Lurking in the shadows of entanglement

Mathematical Methods of Quantum Technologies
Steklov Mathematical Institute, RAS

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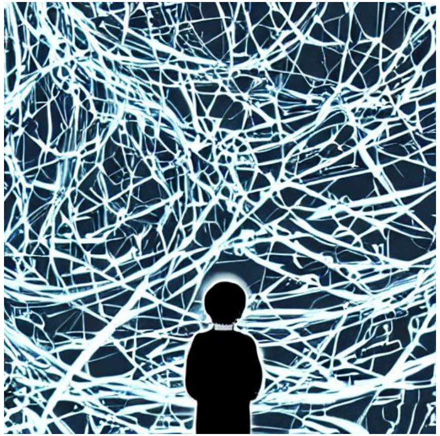
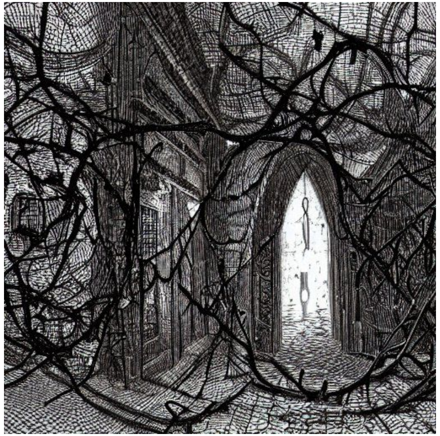
Shaping contours of entanglement islands in BCFT (arXiv:2107.09083) + new things

New Trends in Mathematical Physics
11 November 2022

Versions of how neural network “Stable diffusion” thinks my talk will look like (by title)

Our general motivation – how a more fine-grained entanglement measure can reveal hidden features in quantum systems affected by entanglement island effect? Can we find out some special protected states (corresponding to encoded radiation)?

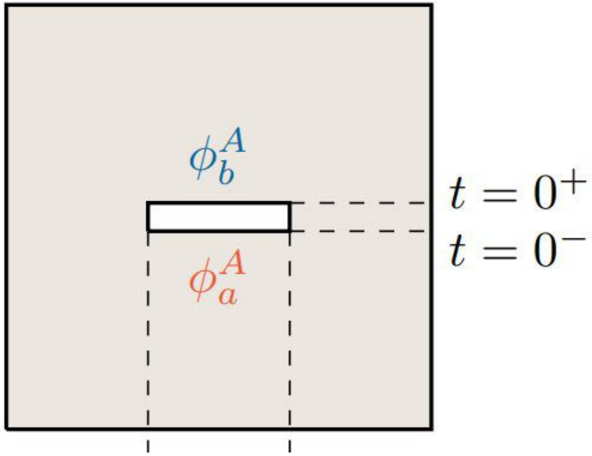
Entanglement entropy in 2d CFT

Related to two-point correlation function (which is simple in 2d CFT)

$$S_A = \frac{c}{3}\log\frac{l}{a}$$

$$S_A^{c.c.} = \frac{c}{3}\cdot\log\left(\frac{L}{\pi a}\sin\left(\frac{\pi l}{L}\right)\right)$$

$$S_A^{f.t.} = \frac{c}{3}\cdot\log\left(\frac{\beta}{\pi a}\sinh\left(\frac{\pi l}{\beta}\right)\right)$$



Generalizations

Effectively entanglement entropy in 2d CFT is related to two-point correlation function. We need more fine-grained measures to see hidden features in quantum systems. First way – more higher-point functions (i.e. negativity, reflected entropy etc.). Another way (one of possible) – spatially/momentum/charge resolved quantities.

Entanglement contour and conditional (partial) entropy

$$S(A) = \int_{x \in A} f_A(x) dx,$$

$$f_A(x) \geq 0.$$

The entanglement contour $f_A(x)$ inherits symmetries of the reduced density matrix ρ_A.

The entanglement contour is invariant under local unitary transformations.

$$s_A(A_2) = \frac{1}{2}\Big(S(A_1 \cup A_2) + S(A_2 \cup A_3) - S(A_1) - S(A_3)\Big)$$

$$A = A_1 \cup A_2 \cup A_3$$

Entanglement contour

Chen, Vidal (2014)

Wen(2018)

$$S(A) = \int_{x\in A} f_A(x)dx, \qquad f_A(x) \geq 0$$

$$S(x_1, x_2) = \frac{c}{3}\log\left(\frac{x_2 - x_1}{\varepsilon}\right), \quad f_A(x) = \frac{c\,(x_2 - x_1)}{6\,(x - x_1)\,(x_2 - x)}$$

$$S(x_1, x_2) = \frac{c}{3}\log\left(\frac{\sinh(\pi T(x_2 - x_1))}{\pi T\varepsilon}\right),$$

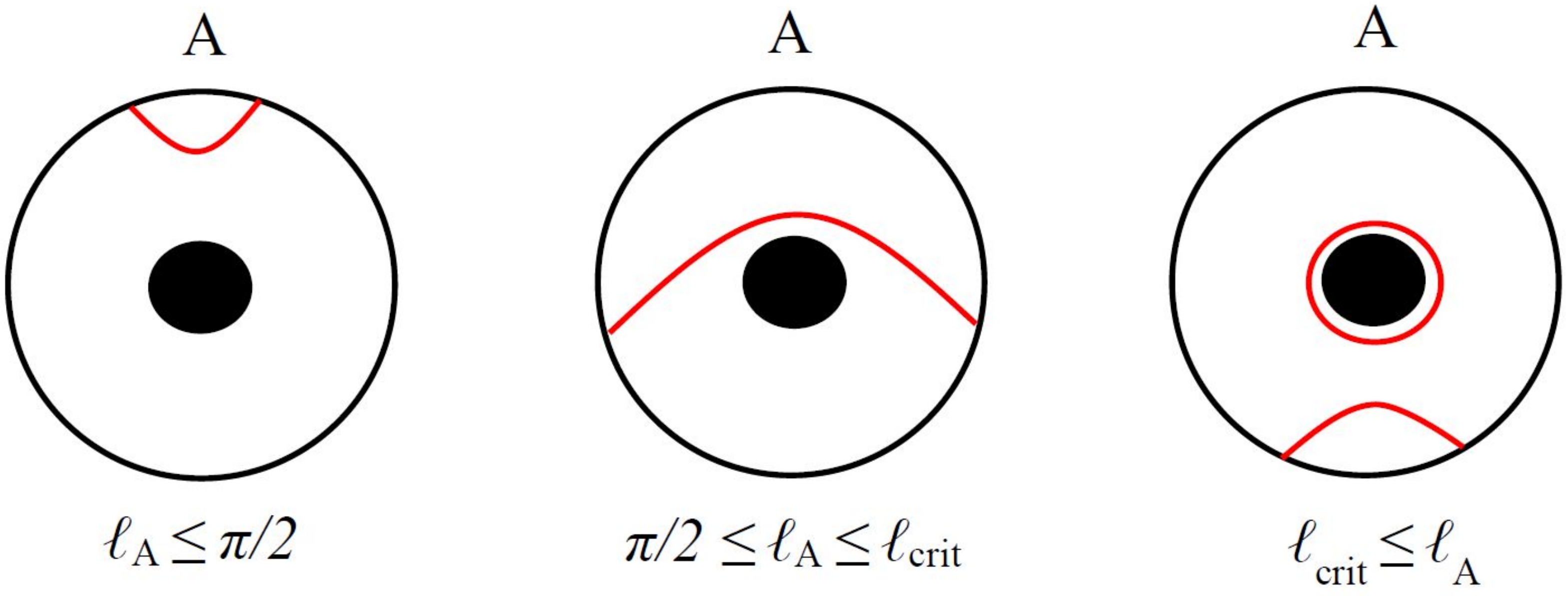
$$f_A(x) = \frac{\pi c T}{6}\left(\coth\left(\pi T\left(x - x_1\right)\right) + \coth\left(\pi T\left(x_2 - x\right)\right)\right)$$

Prototypical island-like phenomena

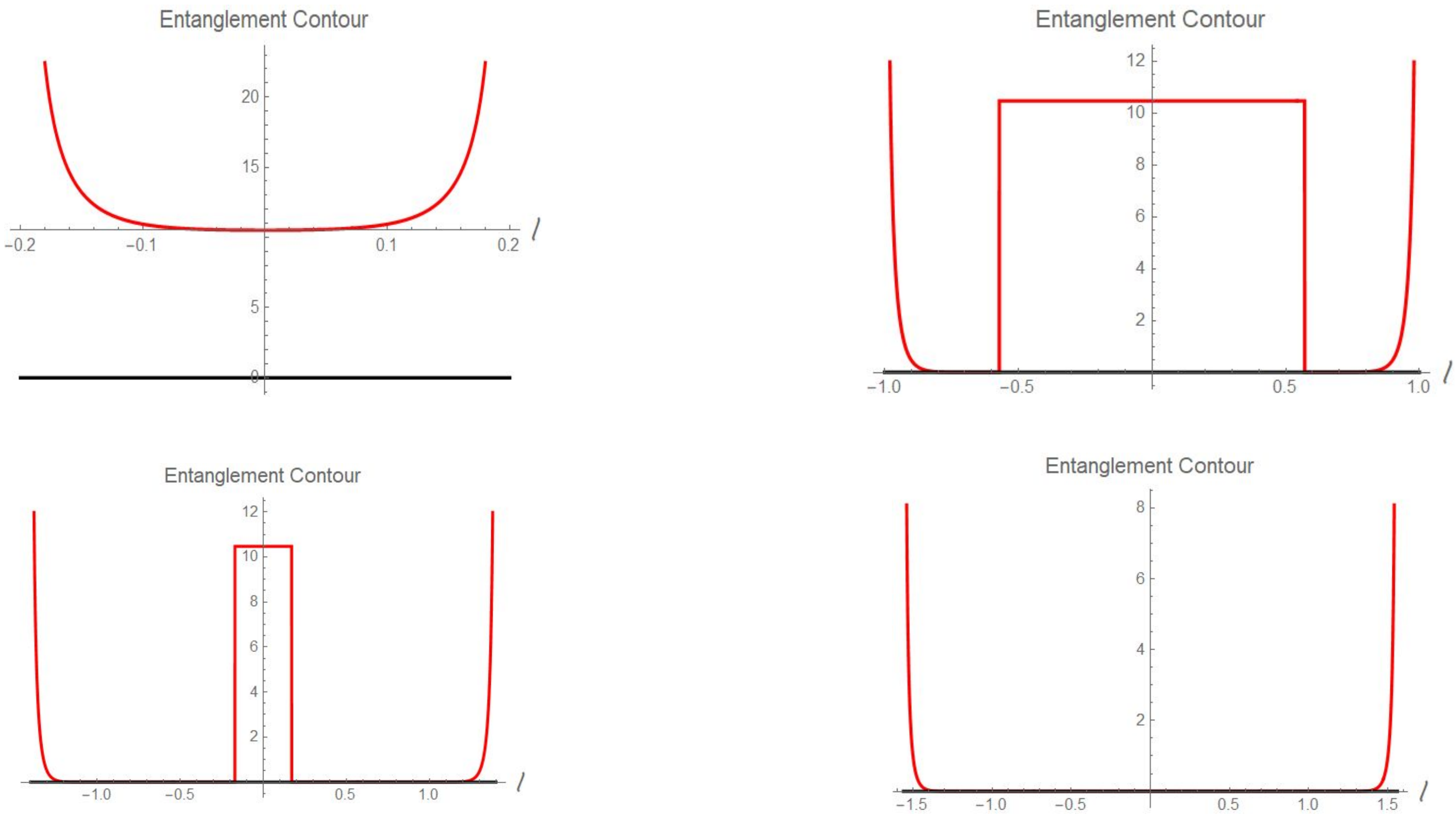
Naivley: Some “bath” + some microstates system = Black hole

Black hole microstates: entanglement plateaux

Hubeny, Maxfield,Rangamani' 13; Bao, Ooguri' 17



Different phase transitions



Why entanglement contour is useful?

More fine-grained quantity which captures tricky and fine-grained features which could be missed by entanglement.

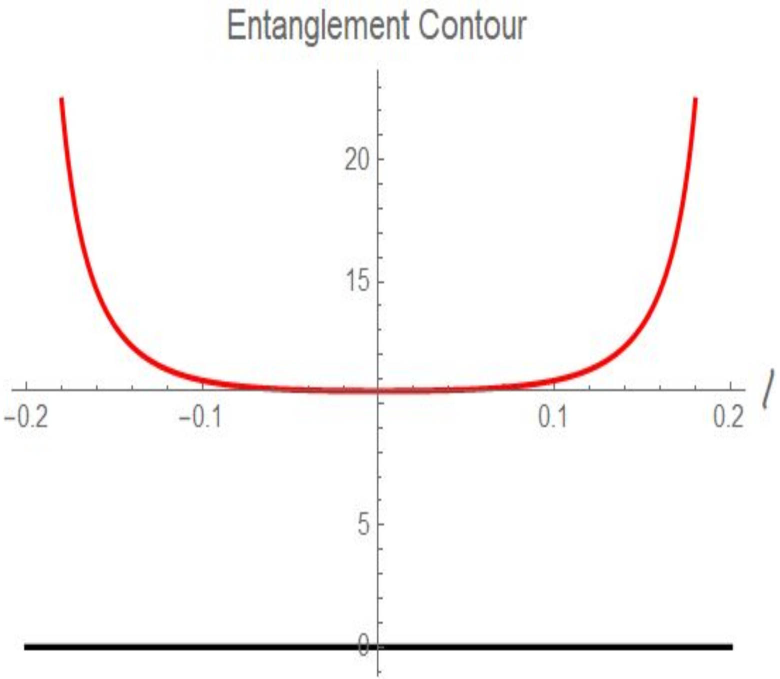
Add knowledge about quantum error correction (important in black holes microstates and information paradox).

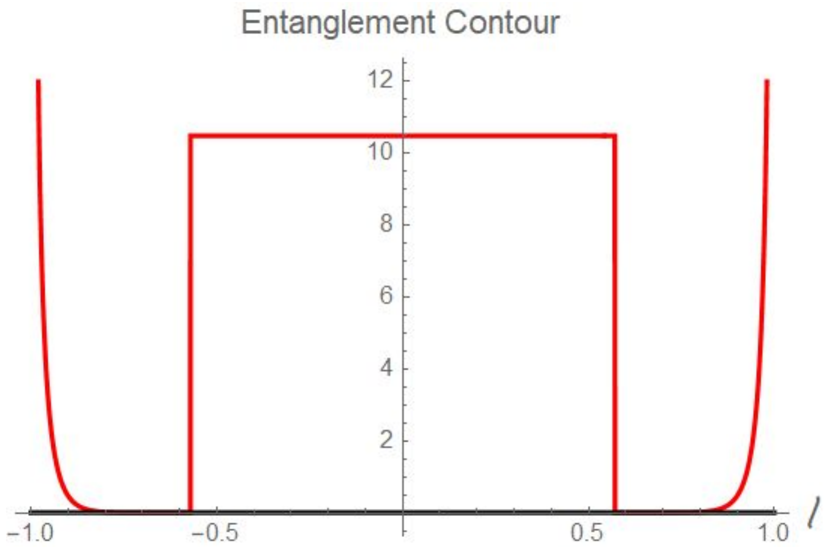
O. Fawzi and R. Renner, “Quantum conditional mutual information and approximate markov chains”, arXiv:1410.0664

The smaller entanglement contour (partial mutual information) = exists recovery map

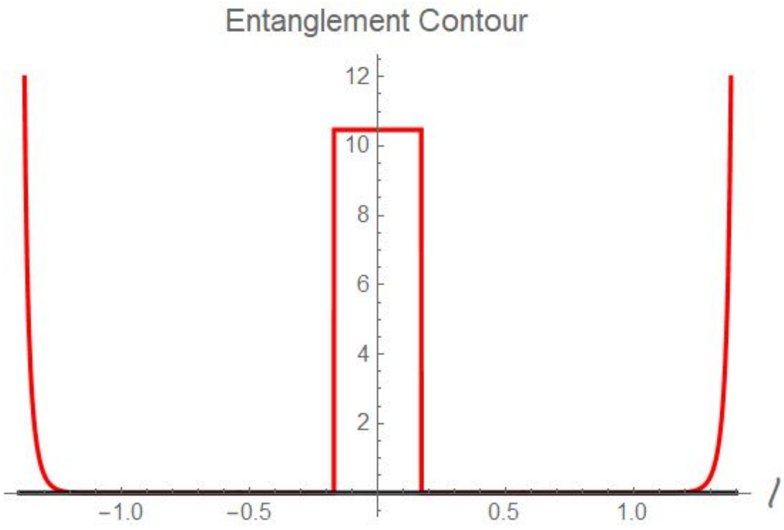
with better fidelity (state dependent). Vanishing = perfect recovery

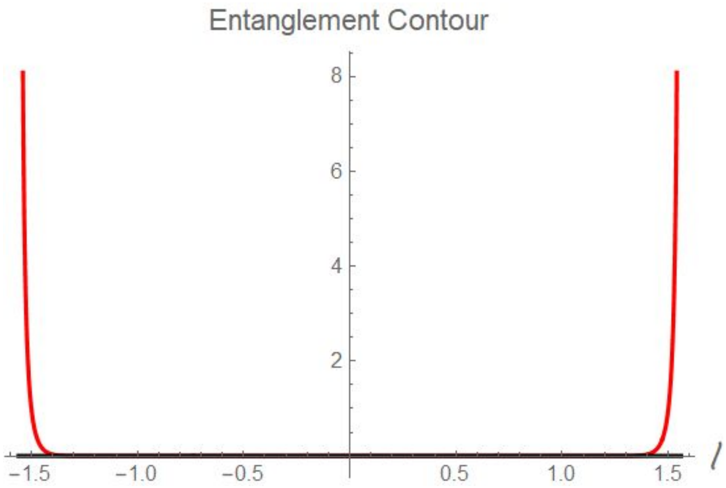
Small subsystems in BH are harder to recover





Large subsystems in BH are simpler to recover





Page Curve and Islands?

Perfect recover (shadow of entanglement)?

Entanglement entropy in BCFT

One-point correlator in CFT = 0 (vac)

One-point correlator in BCFT is not zero

Entanglement in 2d CFT = two-point function -easy

Entanglement in 2d BCFT = four point function - complicated!

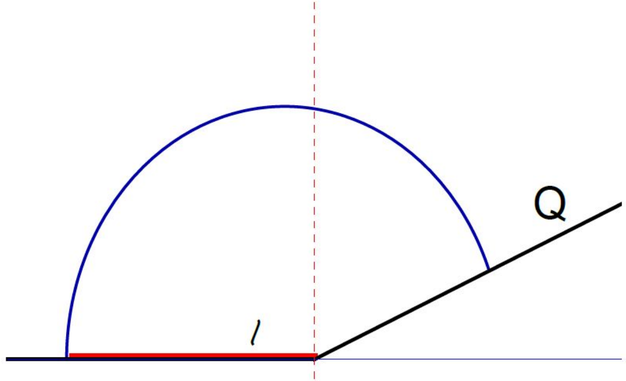
Gravity helps!

BCFT dual = CFT dual + End-of-the-world Brane

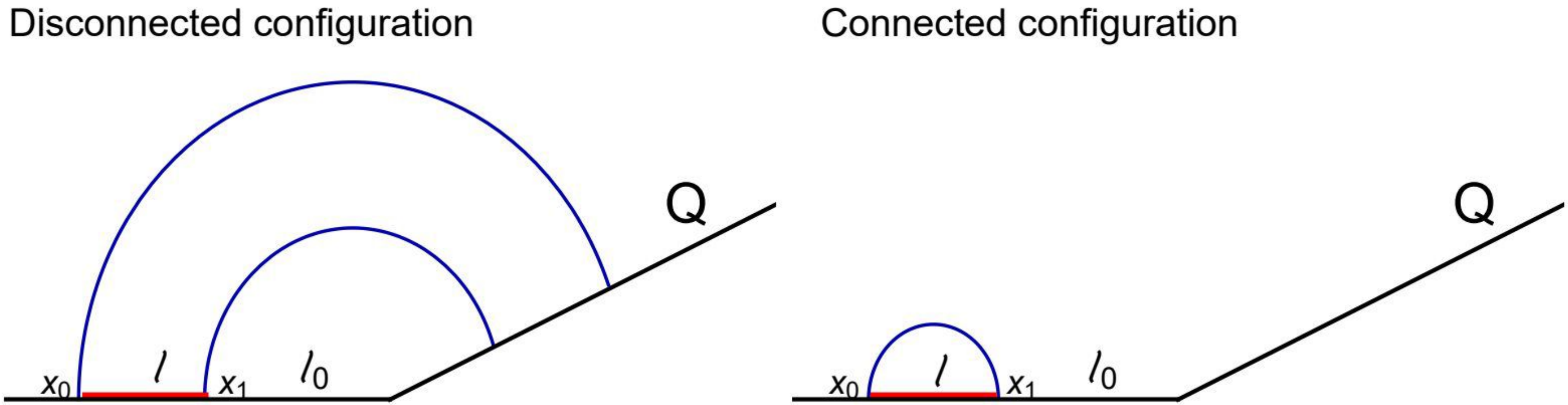
$$I = \frac{1}{16\pi G_N}\int_N \sqrt{-g}(R-2\Lambda) + \frac{1}{8\pi G_N}\int_Q \sqrt{-h}(K-T_{\text{br}})$$

Takayanagi' 11

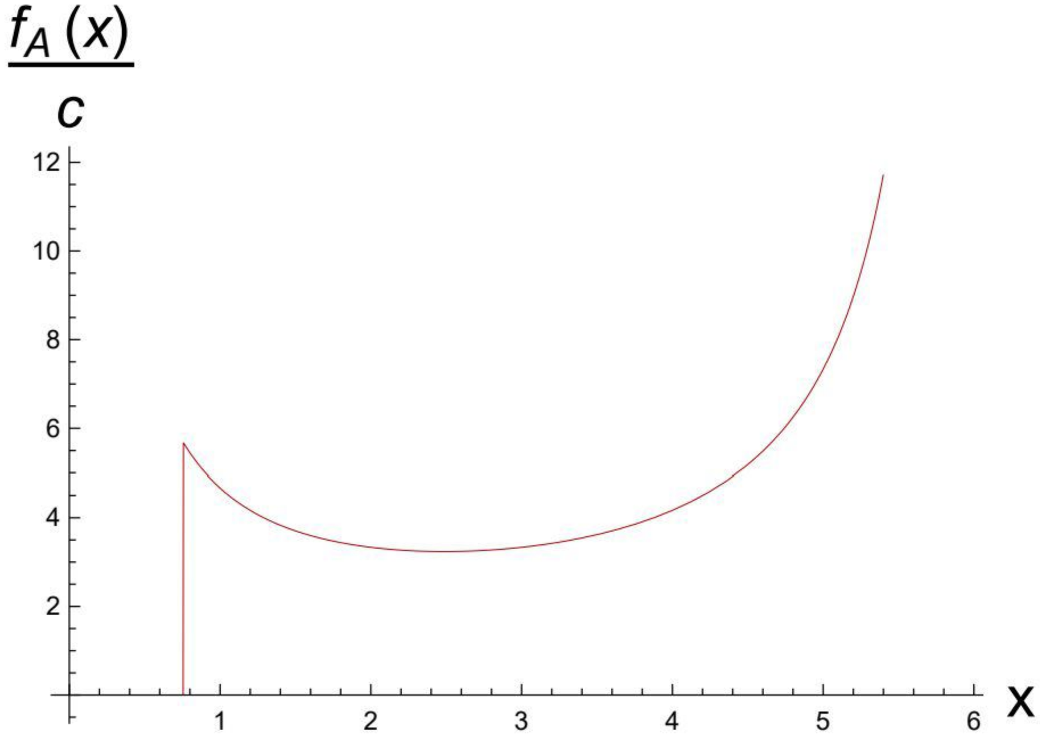
$$S(\ell) = \frac{c}{6}\log\left(\frac{2\ell}{\epsilon}\right) + \log g_b$$



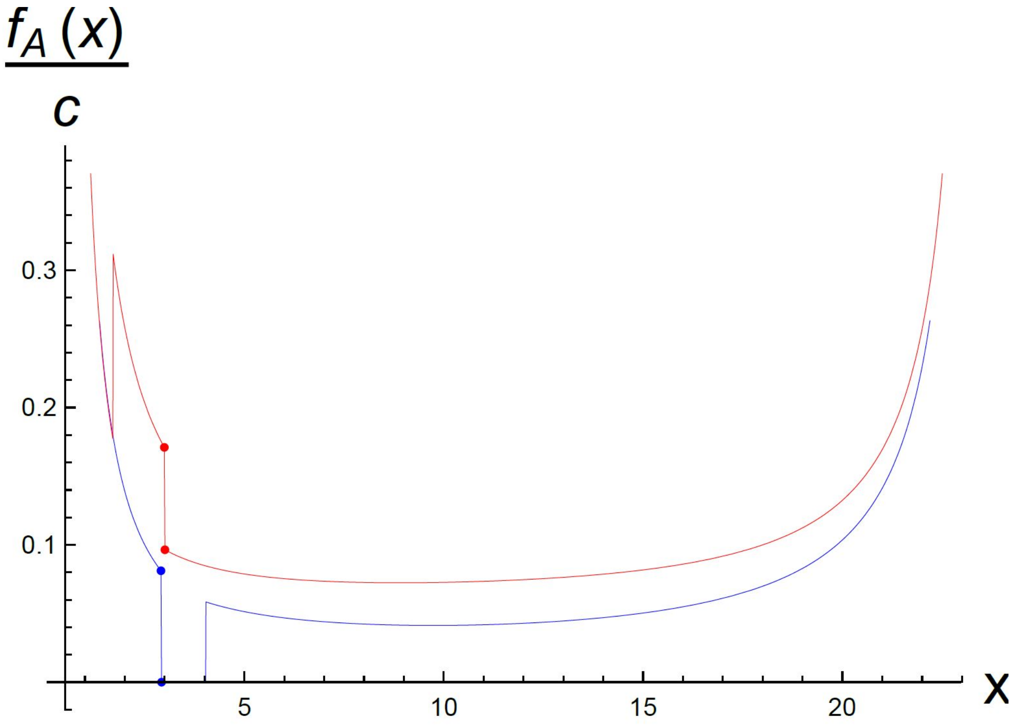
Dual space and geodesics with different topologies



Shadow in entanglement contour

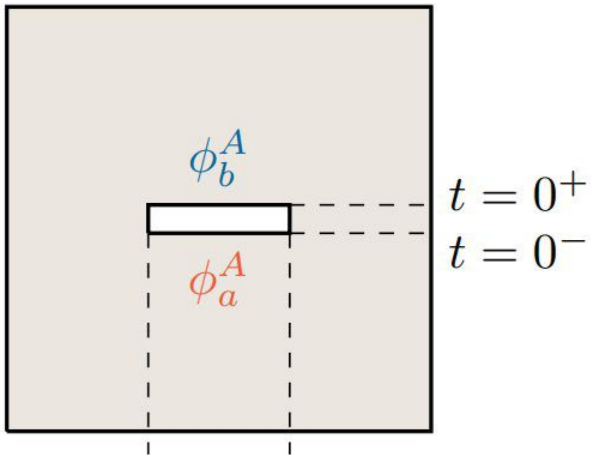


Holes in the entanglement

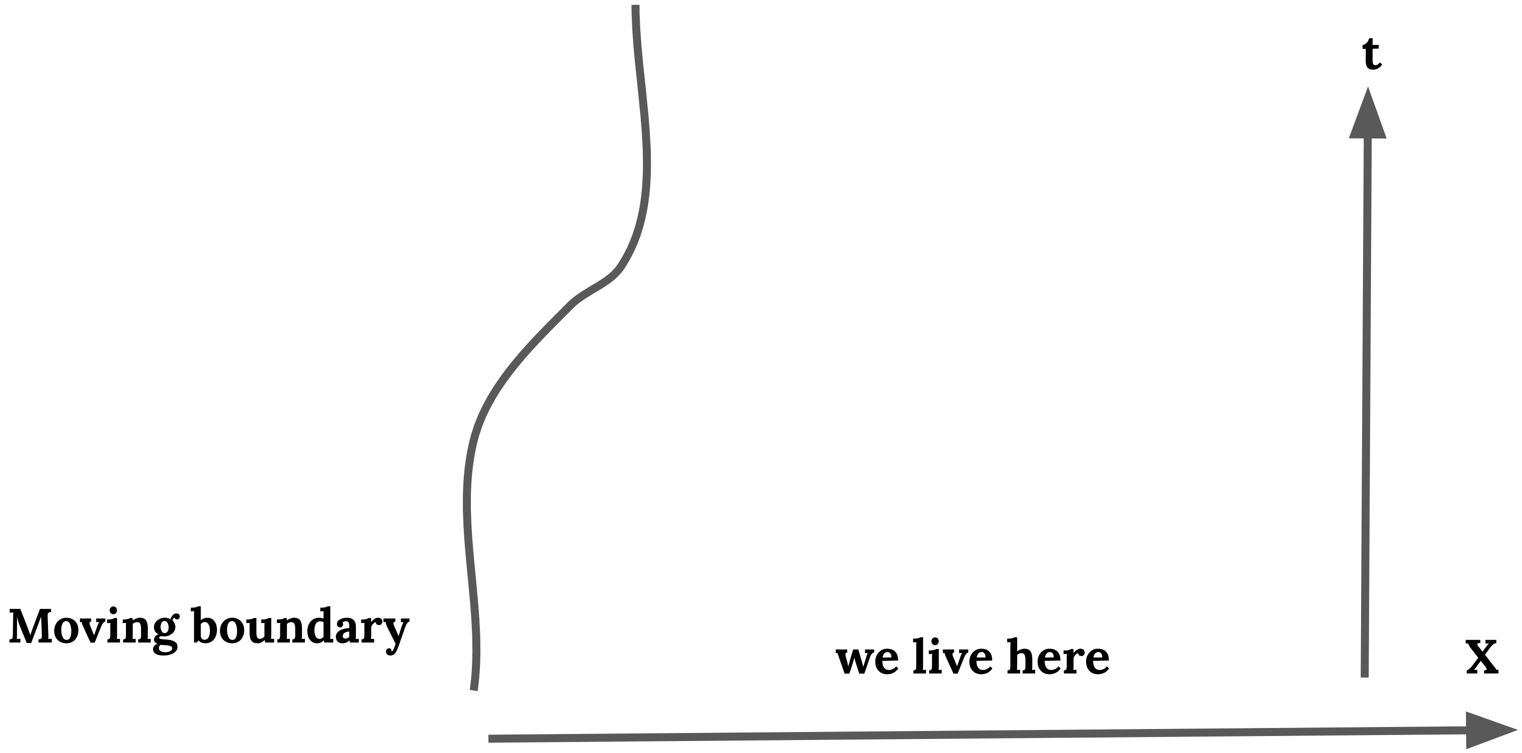


Entanglement islands: the same discontinous behaviour

Gravity +



Moving mirror



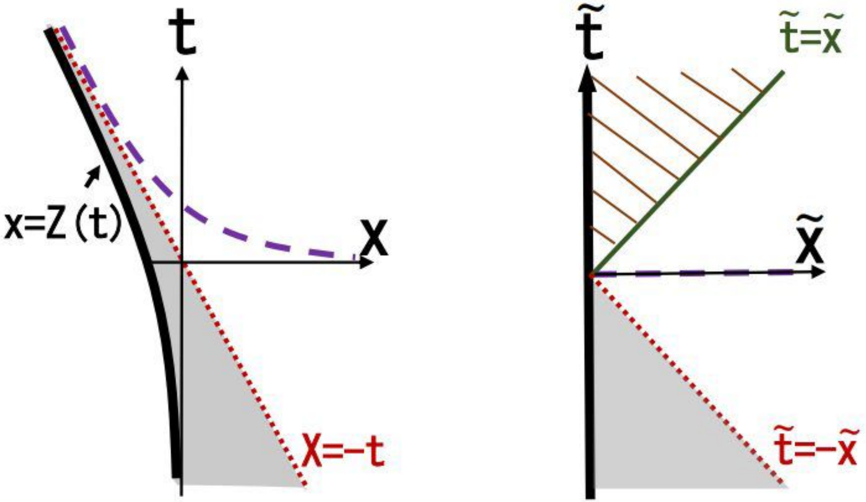
Conformal mapping to static boundary

From 2011.12005

$$p(u) = -\beta \log(1 + e^{-\frac{u}{\beta}}),$$

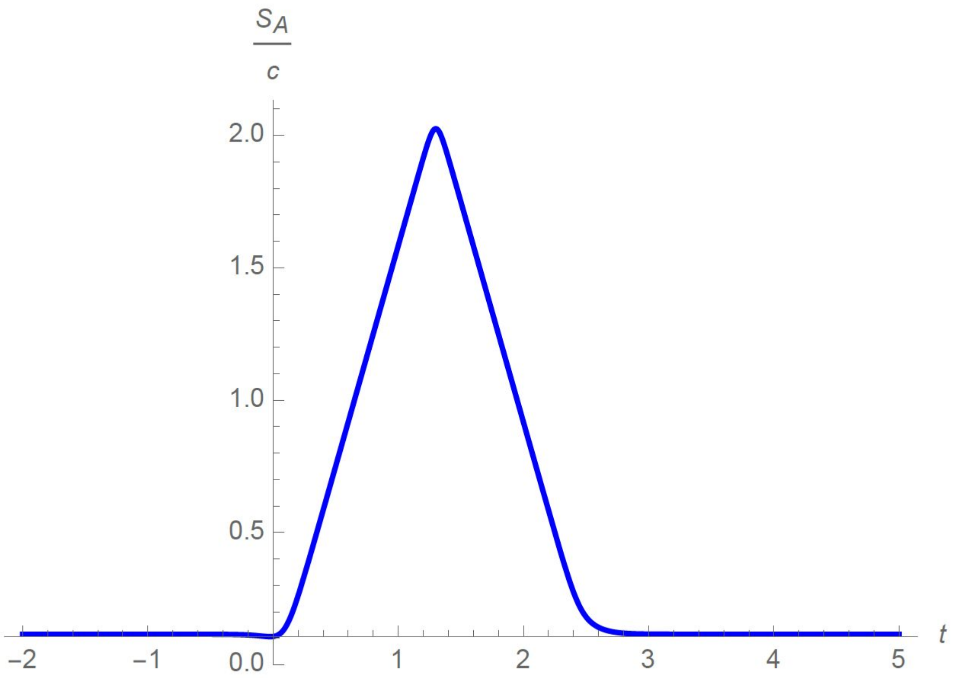
$$T_{uu} = \frac{c}{48\pi\beta^2}\left(1 - \frac{1}{(1+e^{u/\beta})^2}\right)$$

$$S_A = \frac{c}{6}\log\frac{t + x_0 - p(t - x_0)}{\epsilon\sqrt{p'(t - x_0)}} + S_{\text{bdy}}.$$

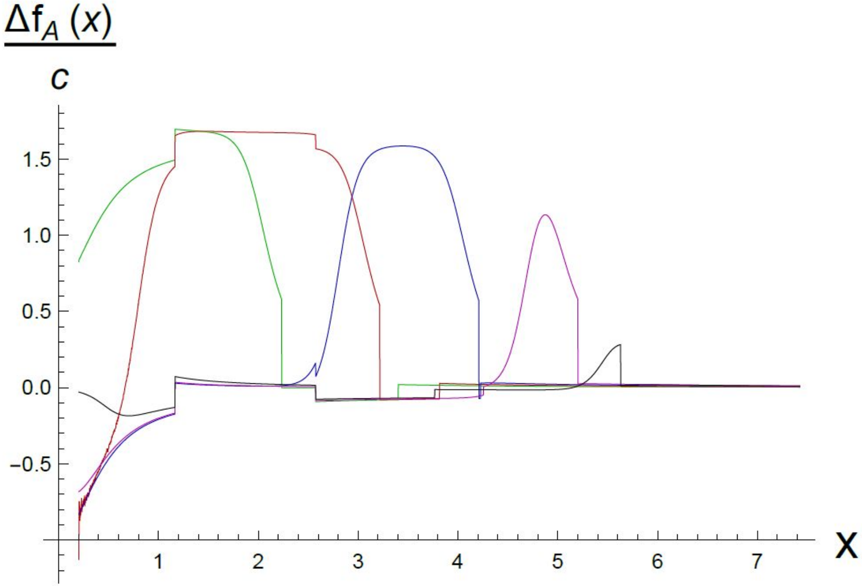


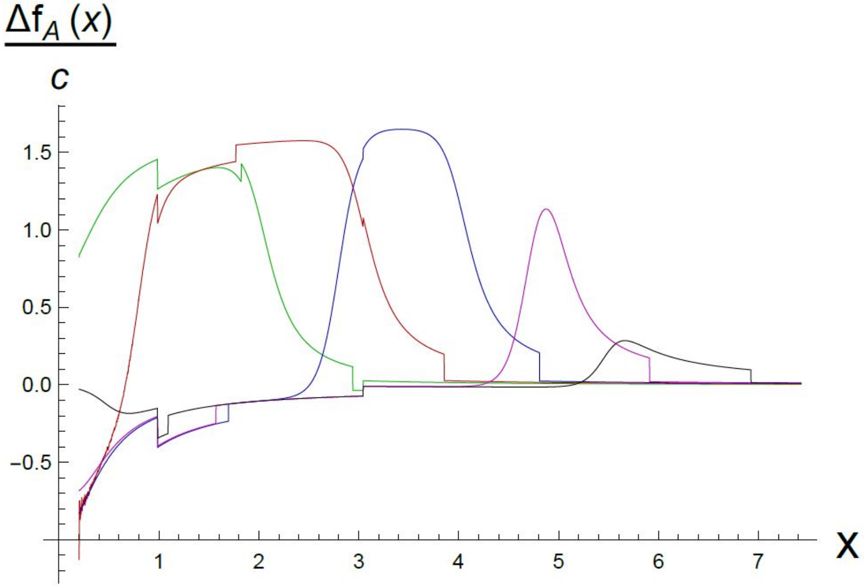
Page curve:holy grail of recent two years

From 2011.12005



Decomposing in entanglement contour Page curve





$$x \in (0.2, 15)$$

Conclusion

Entanglement contour reveals different hidden features

It can reveals where the states are protected and hard to extract information

In systems with boundary there exists effect of “entanglement shadow” (irrespectively of quantum information and black holes)