + \mathfrak{g}_3(z)dx_3^2 + \frac{dz^2}{g_0(z)} \right] (**)$$

$$g_{00} = \mathfrak{B}^2 g_0, \quad g_{11} = \mathfrak{B}^2 \mathfrak{g}_1, \quad g_{22} = \mathfrak{B}^2 \mathfrak{g}_2, \quad g_{33} = \mathfrak{B}^2 \mathfrak{g}_3, \quad g_{44} = \mathfrak{B}^2 / g_0$$

All EOMs (5 Einstein equations, 1 for scalar field ϕ, 1 for electric potential A_0)

$$A_0'' = -A_0' \left(\frac{\mathfrak{B}'(z)}{\mathfrak{B}(z)} + \frac{\mathfrak{g}_1'}{2\mathfrak{g}_1} + \frac{\mathfrak{g}_2'}{2\mathfrak{g}_2} + \frac{\mathfrak{g}_3'}{2\mathfrak{g}_3} + \frac{f_0'}{f_0} \right) \quad (1)$$

$$g_0'' + g_0' \left(\frac{3\mathfrak{B}'}{\mathfrak{B}} - \frac{\mathfrak{g}_1'}{2\mathfrak{g}_1} + \frac{\mathfrak{g}_2'}{2\mathfrak{g}_2} + \frac{\mathfrak{g}_3'}{2\mathfrak{g}_3} \right) +$$
$$+ g_0 \left(-\frac{3\mathfrak{B}' g_1'}{\mathfrak{B}\mathfrak{g}_1} - \frac{\mathfrak{g}_1''}{\mathfrak{g}_1} - \frac{\mathfrak{g}_1'\mathfrak{g}_2'}{2\mathfrak{g}_1\mathfrak{g}_2} - \frac{\mathfrak{g}_1'\mathfrak{g}_3'}{2\mathfrak{g}_1\mathfrak{g}_3} + \frac{\mathfrak{g}_1'^2}{2\mathfrak{g}_1^2} \right) - \frac{f_2}{\mathfrak{B}^2\mathfrak{g}_1\mathfrak{g}_3} - \frac{f_3}{\mathfrak{B}^2\mathfrak{g}_1\mathfrak{g}_2} - \frac{A_0'^2 f_0}{\mathfrak{B}^2} = 0 \quad (2)$$

$$\frac{6\mathfrak{B}''}{\mathfrak{B}} - \frac{12\mathfrak{B}'^2}{\mathfrak{B}^2} + \frac{\mathfrak{g}_1''}{\mathfrak{g}_1} - \frac{\mathfrak{g}_1'^2}{2\mathfrak{g}_1^2} + \frac{\mathfrak{g}_2''}{\mathfrak{g}_2} - \frac{\mathfrak{g}_2'^2}{2\mathfrak{g}_2^2} + \frac{\mathfrak{g}_3''}{\mathfrak{g}_3} - \frac{\mathfrak{g}_3'^2}{2\mathfrak{g}_3^2} + \phi'^2 = 0 \quad (3)$$

In this case we can solve (1) and (2) equations separately, from (1) we determine A_0. From (2) we determine g_0, if we substitute expressions for f_2 (4) and f_3 (5) in (2). After that we determine ϕ from (3), f_2 from (4), f_3 from (5), V from (6).

Conclusion

- The general anisotropic Einstein-dilaton-four-Maxwell holographic model was considered
- It was shown Bianchi identity and EOM connection (only 6 independent equations out of 7 in the fully anisotropic case)
- The scheme of finding a solution was presented

What's next?
Fully anisotropic hybrid model with external magnetic field
To solve the resulting system for the specific warp-factors and anisotropy functions to obtain the phase transitions structure

Thank you for your attention!