Delicate windows into evaporating black holes

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based on 2209.15477 with Ben Craps, Juan Hernandez and Maria Knysh (VUB)

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Introduction

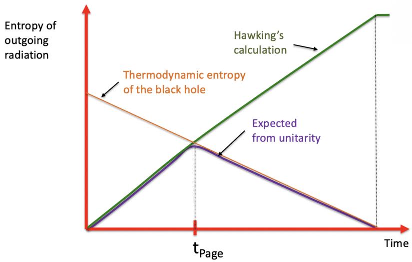

Figure: from Almheiri et. al., 2006.06872

- The QES / island formula for the evaporating black holes reproduces the Page curve for the Hawking radiation consisstent with the unitary evolution of a pure state and providing the access to the black hole interior after the Page time.

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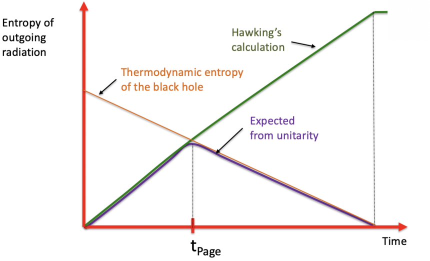

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- The QES / island formula for the evaporating black holes reproduces the Page curve for the the Hawking radiation consisstent with the unitary evolution of a pure state and providing the access to the black hole interior after the Page time.
- This Page curve is robust to unitary quantum dynamics within the observer / bath.

Motivation

However, operationally in practice the observer will not be able to collect infinite amount of radiation.

Questions:

- What is the Page curve for finite portions of Hawking radiation?
- How robust is it to unitary dynamics in the bath?
- How much (and when) of the black hole interior can we access from a fixed portion of the bath throughout the black hole evaporation?

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In this work we study these questions in the model of the evaporating AdS_2 black hole in JT gravity coupled to a flat reservoir (AEM^4Z).
Almheiri et. al. 1905.08762, 1908.10996; Chen et. al., 1911.03402, 2007.11658

Eternal black hole case was studied in Balasubramanian et. al., 2107.14746; Hollowood et. al., 2109.01895

Outline

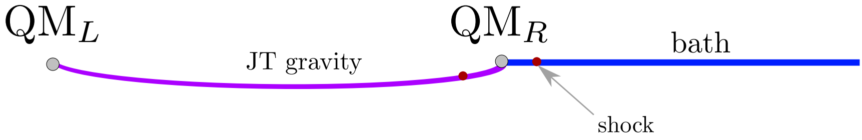
- Review of the AdS_2 black hole evaporating into an external bath (AEM^4Z model)
- Revisiting the Page curve of a semi-infinite radiation segment
- Entanglement entropy dynamics of finite segments of the radiation: dynamics of the interior access

Evaporating black hole with doubly hologrpahic radiation

- Microscopic perspective

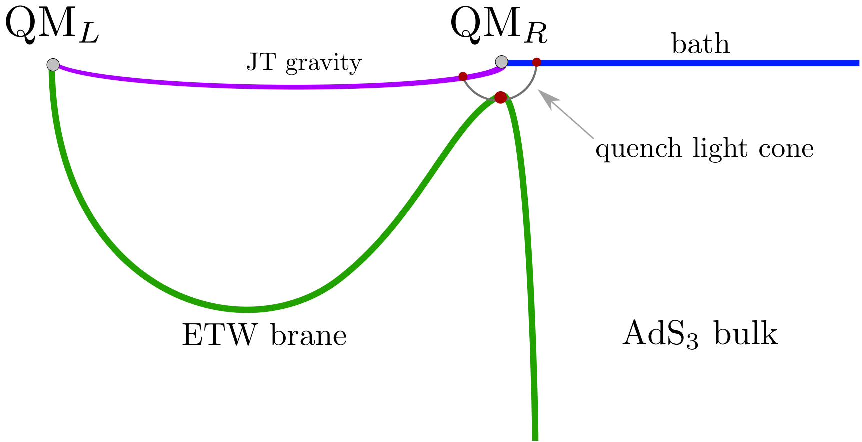
Evaporating black hole with doubly hologrpahic radiation

- Gravity perspective



Evaporating black hole with doubly hologrpahic radiation

- Doubly hologrpahic perspective



Evaporating AdS_2 black hole

JT gravity:

$$I = \frac{1}{16\pi G}\left[\int_{\mathcal{M}} d^2x\sqrt{-g}\{\phi_0 R + \phi(R+2)\} + 2\int_{\partial\mathcal{M}}\{(\phi_0+\phi_b)K\}\right] + I_{CFT}.$$

Poincaré metric in the gravitational region

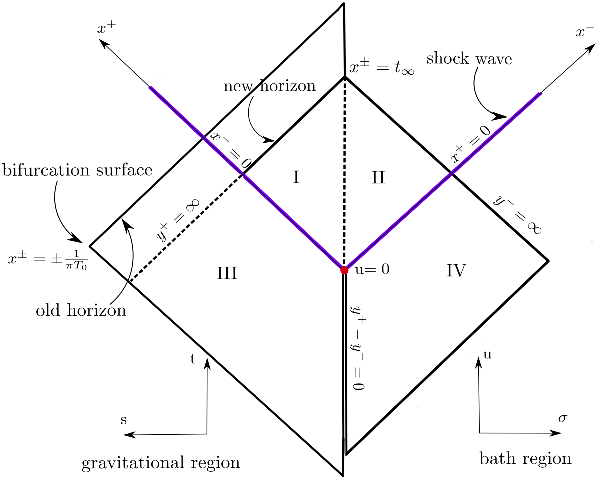
$$ds^2_{\text{AdS}} = -\frac{4dx^+dx^-}{(x^+-x^-)^2},$$

where $x^\pm = t \pm s$; $\quad t = \frac{x^+ + x^-}{2} \in \mathbb{R}$; $\quad s = \frac{x^+ - x^-}{2} > 0$. Bath region:

$$ds^2_{\text{bath}} = -\frac{dy^+dy^-}{\epsilon^2},$$

where $y^\pm = u \mp \sigma$; $\quad u = \frac{y^+ + y^-}{2} \in \mathbb{R}$; $\quad \sigma = \frac{y^- - y^+}{2} > 0$.
We extend the x coordinates into the bath and similarly the y coordinates into the AdS region by gluing $x^\pm = f(y^\pm)$.

Poincare patch of evaporating black hole glued to the bath



The quench data

- Before the quench:

$$
\begin{aligned}
f(u) &= \frac{1}{\pi T_0}\tanh(\pi T_0 u)\,; \\
\phi(y) &= 2\phi_r \pi T_0 \coth\left(\pi T_0 (y^+ - y^-)\right)\,.
\end{aligned}
$$

T_0 - initial black hole temperature

- After the quench: production of a shock with energy $E_S = \frac{\bar{\phi}_r \pi}{4G}(T_1^2 - T_0^2)$. The JT solution:

$$
\begin{aligned}
f(u) &= \frac{1}{\pi T_1}\frac{I_0(\frac{2\pi T_1}{k})K_0(\frac{2\pi T_1}{k}\mathrm{e}^{-\frac{ku}{2}}) - K_0(\frac{2\pi T_1}{k})I_0(\frac{2\pi T_1}{k}\mathrm{e}^{-\frac{ku}{2}})}{I_1(\frac{2\pi T_1}{k})K_0(\frac{2\pi T_1}{k}\mathrm{e}^{-\frac{ku}{2}}) + K_1(\frac{2\pi T_1}{k})I_0(\frac{2\pi T_1}{k}\mathrm{e}^{-\frac{ku}{2}})}\,; \\
\phi &= \phi_r\left(\frac{f''(y^-)}{f'(y^-)} + 2\frac{f'(y^-)}{x^+ - x^-}.\right), \quad x^{\pm} = f(y^{\pm}),.
\end{aligned}
$$

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The island formula for holographic radiation

$$\mathcal{S}(A) = \min \underset{\mathcal{I}}{\text{ext}} \left[S_{\mathsf{CFT}}(A \cup \mathcal{I}) + \text{Area}(\mathrm{d}\mathcal{I}) \right] ,$$

(Almheiri, Mahajan, Maldacena, Zhao; Penington, Shenker, Stanford, Yang; Almheiri, Hartman, Maldacena, Shaghoulian)

Algorithm:

1. Compute the dilaton profile for the area term contribution to the generalized entropy.
2. Compute the von Neumman entropy S_{vN} of the CFT matter. Since the CFT is holographic, this part of the generalized entropy is given by the (H)RT formula. It involves competition between configurations of geodesics in AdS_3 bulk.
3. Extremize the resulting generalized entropy $S_{gen} = S\phi + S_{vN}$ for each configuration independently, and pick the one for which the resulting entropy is smallest.

CFT entanglement entropy

For a holographic BCFT on a half-plane one has

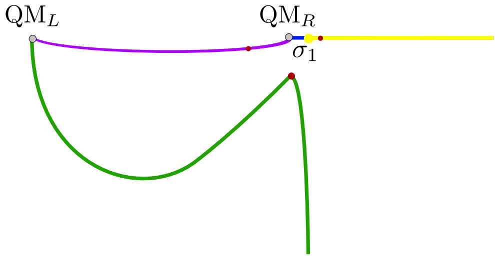
$$S_{\mathrm{CFT}}(z_1,z_2)=\begin{cases}\frac{c}{6}\log\left(\frac{|z_1-\bar z_1||z_2-\bar z_2|}{\delta^2}\right)+2\log g & \eta<\eta^*,\\ \frac{c}{6}\log\left(\frac{|z_1-z_2||\bar z_1-\bar z_2|}{\delta^2}\right) & \eta>\eta^*.\end{cases}$$

where δ is the UV divergence, $\eta^*=\frac{1}{1+g^{12/c}}$ and in the doubly holographic model g has the meaning of ETW brane tension. We assume $g=0$.
Half plane is mapped to the physical AdS_2 + bath background using

$$z(y^-)=\begin{cases}-\left(\frac{c}{12\pi E_s}\right)^2\frac{i}{f(y^-)}\,, & y^-<0\,,\\ iy^-\,, & y^->0\,,\end{cases}$$

$$\bar z(y^+)=\begin{cases}-\left(\frac{c}{12\pi E_s}\right)^2\frac{i}{f(y^+)}\,, & y^+>0\,,\\ iy^+\,, & y^+<0\,,\end{cases}$$

QES / island formula for the semi-infinite interval



Possible phases Chen et. al., 1911.03402:

- Quench phase: trivial QES, RT geodesic ending on the ETW brane
- Scrambling phase: nontrivial QES to the past of the shock, close to the horizon
- Late time phase: nontrivial QES to the future of the shock, inside the horizon

Generalized entropies

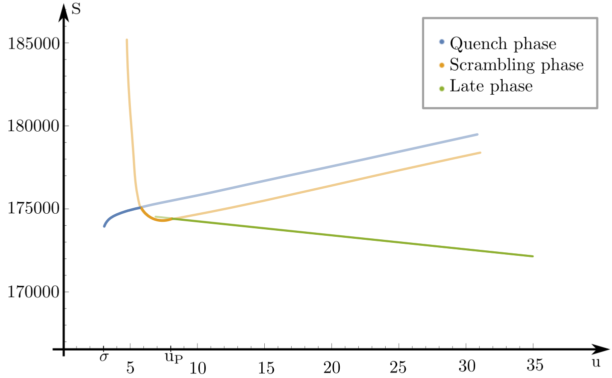
quench: $$S^Q_{gen} = \frac{c}{6}\log\left(\frac{12\pi \mathrm{Es}}{c\delta\epsilon} y^- \frac{f(y^+)}{\sqrt{f'(y^+)}}\right) + \frac{c}{6}\log\frac{2}{\delta} + \frac{2\pi T_0\phi_r + \phi_0}{4G},$$

scrambling: $$S^S_{gen} = \frac{c}{6}\log\left(\frac{24\pi \mathrm{Es} x^-_{QS} y^-(x^+_{QS} - f(y^+))}{c\delta^2\epsilon(x^-_{QS} - x^+_{QS})\sqrt{f'(y^+)}}\right) + \frac{\phi_0 + \phi(x^+_{QS}, x^-_{QS})}{4G} + \frac{c}{6}\log\frac{2}{\delta} + \frac{2\pi T_0\phi_r + \phi_0}{4G},$$

late: $$S^L_{gen} = \frac{c}{6}\log\left(\frac{2(y^- - y^-_{QL})(x^+_{QL} - f(y^+))}{\delta^2\epsilon(x^-_{QL} - x^+_{QL})}\sqrt{\frac{f'(y^-_{QL})}{f'(y^+)}}\right) + \frac{\phi_0 + \phi(x^+_{QL}, x^-_{QL})}{4G} + \frac{c}{6}\log\frac{2}{\delta} + \frac{2\pi T_0\phi_r + \phi_0}{4G}.$$

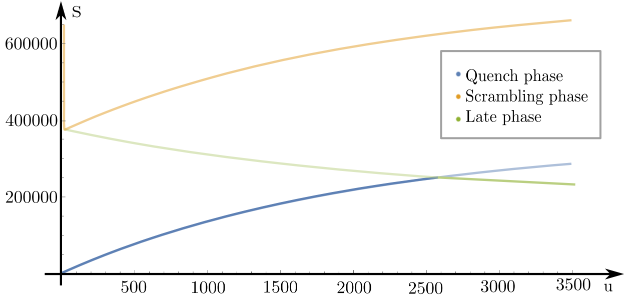
Green terms are included if QM_L included, and blue terms are included if QM_L *not* included.

Page curve for semi-infinite radiation segment plus QM_L



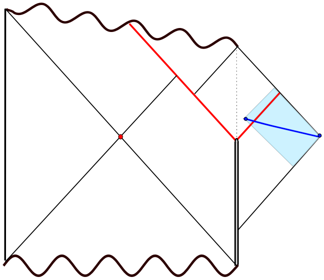
Time evolution of the entanglement entropy of a QM_L+semi-infinite segment of radiation in the bath. The scrambling and late phases both have nontrivial QES.

Page curve for semi-infinite radiation segment (without QM_L)

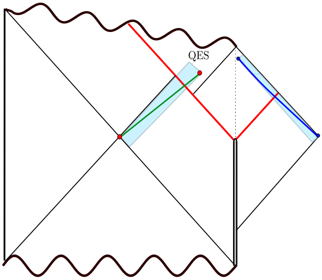


Time evolution of the entanglement entropy of a semi-infinite segment of radiation in the bath with $\phi_0 = 0$. The scrambling phase is gone.

Entanglement wedge evolution of semi-infinite segments



(a) Quench phase

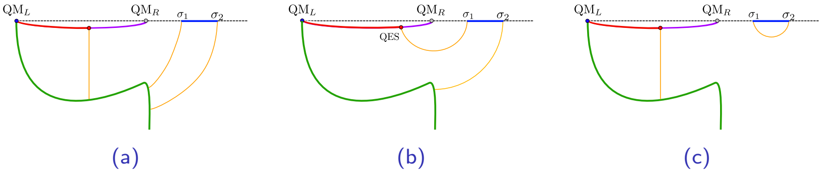


(b) Late phase

The page time is given by $u_{\text{Page}} \sim \frac{1}{k} \log\left(\frac{3\pi T_1 \phi_r}{\pi \phi_r (2T_1 - T_0) - \phi_0}\right) + O(k)$.

Islands appear within the regime of applicability only for small ϕ_0.

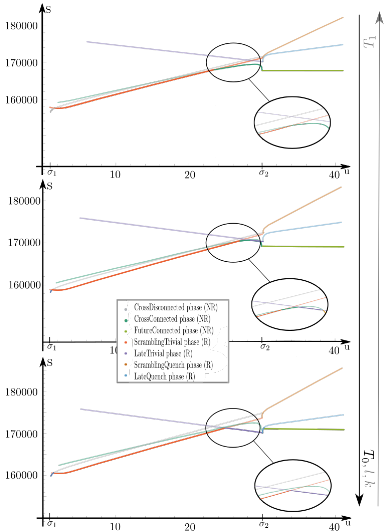
QES formula for finite segments of radiation + QM_L



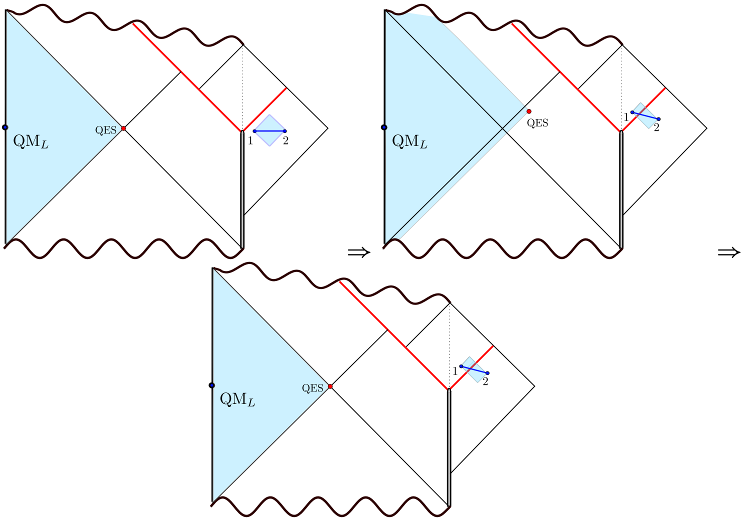
Possible phases: many. Relevant ones at intermediate times are

- CrossDisconnected phase: 2 RT geodesics ending on ETW brane, trivial QES
- CrossConnected phase: RT geodesic connecting the segment endpoints (with the shock in between), trivial QES
- ScramblingTrivial phase: one nontrivial QES outside the horizon, second endpoint has RT geodesic ending on the brane
- LateTrivial phase: one nontrivial QES inside the horizon, second endpoint has RT geodesic ending on the brane

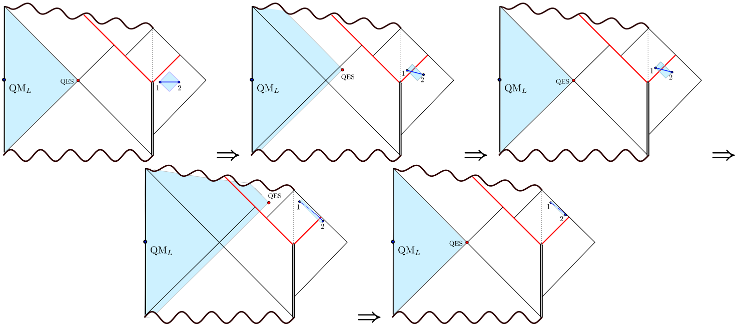
Page curve possibilities for finite segments



Scenario 1 of EW evolution

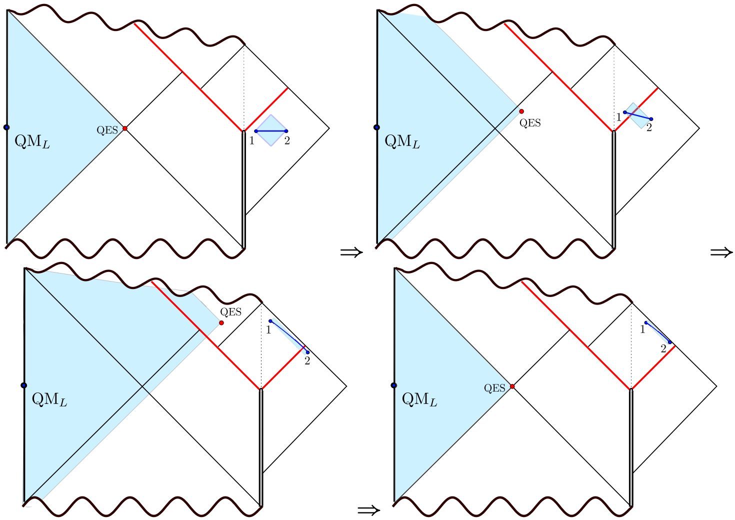


Scenario 2 of EW evolution

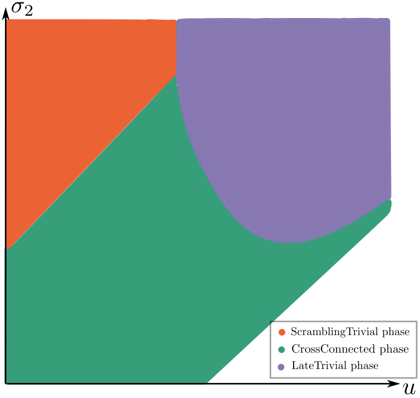


The amount of reconsctructible interior information is non-monotonic

Scenario 3 of EW evolution

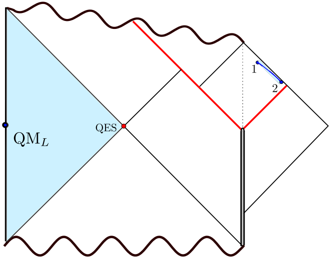


Phase diagram for the non-monotonic reconstruction windows in t, σ_2 variables

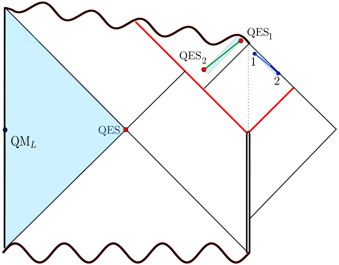


- in t-direction, the reconstruction window is generally nonmonotonic
- in σ_2-direction, however, the entanglement wedge nesting is respected

Comment on late times



(a) FutureConnected phase



(b) LateLate phase

Figure: A sketch of the entanglement wedges for the possible phases of the entanglement entropy for QM$_L$+finite segment during very late times.

The LateLate phase (on the right) is not realized for finite segments within the regime of applicability of the model.

Conclusions

- Semi-infinite segments of radiation show islands within regime of applicability for small ϕ_0. For large ϕ_0, the island transition gets moved to times beyond the regime of applicability of the model.
- Finite segment Page curves are highly sensitive to the segment location and other parameters. The entanglement wedge of the segments can evolve through one of 3 scenarios. The size of the accessible potion of the black hole interior can be non-monotonic.
- For late times, the interior access is lost for finite segments, and the entanglement entropy relaxes back to the vacuum value.

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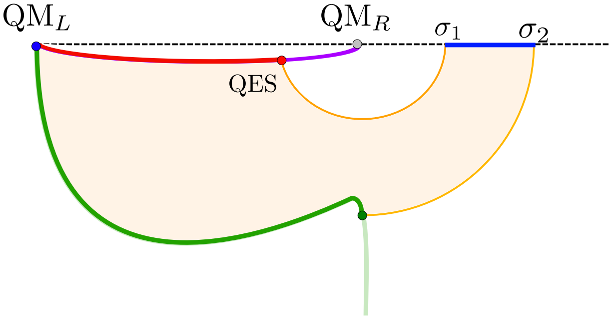
Thank you for attention!

Complexity = volume for radiation subregions



Typical volume which computes complexity of the bath subregion $[\sigma_1, \sigma_2] \cup \mathrm{QM}_L$ in a reconstructing phase.

Subregion complexity for the radiation segments