V. Pushkarev

Steklov Mathematical Institute,
Russian Academy of Sciences

Regular and irregular dynamics after local quenches in massive scalar field theory

arXiv:2205.12290 [hep-th],
D. Ageev, A. Belokon’, VP

New Trends in Mathematical Physics, November 2022

Outline

1. Motivation
2. Operator local quench formalism
3. Local quench in 2d CFT
4. Local quench in non-conformal setups:
   a. Massive scalar field theory in flat space
   b. Massive scalar field theory in finite volume
5. Conclusions and future directions

Out-of-equilibrium dynamics: prototypical quenches

Quantum quench is a non-equilibrium process following a sudden change (smooth or instantaneous) of parameters of a system.

Global quench is a homogeneous change of the parameters: e.g. the coupling constant or the chemical potential. [Calabrese and Cardy, `06; Das, Galante, Myers, `15]

Local quench is a local change of the parameters or a local interaction/perturbation. [Calabrese and Cardy, `07]

Motivation

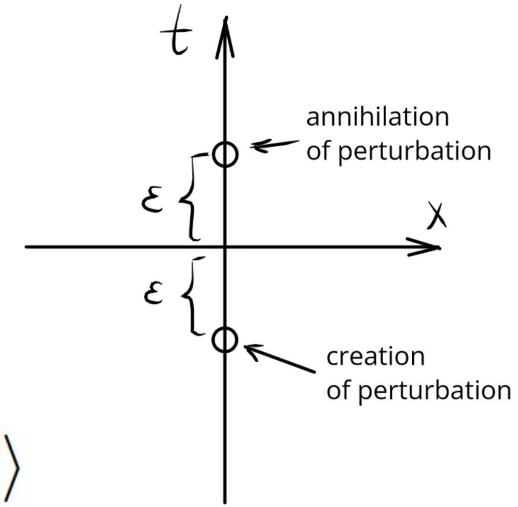
- General probe of non-equilibrium QFT [Das, Galante, Myers, `15]
- Applications in:
  condensed matter theory [Katsnelson, Dobrovitski, Harmon,`00; Ganahl et al, `12],
  HEP [Zhang et al, `22], information theory [He et al, `14; He, Shu, `20],
  physics of the Early Universe [Carrilho and Ribeiro, `17]
- Duals in AdS/CFT or dS/CFT (entanglement entropy, thermalization)
  [Balasubramanian, `11; Nozaki, Numasawa, Takayanagi, `13;
  Hartman, Maldacena, `13;
  Ageev and Aref'eva, `16, `18, `19;
  Aref'eva, Khramtsov, Tikhanovskaya, `17;
  Ageev, `19 and many other]

CFT	AdS
global quench	black hole formation
local quench	falling particle

We would like to study the dynamics of energy density after local quench in the theory of free massive scalar field.

Operator local quench protocol

[Nozaki, Numasawa, Takayanagi, `14]



Convenient and universal way to introduce and consider localized perturbations.

A local operator acting on a ground state gives a locally excited state:

$$|\Psi(t)\rangle = \mathcal{N}_O\, e^{-iH(t-t_0)} O(t_0, x_0)|0\rangle$$

quenching operator

observable

$$\langle \mathcal{O}(t,x)\rangle_\Psi = \frac{\langle \Psi|\mathcal{O}(t,x)|\Psi\rangle}{\langle\Psi|\Psi\rangle} \longrightarrow \langle \mathcal{O}(t,x)\rangle_\Psi = \frac{\langle 0|O(i\varepsilon,0)\mathcal{O}(x,t)O(-i\varepsilon,0)|0\rangle}{\langle 0|O(i\varepsilon,0)O(-i\varepsilon,0)|0\rangle}$$

Energy density dynamics: two-dimensional conformal field theory

[Nozaki, Numasawa, Takayanagi, `13]

Conformal Ward identities:

$$\langle O(z_0,\bar{z}_0)T(z)O(z_1,\bar{z}_1)\rangle \sim \sum_{k=0}^{1}\left(\frac{h}{(z-z_k)^2}+\frac{1}{z-z_k}\partial_{z_k}\right)\langle O(z_0,\bar{z}_0)O(z_1,\bar{z}_1)\rangle$$

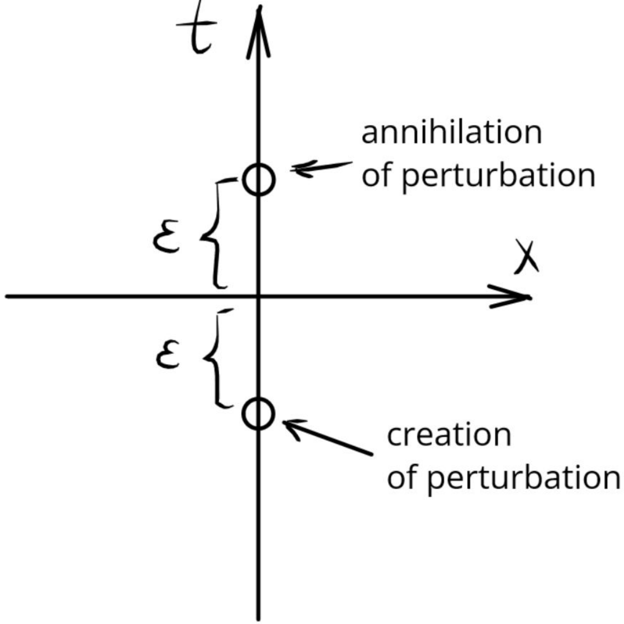
$$\langle \mathcal{E}(t,x)\rangle_{O_{(1,0)}} = -\frac{\langle O(z_0,\bar{z}_0)\left(T(z)+T(\bar{z})\right)O(z_1,\bar{z}_1)\rangle}{\langle O(z_0,\bar{z}_0)O(z_1,\bar{z}_1)\rangle} =$$

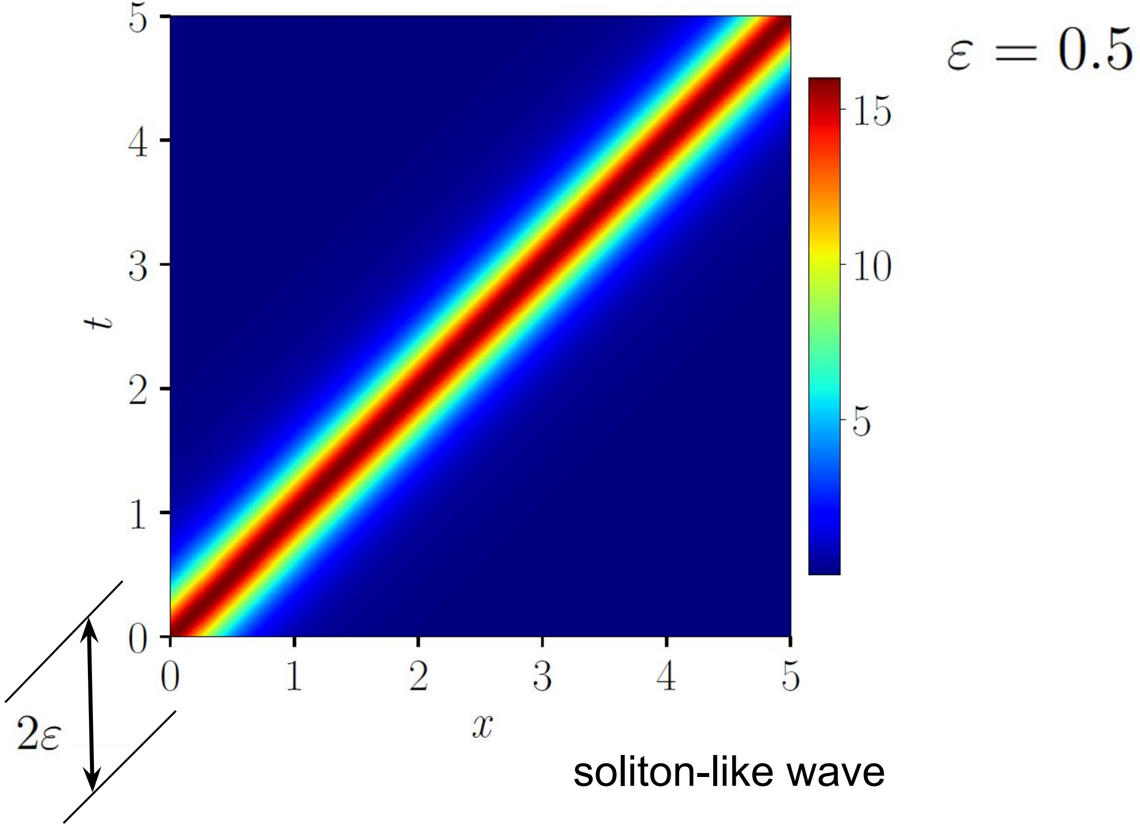
$$= \frac{D\left[\langle O(z_0,\bar{z}_0)O(z_1,\bar{z}_1)\rangle\right]}{\langle O(z_0,\bar{z}_0)O(z_1,\bar{z}_1)\rangle} = \frac{4\varepsilon^2}{\left((x-t)^2+\varepsilon^2\right)^2}$$

$$z = x + i\tau,$$
$$\bar{z} = x - i\tau$$

$\partial_z\phi(z,\bar{z})$ is a primary field with conformal weights (1,0).

CFT local quench in background Minkowski spacetime





soliton-like wave

Straightforward calculation of energy density dynamics

1) Write down the Euclidean coordinate-space propagator;

2) regularize the composite operators, e.g. $\phi^2(x)$, by separation of the inner spacetime point into two different ones,

$$\phi^2(x) \to \lim_{y \to x} \phi(x)\phi(y);$$

3) calculate 4-point functions using Wick's theorem;

4) take the limit of merging points in the composite operator and extract the finite part;

5) perform an inverse Wick rotation to get a result in Minkowski spacetime;

6) take points of the quenching operator to be $(-i\varepsilon, 0)$ and $(i\varepsilon, 0)$.

CFT revisited: without use of Ward Identities

$$\langle\partial\phi(z_1,\bar{z}_1)|T(z)|\partial\phi(z_0,\bar{z}_0)\rangle = -\frac{1}{2}\lim_{\substack{w\to z\\ \bar{w}\to\bar{z}}}\partial_{z_1}\partial_{z_0}\partial_w\partial_z\langle\phi(z_1,\bar{z}_1)\phi(w,\bar{w})\phi(z,\bar{z})\phi(z_0,\bar{z}_0)\rangle =$$

$$= -\frac{1}{2}\lim_{\substack{w\to z\\ \bar{w}\to\bar{z}}}\left[\frac{1}{(w-z_0)^2(z-z_1)^2}+\frac{1}{(w-z_1)^2(z-z_0)^2}+\frac{1}{(w-z)^2(z_0-z_1)^2}\right] =$$

$$= -\frac{1}{(z-z_0)^2(z-z_1)^2} - \frac{1}{2}\lim_{\substack{w\to z\\ \bar{w}\to\bar{z}}}\left[\frac{1}{(w-z)^2(z_0-z_1)^2}\right]$$

The same answer:

$$\frac{\langle\partial\phi(i\varepsilon,0)|\mathcal{E}(t,x)|\partial\phi(-i\varepsilon,0)\rangle}{\langle\partial\phi(i\varepsilon,0)\partial\phi(-i\varepsilon,0)\rangle} = \frac{4\varepsilon^2}{\left((x-t)^2+\varepsilon^2\right)^2}$$

Free massive scalar field

$$S = \frac{1}{8\pi}\int d\tau\, dx\left((\partial_\tau\phi)^2 + (\partial_x\phi)^2 + m^2\phi^2\right)$$

$$\mathcal{E}(\tau, x) = \frac{1}{4}\left(-(\partial_\tau\phi)^2 + (\partial_x\phi)^2 + m^2\phi^2\right) = -T(z) - \bar{T}(\bar{z}) + \frac{1}{4}m^2\phi^2(z, \bar{z})$$

three composite operators

The quenching operator as in CFT,

$$O = \partial\phi$$

Use the point-splitting regularization:

$$\phi^2(x) \to \lim_{y\to x}\phi(x)\phi(y)$$

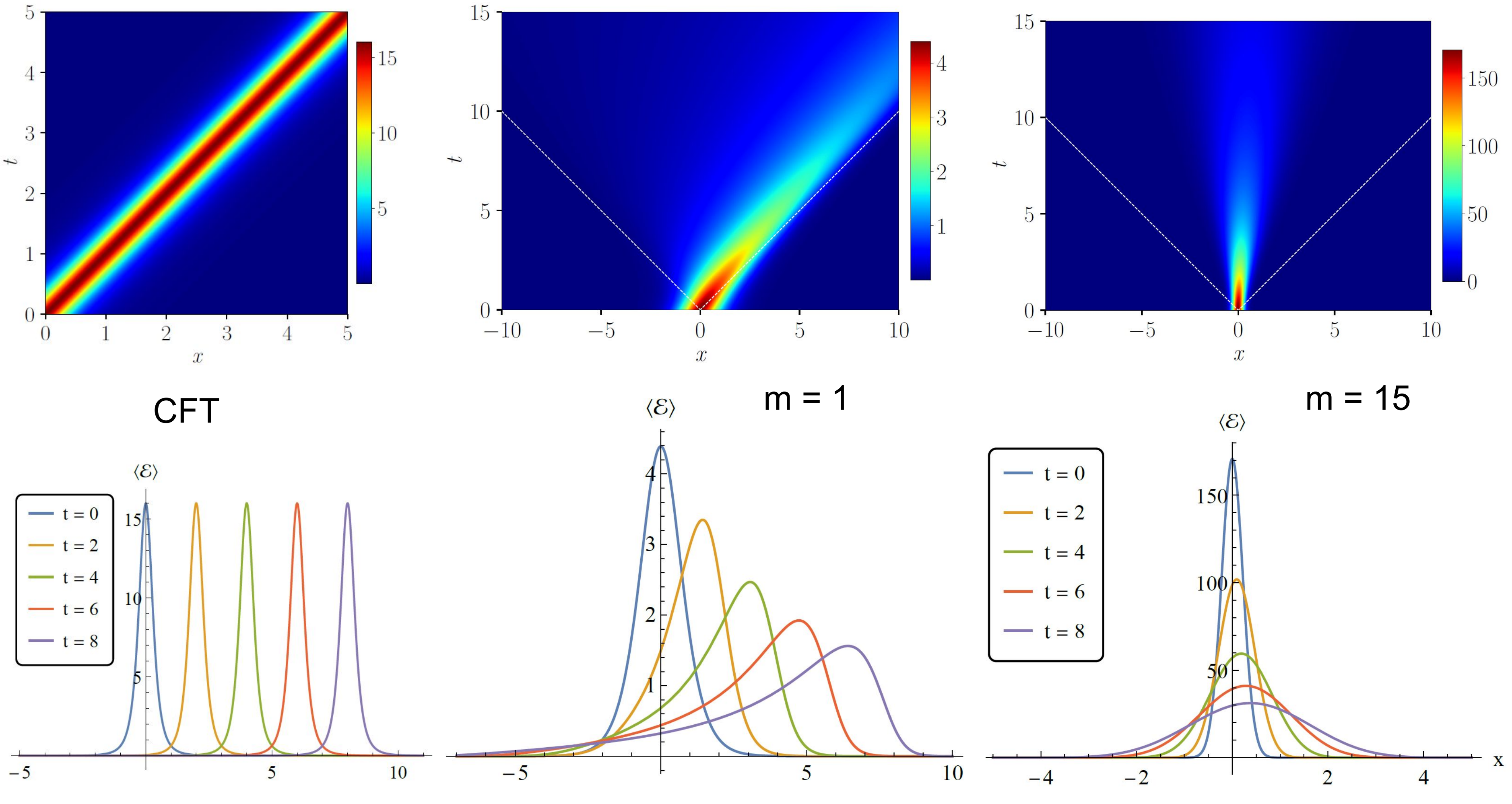
Adding mass in the CFT case: exactly solvable case

$$\frac{\langle \partial\phi(i\varepsilon,0)|\mathcal{E}(t,x)|\partial\phi(-i\varepsilon,0)\rangle}{\langle \partial\phi(i\varepsilon,0)\partial\phi(-i\varepsilon,0)\rangle} =$$

$$= \frac{m^2}{2K_2(2\varepsilon m)}\left[\frac{2}{\varepsilon^2+(t-x)^2}\left|\sqrt{(\varepsilon-it)^2+x^2}\,K_1\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2 + \right.$$

$$\left. + \frac{\varepsilon^2+(t+x)^2}{\varepsilon^2+(t-x)^2}\left|K_2\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2 + \left|K_0\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2\right]$$

Note: **real-valued** function.



CFT
m = 1
m = 15
t
x
⟨ℰ⟩
t = 0
t = 2
t = 4
t = 6
t = 8
x

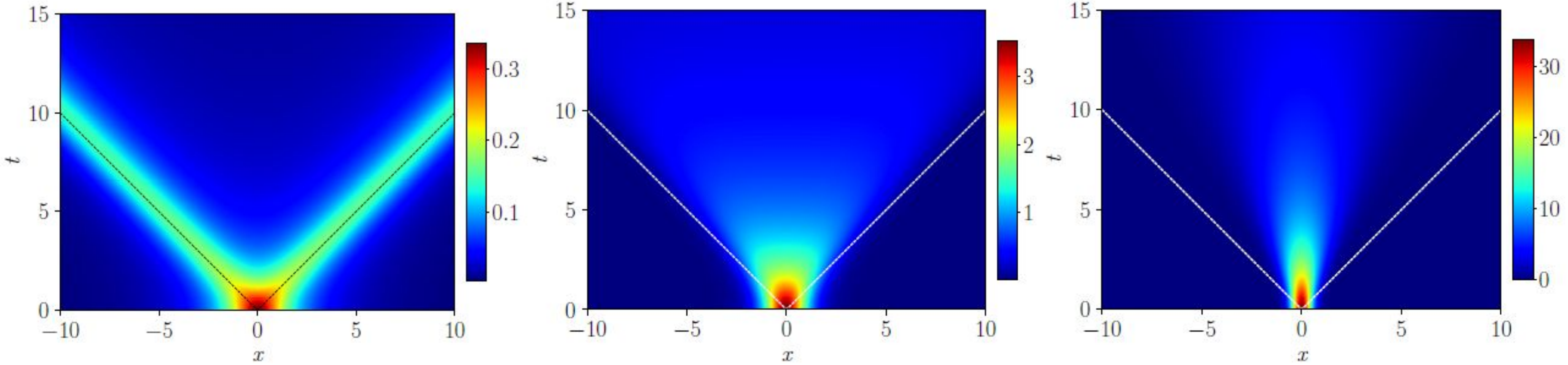
Massive scalar theory in flat space: different quenching operator

Now the quenching operator is $O = \phi$.

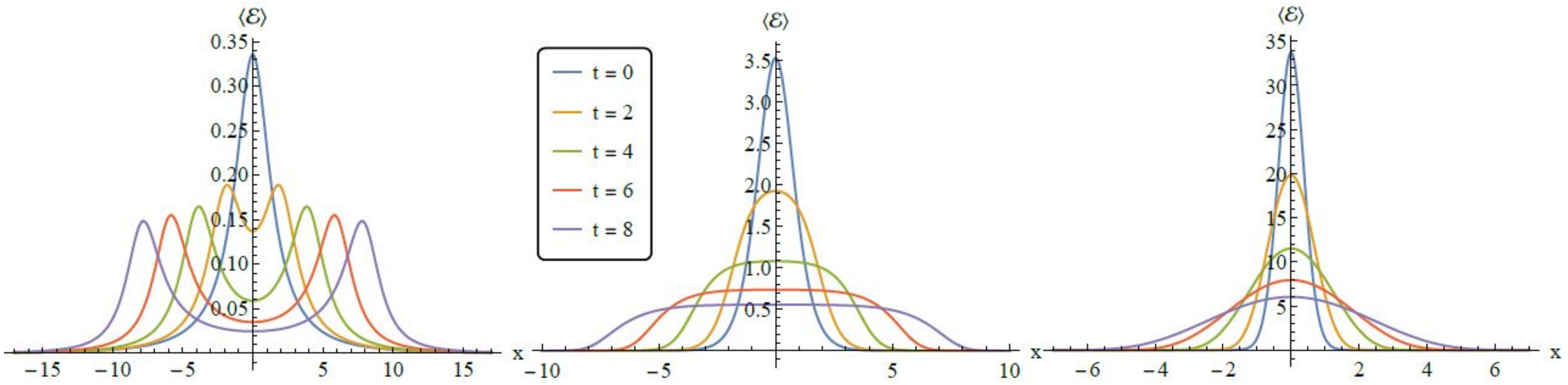
$$\frac{\langle \phi(i\varepsilon,0)|\mathcal{E}(t,x)|\phi(-i\varepsilon,0)\rangle}{\langle \phi(i\varepsilon,0)\phi(-i\varepsilon,0)\rangle} =$$

$$\frac{m^2}{K_0(2\varepsilon m)}\left[(\varepsilon^2+t^2+x^2)\left|\frac{K_1\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)}{\sqrt{(\varepsilon-it)^2+x^2}}\right|^2+\left|K_0\left(m\sqrt{(\varepsilon-it)^2+x^2}\right)\right|^2\right]$$

Massive scalar theory in flat space



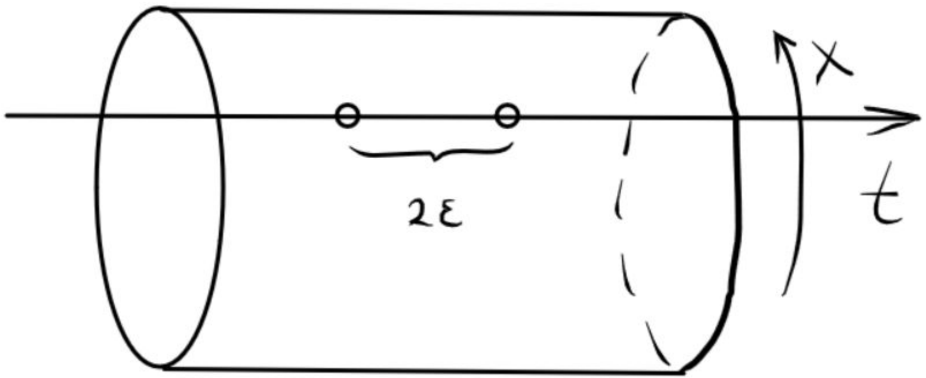
(a) $m = 0.1$ (b) $m = m_{\text{crit}} = 1$ (c) $m = 5$



(d) $m = 0.1$ (e) $m = m_{\text{crit}} = 1$ (f) $m = 5$

$\varepsilon = 1.5$

Theory in finite volume: is the dynamics trivial after mass inclusion?



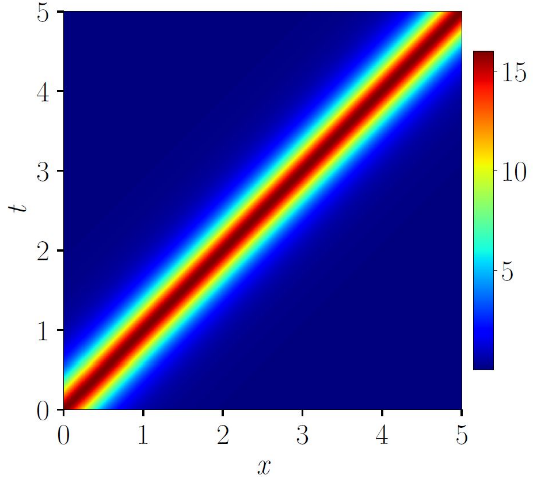
$$\phi(t, x + L) = \phi(t, x)$$

Massless case: CFT on a cylinder

[Caputa et al, `15]

Quenching operator: $O = \partial\phi$

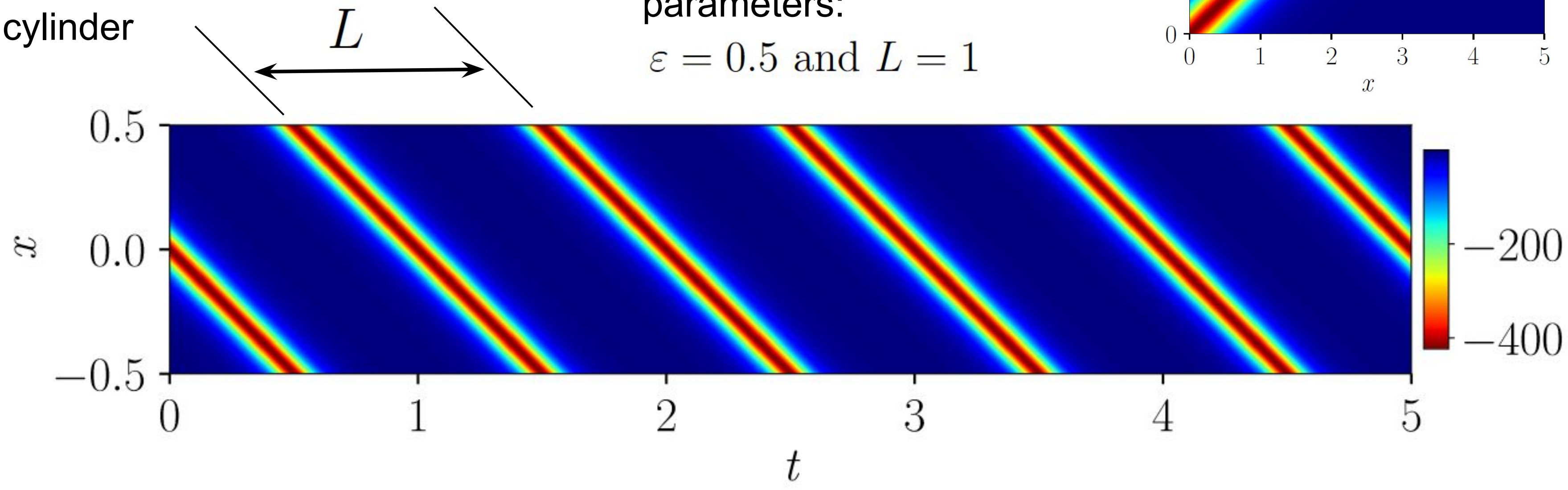
flat space, one dimensional parameter



two dimensional parameters:

$\varepsilon = 0.5$ and $L = 1$

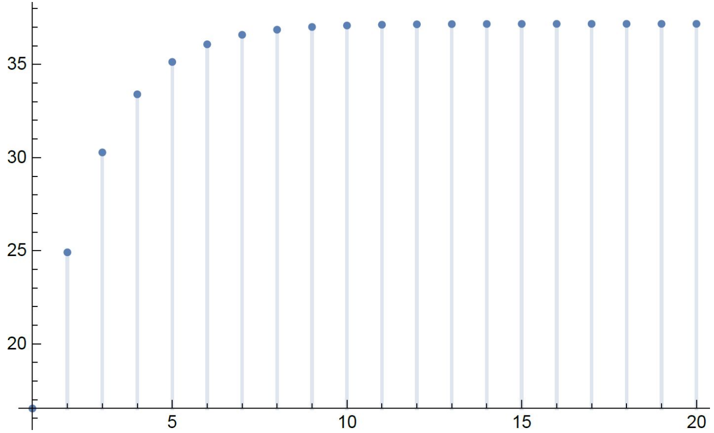
cylinder



Massive scalar case, quenching operator $O = \phi$

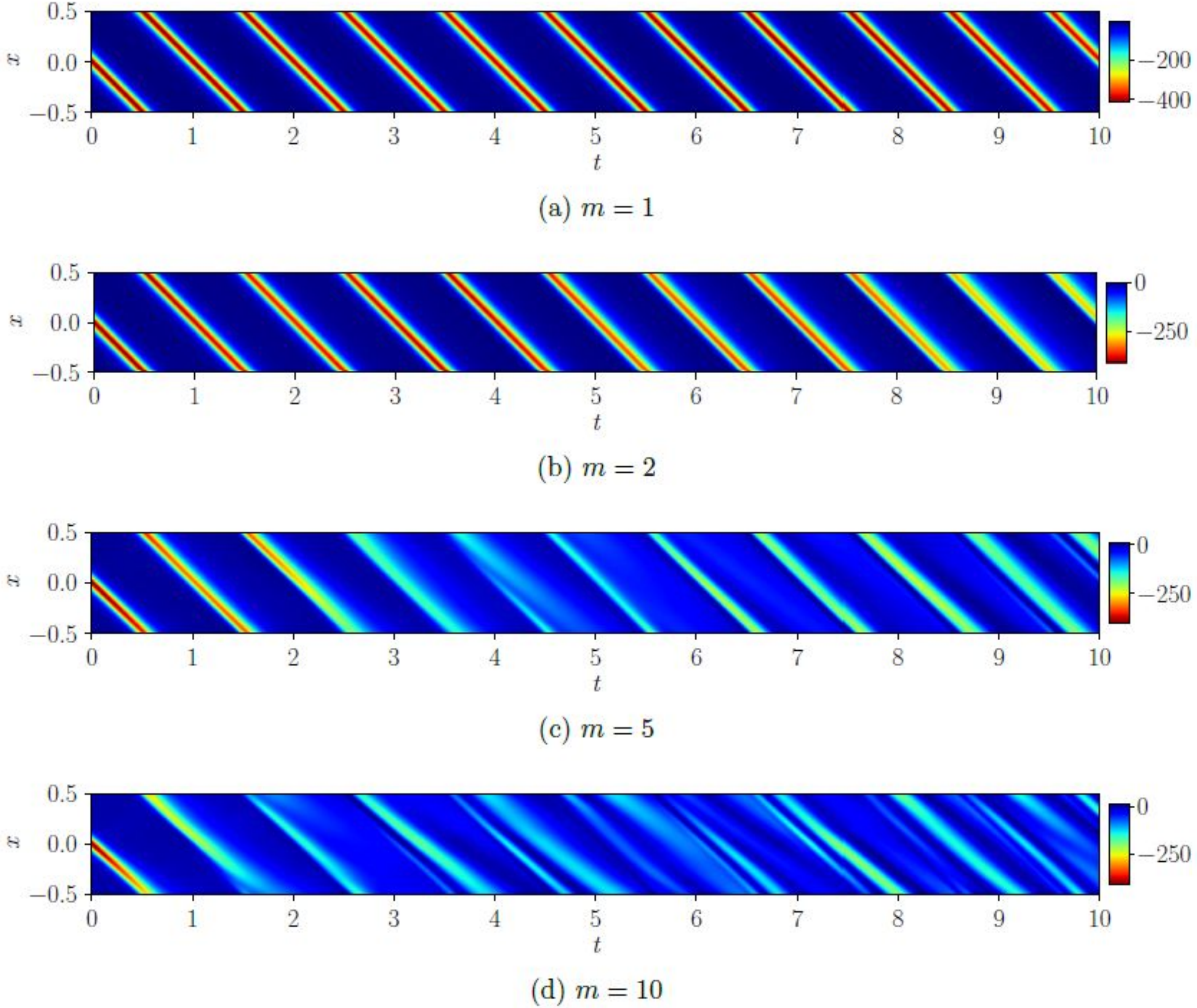
$$\begin{cases} A\left(-\Delta^2 + m^2\right) K(\vec{x}_1 - \vec{x}_2) = \delta^{(2)}(\vec{x}_1 - \vec{x}_2), \\ K(\tau, x + L) = K(\tau, x) \end{cases}$$

$$\langle \phi(\tau, x)\phi(0,0)\rangle = \frac{1}{AL}\sum_n \int_q \frac{dq}{2\pi} \frac{e^{i\omega_n x + iq\tau}}{q^2 + \omega_n^2 + m^2}\Bigg|_{\omega_n = \frac{2\pi n}{L}}$$



$\langle \mathcal{E}(t, x)\rangle_\phi$ at an arbitrary point: the series is quickly converging.

Massive scalar case, $O = \partial\phi$



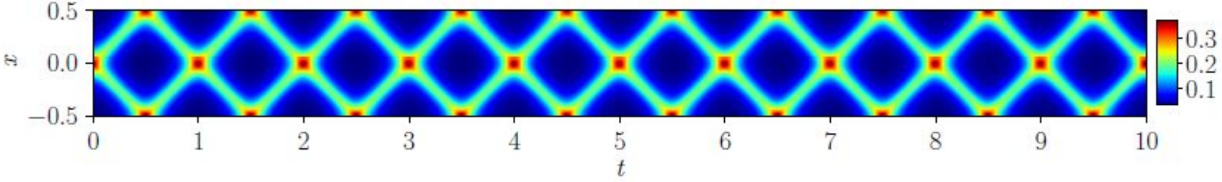
(a) $m = 1$

(b) $m = 2$

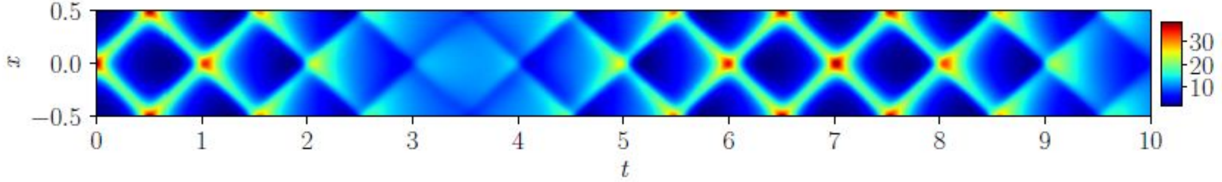
(c) $m = 5$

(d) $m = 10$

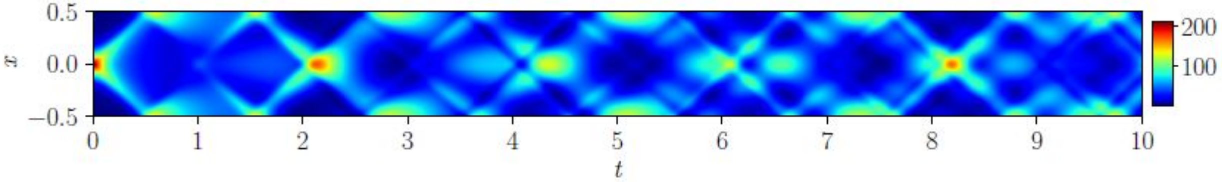
Massive scalar case, $O = \phi$



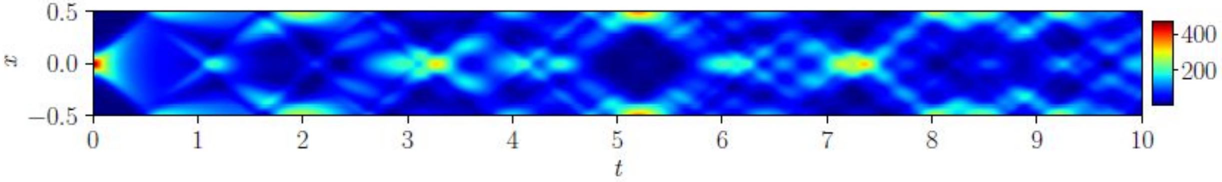
(a) $m = 0.01$



(b) $m = 1$



(c) $m = 5$



(d) $m = 10$

Applications and future investigations

- Dynamics of observables in curved spacetimes (AdS and dS) — relation to studying of field dynamics in problems of the physics of the Early Universe and the physics of black holes.
- Canonical interacting fields (scalar fields, gauge fields, etc.)
- Condensed matter: canonical (graphene, Hubbard model) and exotic examples (fractons, unparticles), quantum Hall effect, …
- Relation to quantum gravity via holography?

Thank you for your attention!