Schwarzschild Black Holes, Islands and Virasoro algebra

Based on arXiv:2211.03153 with Igor Volovich

New Trends in Mathematical Physics

Daniil Stepanenko

Steklov Mathematical Institute

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Overview

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Introduction

The black hole information problem is often formulated as an inconsistency between the unitary evolution and transformation of the initial pure state of the collapsing matter into a mixed state of the Howking radiation.

Information paradox

- Evolution of a closed system must be unitary. This can be interpreted as conservation of information.
- Imagine that there is a collapsing matter in a pure quantum state. The unitary evolution will transform our initial vector to another pure state, but not to a mixed state.
- However, Howking has argued that black holes emit particles like black bodies with temperature

$$T_H = \frac{1}{8\pi M} \tag{1}$$

- Black body is in a mixed state that is discribed by the Planck density operator. Moreover, after the black hole completely evaporated one gets the system in a mixed state.

Island formula

The entropy of Hawking radiation of black holes grows up to infinity during evaporation and it is the information paradox. This increase contrasts with Page's curve, in which entropy decreases after the t_{Page} Page time.
The entanglement entropy of the Hawking radiation is given by

$$S(R) = \min\left\{ \text{ext}\left[\frac{\text{Area}(\partial I)}{4G} + S^{(\text{finite})}_{\text{matter}}(R \cup I) \right] \right\} \tag{2}$$

The island rule was derived by using the replica method for the gravitational path integral. Here I is a region, called the "island" and S_{matter} is the von Neumann entropy $S_{vN}(R \cup I)$ of union of the island.

Overview

Setup

The Schwarzschild metric allows the limit of masses to zero, it defines a flat Minkowski spacetime.

- We consider the conformal quantum field in the classical Schwarzschild black hole background.
- We interpret the classical background as an eternal black hole without taken into account evaporation and backraction.
- The Schwarzschild metric has a fixed mass, but we can choose any mass, including the case of the mass which tends to zero.
- In such a model, we can say that there is a process of evaporation, but the mass of a given black hole is constant, because of the flow of matter from the reservoir.

Schwarzschild black hole

- The Schwarzschild black hole metric we consider as

$$ds^2 = -\frac{r - r_\mathrm{h}}{r} dt^2 + \frac{r}{r - r_\mathrm{h}} dr^2 + r^2 d\Omega^2 \quad (3)$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ and the Schwarzschild horizon radius $r_\mathrm{h} = 2GM$, where M is mass of the black hole and G is the Newton constant.

- We choose mass as an arbitrary parameter.
- This consideration avoids the backreaction problem and Planckian scales. Moreover, in this case we obtain the eternal black holes, which is in agreement with the calculations made in the paper Hashimoto et al. (2020).
- The Hawking temperature is

$$T_H = \frac{1}{4\pi r_\mathrm{h}} = \frac{1}{8\pi GM} \quad (4)$$

Islands configuration for Schwarzschild black holes

In paper Hashimoto et al. (2020), an expression for the entanglement entropy in Schwarzschild black holes in four spacetime dimension with only free massless matter fields was derived, given by the formula:

- $$S_I = \frac{2\pi r_h^2}{G} + \frac{c}{6}\frac{b - r_h}{r_h} + \frac{c}{6}\log\frac{16 r_h^3 (b - r_h)^2}{G^2 b} \quad (5)$$
- Here b is the boundaries of the radiation region and c is the central charge of the matter fields.

The problem with the island formula for the Schwarzschild black hole

It has been demonstrated that the island configuration cannot provide a zero entropy value at the end of Schwarzschild black hole evaporation. Aref'eva and Volovich (2021)

- Island configurations do not provide a bounded entanglement entropy
- The formula (5) leads to the explosive entropy growth due to the term

$$\frac{cb}{r_h} = \frac{cb}{2MG} \tag{6}$$

  which becomes dominant for small M.
- We suggested a way to improve this situation by making the central charge dependent on the black hole mass.

Overview

A possible solution to the entropy explosion problem

We consider a special case with relationship between the central charge and mass in the form

- $$c(M) = AM^{1+\epsilon} \tag{7}$$

  Where A is a constant and ϵ is a positive number parameter, $\epsilon > 0$.

- In that case, we have:

$$S_I = \frac{2\pi(2GM)^2}{G} + \frac{AM^{1+\epsilon}}{6}\frac{b-2GM}{2GM} + \frac{AM^{1+\epsilon}}{6}\log\frac{16(2GM)^3(b-2GM)^2}{G^2 b} \tag{8}$$

- Then at the late time (the end of black hole lifetime) as M $\to$ 0.

$$\lim_{M\to 0} S_I = 0 \tag{9}$$

Entropy corresponding to non-island configuration

Entanglement entropy at late times, in the case of the absence of the island, been derived in Hashimoto et al. (2020).

$$S_{NI} = \frac{c}{6}\frac{t}{r_h} \tag{10}$$

At later times the expression (10) goes to infinity. To solve this problem, let us apply the mechanism of dependence of the central charge on the black hole mass.

$$c = AM^{1+\epsilon} = \left(\frac{M}{M_0}\right)^{1+\epsilon} \quad \rightarrow \quad S_{NI} = \frac{\left(\frac{M}{M_0}\right)^{1+\epsilon} t}{12GM} = \frac{M^{\epsilon} t}{12M_0^{1+\epsilon} G} \tag{11}$$

We choose constant $A = (\frac{1}{M_0})^{1+\epsilon}$, where M_0 is initial black mass, so that the value of entropy was a dimensionless quantity.

- Entropy without island at fixed time and mass equals zero, becomes finite.

Entropy corresponding to non-island configuration

What happens if we compare this result for an evaporating black hole, where mass is a dynamic variable ?

- We apply the time evolution formula for mass from Page (1976).

$$\frac{dM}{dt} = -\frac{1}{G^2}\frac{\alpha}{M^2} \tag{12}$$

- The solution of this equation is obtained as

$$M(t) = \left(\frac{3(G^2B - \alpha t)}{G^2}\right)^{1/3} = \left(\frac{3(G^2M_0^3 - \alpha t)}{G^2}\right)^{1/3} \tag{13}$$

- Where constant of integration was chosen as follows $B = M_0^3$ and α is a numerical coefficient.

This gives the final expression for entropy without an island.

$$S_{NI} = \frac{\left(\frac{3(G^2 M_0^3 - \alpha t)}{G^2}\right)^{\epsilon/3} t}{12 M_0^{1+\epsilon} G} = \frac{\left(3(M_0^3 - \frac{\alpha t}{G^2})\right)^{\epsilon/3} t}{12 M_0^{1+\epsilon} G^{\frac{1+2\epsilon}{3}}} \tag{14}$$

- The behavior of entropy under a non-island configuration is shown in Fig.A.
- We can clearly see that the entropy without an island becomes finite if we allow the dependence of the central charge on the mass of the black hole.
- Where the initial black hole mass was chosen as $M_0 = 1 \times 10^{14}$, $G = 1$ and the parameter $\alpha = 3.6 \times 10^{-4}$.

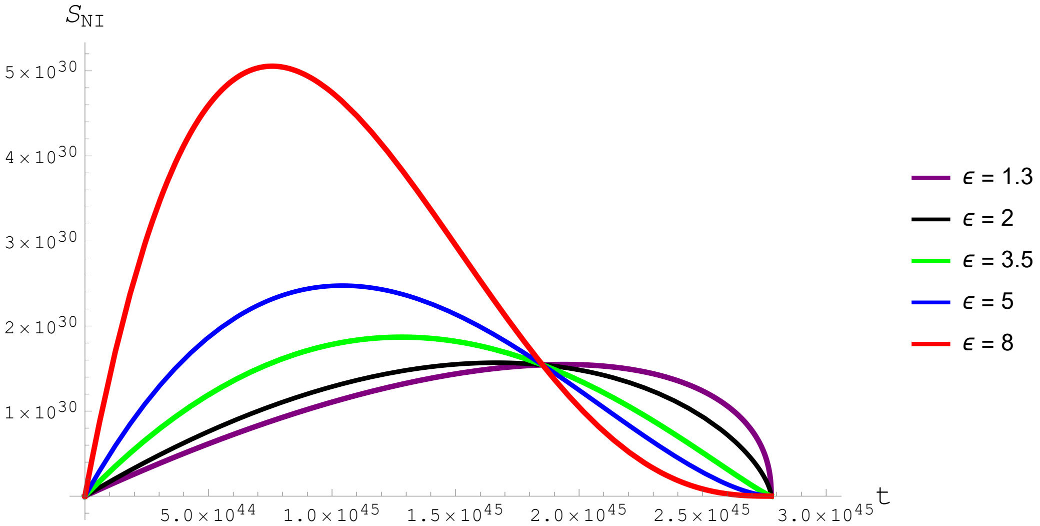

Figure A. We observe an increase in entropy without an island, but then decreasing to zero at the end of the black hole lifetime.

Entropy corresponding to island configuration

Let's look at the behavior of entropy under the island configuration (8) and apply the mechanism of dependence of the central charge on the black hole mass (5).

$$S = \frac{2\pi(2GM)^2}{G} + \frac{M^{\epsilon}}{6M_0^{1+\epsilon}}\frac{b-2GM}{2G} + \frac{M^{1+\epsilon}}{6M_0^{1+\epsilon}}\log\frac{16(2GM)^3(b-2GM)^2}{G^2 b} \tag{15}$$

An expression describing the evolution of mass over time (13).

$$M(t) = \left(3(M_0^3 - \frac{\alpha t}{G^2})\right)^{1/3} \tag{16}$$

Entropy corresponding to island configuration

The final expression for entropy with an island can be written in the form

$$S_I = \frac{\left(3\left(M_0^3 - \frac{\alpha t}{G^2}\right)\right)^{\frac{\epsilon+1}{3}}}{6M_0^{\epsilon+1}} \log \frac{128G\left(3\left(M_0^3 - \frac{\alpha t}{G^2}\right)\right)\left(b - 2G\sqrt[3]{3\left(M_0^3 - \frac{\alpha t}{G^2}\right)}\right)^2}{b} + \frac{\left(b - 2G\sqrt[3]{3\left(M_0^3 - \frac{\alpha t}{G^2}\right)}\right)\left(3\left(M_0^3 - \frac{\alpha t}{G^2}\right)\right)^{\epsilon/3}}{12GM_0^{\epsilon+1}} + 8\pi G\left(3\left(M_0^3 - \frac{\alpha t}{G^2}\right)\right)^{2/3} \tag{17}$$

- The behavior of entropy with an island configuration is shown in Fig.B. The graph shows that the island formula using the mechanism of dependence of the central charge on the black hole mass provides a decrease of entropy to zero value.
- Where the initial black hole mass was chosen as $M_0 = 1 \times 10^{14}$, $G = 1$,the parameter $\alpha = 3.6 \times 10^{-4}$ and island boundaries $b = 5$.

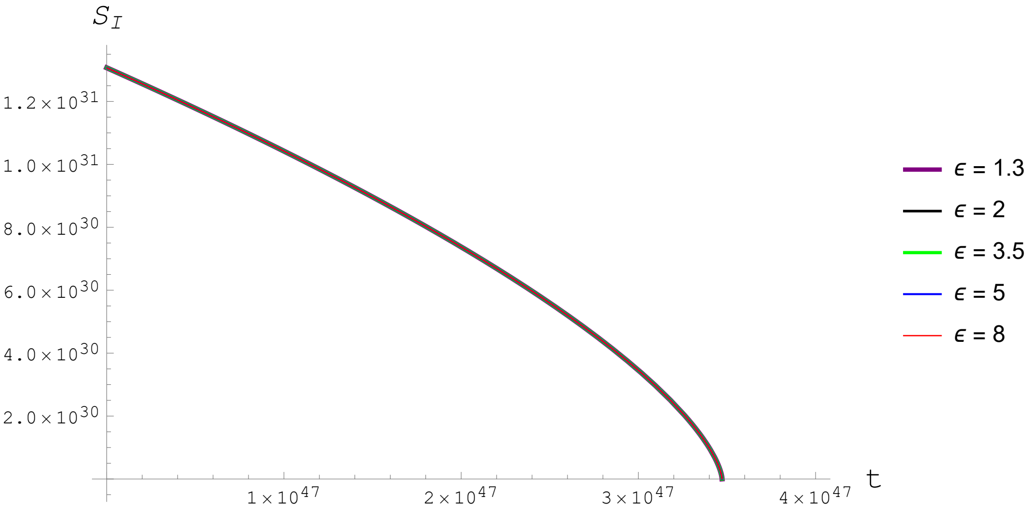

Figure B. The entropy with island decreases and becomes finite. And this expression do not depend of the choice ϵ.

Overview

We consider a special constrain on central charge dependence of black hole mass, for the possibility to resolve the singularity of the entanglement entropy using Island configurations for Schwarzschild black holes. Central charge goes to zero for small black hole mass.

- This approach solves the problem of the entropy explosion raised by Aref'eva and Volovich (2021). We present a mechanism based on the identification of central charge for algebra Virasoro and black hole mass. This makes it possible to, avoiding the singular term for expressing entanglement entropy with island configuration (5).
- In other word, we conclude island formula under constrain (7) do solve the blow up of the entropy for the Schwarzschild black holes.
- And not only that, application of such mechanism for the none island formula, so provides a finite value of entropy at the end of black hole lifetime.

References I

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